MATH 16B

SOLUTIONS - SEC. 6.6

⑦ $\displaystyle\int_5^\infty \frac{x}{\sqrt{x^2-16}}\,dx = \lim_{T\to\infty}\int_5^T \frac{x}{\sqrt{x^2-16}}\,dx$ LET $\underline{u = x^2-16}$, $du = 2x\,dx$

$$= \lim_{T\to\infty}\frac{1}{\enclose{circle}{2}}\int_5^T \frac{1}{\sqrt{x^2-16}}\cdot\enclose{circle}{2}x\,dx = \lim_{T\to\infty}\frac{1}{2}\int_{x=5}^{x=T}\frac{1}{\sqrt{u}}\,du = \lim_{T\to\infty}\frac{1}{2}\int_{x=5}^{x=T}u^{-1/2}\,du = \lim_{T\to\infty}\frac{1}{\cancel{2}}\Big[\cancel{2}u^{1/2}\Big]_{x=5}^{x=T}$$

$= \displaystyle\lim_{T\to\infty}\Big[\sqrt{x^2-16}\Big]_5^T = \lim_{T\to\infty}\left(\sqrt{T^2-16}-3\right) = \infty$, so THE INTEGRAL $\boxed{\text{DIVERGES}}$

⑬ $\displaystyle\underline{\int_{-\infty}^{\infty} 2x e^{-3x^2}dx = \int_{-\infty}^{0} 2x e^{-3x^2}dx + \int_0^\infty 2x e^{-3x^2}dx}$

1) $\displaystyle\int_0^\infty 2x e^{-3x^2}dx = \lim_{T\to\infty}\int_0^T 2x e^{-3x^2}dx$ $\underline{u=-3x^2}$, $du = -6x\,dx$

$$= \lim_{T\to\infty}\frac{2}{\enclose{circle}{-6}}\int_0^T e^{-3x^2}(\enclose{circle}{-6})x\,dx = \lim_{T\to\infty}-\frac{1}{3}\Big[e^{-3x^2}\Big]_0^T = \lim_{T\to\infty}-\frac{1}{3}\left(e^{-3T^2}-1\right)$$

$$= \lim_{T\to\infty}-\frac{1}{3}\left(\frac{1}{e^{3T^2}}-1\right) = -\frac{1}{3}(0-1) = \underline{\frac{1}{3}}$$

2) $\displaystyle\int_{-\infty}^{0} 2x e^{-3x^2}dx = \lim_{T\to-\infty}\int_T^0 2x e^{-3x^2}dx$

$$= \lim_{T\to-\infty}-\frac{1}{3}\Big[e^{-3x^2}\Big]_T^0 = \lim_{T\to-\infty}-\frac{1}{3}\left(1-e^{-3T^2}\right) = \lim_{T\to-\infty}-\frac{1}{3}\left(1-\frac{1}{e^{3T^2}}\right) = -\frac{1}{3}(1-0)$$

$$= \underline{-\frac{1}{3}}$$

THEREFORE $\displaystyle\int_{-\infty}^{\infty} 2x e^{-3x^2}dx = \left(-\frac{1}{3}\right)+\frac{1}{3} = \boxed{0}$

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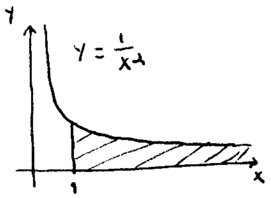


a) $\displaystyle\underline{A = \int_1^\infty \frac{1}{x^2}\,dx} = \lim_{T\to\infty}\int_1^T x^{-2}\,dx = \lim_{T\to\infty}\Big[-\frac{1}{x}\Big]_1^T$

$$= \lim_{T\to\infty}\left(-\frac{1}{T}-(-1)\right) = 0+1 = \boxed{1}$$

b) (REVOLVE AROUND X-AXIS):

$$\underline{V = \int_1^\infty \pi\left(\frac{1}{x^2}\right)^2 dx} = \lim_{T\to\infty}\int_1^T \pi\left(\frac{1}{x^4}\right)dx = \lim_{T\to\infty}\pi\int_1^T x^{-4}\,dx = \lim_{T\to\infty}\pi\Big[-\frac{1}{3}x^{-3}\Big]_1^T$$

$$= \lim_{T\to\infty}-\frac{\pi}{3}\left(T^{-3}-1\right) = \lim_{T\to\infty}-\frac{\pi}{3}\left(\frac{1}{T^3}-1\right) = -\frac{\pi}{3}(0-1) = \boxed{\frac{\pi}{3}}$$

㊴ a) $\displaystyle PV = \int_0^{20} 500{,}000\,e^{-.09T}\,dT = 500{,}000\Big[\frac{1}{-.09}e^{-.09T}\Big]_0^{20} = \frac{500{,}000}{-.09}\left(e^{-1.8}-1\right)$

$$= \boxed{\$\frac{50{,}000{,}000}{9}\left(1-e^{-1.8}\right)} \approx \underline{\$4{,}637{,}228}$$

b) $\displaystyle PV = \int_0^\infty 500{,}000\,e^{-.09T}\,dT = \lim_{b\to\infty}\int_0^b 500{,}000\,e^{-.09T}\,dT$

$$= \lim_{b\to\infty}500{,}000\Big[\frac{1}{-.09}e^{-.09T}\Big]_0^b = \lim_{b\to\infty}\frac{500{,}000}{-.09}\left(e^{-.09b}-1\right)$$

$$= \lim_{b\to\infty}\frac{50{,}000{,}000}{9}\left(1-\frac{1}{e^{.09b}}\right) = \frac{50{,}000{,}000}{9}(1-0) = \boxed{\$\frac{50{,}000{,}000}{9}}$$

$$\approx \underline{\$5{,}555{,}556}$$

MATH 16B SOLUTIONS - PINK SHEET (CH.6 AND SEC. 8.5)

① $$\int \frac{x^3}{x^2-x-6}\,dx = \int \left(x+1+\frac{7x+6}{x^2-x-6}\right)dx \longleftarrow \quad \begin{array}{r|l} & x+1 \\ \hline x^2-x-6 & x^3 \\ & \underline{x^3-x^2-6x} \\ & x^2+6x \\ & \underline{x^2-x-6} \\ & 7x+6 \end{array}$$

$$= \frac{x^2}{2} + x + \int \frac{7x+6}{x^2-x-6}\,dx$$

$$= \frac{x^2}{2} + x + \int \left(\frac{27/5}{x-3} + \frac{8/5}{x+2}\right)dx \longleftarrow \quad \frac{7x+6}{x^2-x-6} = \frac{7x+6}{(x-3)(x+2)} = \frac{A}{x-3} + \frac{B}{x+2}$$

$$7x+6 = A(x+2) + B(x-3)$$

$\underline{x=3}$: $27 = 5A$ $\underline{A = 27/5}$

$\underline{x=-2}$: $-8 = -5B$ $\underline{B = 8/5}$

$$= \boxed{\frac{x^2}{2} + x + \frac{27}{5}\ln|x-3| + \frac{8}{5}\ln|x+2| + C}$$

② $$\int \underline{\underline{x^3}} \cos x^2\,dx \qquad \text{LET } \underline{t = x^2},\quad dt = 2x\,dx$$

$$= \frac{1}{②}\int \underline{\underline{x^2}} \cos x^2 \cdot ② \underline{\underline{x}}\,dx = \frac{1}{2}\int t\cos t\,dt \qquad \text{LET } \frac{u = t,\quad dv = \cos t\,dt}{du = dt,\quad v = \sin t}$$

$$= \frac{1}{2}\left[t\sin t - \int \sin t\,dt\right] = \frac{1}{2}\left[t\sin t - (-\cos t)\right] + C = \boxed{\frac{1}{2}\left[x^2\sin x^2 + \cos x^2\right] + C}$$

③ $$\int_0^\pi \sin^3\theta\cos^6\theta\,d\theta = \int_0^\pi \underbrace{\sin^2\theta}_{} \cos^6\theta\sin\theta\,d\theta = \int_0^\pi \underline{(1-\cos^2\theta)}\cos^6\theta\sin\theta\,d\theta$$

LET $\underline{u = \cos\theta}$

$du = -\sin\theta\,d\theta$

IF $\theta = 0$, $u = \cos 0 = 1$

$\theta = \pi$, $u = \cos\pi = -1$

$$= -\int_1^{-1}(1-u^2)u^6\,du = \int_{-1}^{1}(\overset{\text{(even)}\downarrow}{u^6 - u^8})\,du = 2\int_0^1 (u^6 - u^8)\,du$$

$$= 2\left[\frac{u^7}{7} - \frac{u^9}{9}\right]_0^1 = 2\left(\frac{1}{7} - \frac{1}{9} - 0\right) = 2\left(\frac{2}{63}\right) = \boxed{\frac{4}{63}}$$

④ $$\int \frac{\ln x}{(x-5)^2}\,dx \qquad \text{LET } \frac{u = \ln x,\quad dv = \frac{1}{(x-5)^2}\,dx}{du = \frac{1}{x}\,dx,\quad v = -\frac{1}{x-5}}$$

$$v = \int (x-5)^{-2}\,dx = \int t^{-2}\,dt \qquad \frac{t = x-5}{dt = dx}$$

$$= -t^{-1} + C = -\frac{1}{t} + C = -\frac{1}{x-5} + C$$

$$= -\frac{\ln x}{x-5} - \int -\frac{1}{x(x-5)}\,dx = -\frac{\ln x}{x+5} + \int \frac{1}{x(x-5)}\,dx \longleftarrow \quad \frac{1}{x(x-5)} = \frac{A}{x} + \frac{B}{x-5}$$

$$1 = A(x-5) + Bx$$

$\underline{x=0}$: $1 = -5A$ $\underline{A = -1/5}$

$\underline{x=5}$: $1 = 5B$ $\underline{B = 1/5}$

$$= -\frac{\ln x}{x-5} + \int\left(\frac{-1/5}{x} + \frac{1/5}{x-5}\right)dx$$

$$= \boxed{-\frac{\ln x}{x-5} - \frac{1}{5}\ln x + \frac{1}{5}\ln|x-5| + C} = -\frac{\ln x}{x-5} + \frac{1}{5}\left(\ln|x-5| - \ln x\right) + C = \boxed{-\frac{\ln x}{x-5} + \frac{1}{5}\ln\frac{|x-5|}{x} + C}$$

⑤ $$\int_0^9 e^{\sqrt{x}}\,dx \qquad \text{LET } \underline{u = \sqrt{x}},\quad x = u^2,\quad \underline{dx = 2u\,du}$$

IF $x = 0$, $u = 0$

$x = 9$, $u = 3$

$$= \int_0^3 e^u \cdot 2u\,du = 2\int_0^3 ue^u\,du \qquad \text{LET } \frac{u = u,\quad dv = e^u\,du}{du = du,\quad v = e^u}$$

$$= 2\left[ue^u - \int e^u\,du\right]_0^3 = 2\left[ue^u - e^u\right]_0^3 = 2\left((3e^3 - e^3) - (0-1)\right) = \boxed{2\left(2e^3+1\right)} = \boxed{4e^3+2}$$

⑥ $$\int \underline{\underline{x^5}} e^{2x^2}\,dx \qquad \text{LET } \underline{t = x^2},\quad dt = 2x\,dx$$

$$= \frac{1}{②}\int \underline{\underline{x^4}} e^{2x^2} \cdot ② \underline{\underline{x}}\,dx = \frac{1}{2}\int t^2 e^{2t}\,dt = \frac{1}{2}\left[\frac{1}{2}t^2e^{2t} - \frac{1}{2}te^{2t} + \frac{1}{4}e^{2t}\right] + C$$

$\underline{u}$		$\underline{dv}$
t^2	⊕	$e^{2t}\,dt$
$2t$	⊖	$\frac{1}{2}e^{2t}$
2	⊕	$\frac{1}{4}e^{2t}$
0		$\frac{1}{8}e^{2t}$

$$= \boxed{\frac{1}{4}x^4e^{2x^2} - \frac{1}{4}x^2e^{2x^2} + \frac{1}{8}e^{2x^2} + C}$$

⑦ $$\int \frac{\sqrt{x}}{x-16}\,dx \qquad \text{LET } \underline{u=\sqrt{x}},\quad x=u^2,\quad dx=2u\,du$$

$$= \int \frac{u}{u^2-16}\cdot 2u\,du = 2\int \frac{u^2}{u^2-16}\,du = 2\int\left(1+\frac{16}{u^2-16}\right)du \longleftarrow \begin{array}{r|l} & 1 \\ \hline u^2-16 & u^2 \\ & \underline{u^2-16} \\ & 16 \end{array}$$

$$= 2\left[u + \int \frac{16}{u^2-16}\,du\right] = 2\left[u+\int\left(\frac{2}{u-4}-\frac{2}{u+4}\right)du\right] \longleftarrow \frac{16}{(u-4)(u+4)} = \frac{A}{u-4}+\frac{B}{u+4}$$

$$16 = A(u+4)+B(u-4)$$

$\underline{u=4}$: $16 = 8A$ $\quad \underline{A=2}$

$\underline{u=-4}$: $16=-8B$ $\quad \underline{B=-2}$

$$= 2\left[u + 2\ln|u-4| - 2\ln|u+4|\right] + C$$

$$= \boxed{2\sqrt{x} + 4\ln|\sqrt{x}-4| - 4\ln(\sqrt{x}+4) + C}$$

$$= 2\sqrt{x} + 4\left(\ln|\sqrt{x}-4| - \ln(\sqrt{x}+4)\right) + C = \boxed{2\sqrt{x} + 4\ln\left|\frac{\sqrt{x}-4}{\sqrt{x}+4}\right| + C}$$

⑧ $$\int \frac{1}{e^{2x}+8e^x}\,dx = \int \frac{1}{e^x(e^x+8)}\,dx \qquad \text{LET } \underline{u=e^x},\ du=e^x\,dx \longleftarrow \left(\underline{\text{OR}} \text{ USE } x=\ln u,\ dx=\tfrac{1}{u}\,du\right)$$

$$= \int \frac{1}{e^x(e^x+8)}\cdot\frac{\enclose{circle}{e^x\,dx}}{e^x} = \int \frac{1}{u(u+8)u}\,du$$

$$= \int \frac{1}{u^2(u+8)}\,du \qquad \frac{1}{u^2(u+8)} = \frac{A}{u}+\frac{B}{u^2}+\frac{C}{u+8}$$

$$1 = Au(u+8)+B(u+8)+Cu^2$$

$\underline{u=0}$: $1 = 8B$ $\quad B=1/8$

$\underline{u=-8}$: $1=64C$ $\quad C = 1/64$

$\underline{\text{COEFF. OF } u^2}$: $0 = A + C$ $\quad \underline{A=-1/64}$

$$\Big\downarrow$$

$$= \int\left(\frac{-1/64}{u}+\frac{1/8}{u^2}+\frac{1/64}{u+8}\right)du$$

$$= -\tfrac{1}{64}\ln|u| - \tfrac{1}{8}u^{-1} + \tfrac{1}{64}\ln|u+8| + C = -\tfrac{1}{64}\ln e^x - \tfrac{1}{8}(e^x)^{-1} + \tfrac{1}{64}\ln(e^x+8)+C$$

$$= \boxed{-\tfrac{1}{64}x - \tfrac{1}{8}e^{-x} + \tfrac{1}{64}\ln(e^x+8)+C}$$

⑨ $$\int \boxed{e^{6x}}\sin(e^{3x})\,dx \qquad \text{LET } \underline{T=e^{3x}},\quad dT = e^{3x}\cdot 3\,dx$$

$$= \enclose{circle}{\tfrac{1}{3}}\int \boxed{e^{3x}}\sin(e^{3x})\cdot\enclose{circle}{3}\boxed{e^{3x}}\,dx$$

$$= \tfrac{1}{3}\int T\sin T\,dT \qquad \text{LET } \underline{u=T,\ dV=\sin T\,dT} \qquad du=dT \quad V=-\cos T$$

$$= \tfrac{1}{3}\left[-T\cos T - \int -\cos T\,dT\right]$$

$$= \tfrac{1}{3}\left[-T\cos T + \sin T\right] + C = \boxed{\tfrac{1}{3}\left[-e^{3x}\cos(e^{3x}) + \sin(e^{3x})\right]+C}$$

⑩ $$\int \sec\theta\, d\theta = \int \frac{1}{\cos\theta}\, d\theta = \int \frac{\cos\theta}{\cos^2\theta}\, d\theta = \underline{\int \frac{\cos\theta}{1-\sin^2\theta}\, d\theta}$$

LET $\underline{u = \sin\theta}$
$du = \cos\theta\, d\theta$

$$= \int \frac{1}{1-\sin^2\theta} \cdot \cos\theta\, d\theta = \underline{\int \frac{1}{1-u^2}\, du} = \int \left(\frac{1/2}{1+u} + \frac{1/2}{1-u} \right) du \longleftarrow \frac{1}{1-u^2} = \frac{1}{(1+u)(1-u)} = \frac{A}{1+u} + \frac{B}{1-u}$$

$1 = A(1-u) + B(1+u)$

$\underline{u=1}$: $1 = 2B$ $\underline{B = 1/2}$

$\underline{u=-1}$: $1 = 2A$ $\underline{A = 1/2}$

$$= \frac{1}{2}\int \left(\frac{1}{1+u} \ominus \frac{\textcircled{-1}}{1-u} \right) du = \frac{1}{2}\Big[\ln|1+u| - \ln|1-u| \Big] + C$$

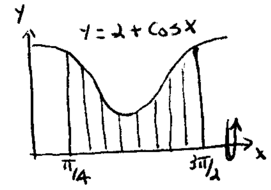
$$= \frac{1}{2} \ln \left| \frac{1+u}{1-u} \right| + C = \frac{1}{2} \ln \left| \frac{1+\sin\theta}{1-\sin\theta} \right| + C = \underline{\frac{1}{2} \ln \left(\frac{1+\sin\theta}{1-\sin\theta} \right) + C}$$ (SINCE $-1 \le \sin\theta \le 1$)

$$= \frac{1}{2} \ln \left(\frac{1+\sin\theta}{1-\sin\theta} \cdot \frac{1+\sin\theta}{1+\sin\theta} \right) + C = \frac{1}{2} \ln \left(\frac{(1+\sin\theta)^2}{1-\sin^2\theta} \right) + C$$

$$= \frac{1}{2} \ln \left(\frac{(1+\sin\theta)^2}{\cos^2\theta} \right) + C = \frac{1}{2} \ln \left(\frac{1+\sin\theta}{\cos\theta} \right)^2 + C = \ln \left| \frac{1+\sin\theta}{\cos\theta} \right| + C$$ (SINCE $\ln x^2 = 2\ln|x|$)

$$= \ln \left| \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} \right| + C = \boxed{\ln|\sec\theta + \tan\theta| + C}$$

⑪



$$V = \int_{\pi/4}^{3\pi/2} \pi (2+\cos x)^2\, dx = \pi \int_{\pi/4}^{3\pi/2} (4 + 4\cos x + \underline{\cos^2 x})\, dx$$

$$= \pi \int_{\pi/4}^{3\pi/2} \left(4 + 4\cos x + \underline{\tfrac{1}{2}(1+\cos 2x)}\right) dx$$

$$= \pi \left[4x + 4\sin x + \frac{1}{2}\left(x + \frac{1}{2}\sin 2x\right) \right]_{\pi/4}^{3\pi/2} = \underline{\pi \left[\frac{9}{2}x + 4\sin x + \frac{1}{4}\sin 2x \right]_{\pi/4}^{3\pi/2}}$$

$$= \pi \left(\left(\frac{27\pi}{4} + 4(-1) + \frac{1}{4}\cdot 0 \right) - \left(\frac{9\pi}{8} + 4\cdot\frac{\sqrt{2}}{2} + \frac{1}{4}\cdot 1 \right) \right) = \boxed{\pi\left(\frac{45\pi}{8} - \frac{17}{4} - 2\sqrt{2} \right)}$$

$$= \boxed{\frac{\pi}{8}\left(45\pi - 34 - 16\sqrt{2} \right)}$$

⑬ $$\int_4^\infty \frac{5}{\sqrt{x}(\sqrt{x}+3)^2}\, dx = \lim_{T\to\infty} \int_4^T \frac{5}{\sqrt{x}(\sqrt{x}+3)^2}\, dx \longleftarrow$$ LET $\underline{u = \sqrt{x} + 3}$, $du = \frac{1}{2\sqrt{x}}\, dx$

$$= \lim_{T\to\infty} 5 \cdot \textcircled{2} \int_4^T \frac{1}{(\sqrt{x}+3)^2} \cdot \frac{1}{\textcircled{2}\sqrt{x}}\, dx = \lim_{T\to\infty} 10 \int_{x=4}^{x=T} \frac{1}{u^2}\, du = \lim_{T\to\infty} 10 \left[-\frac{1}{u} \right]_{x=4}^{x=T}$$

$$= \lim_{T\to\infty} 10 \left[-\frac{1}{\sqrt{x}+3} \right]_4^T = \lim_{T\to\infty} 10 \left(-\frac{1}{\sqrt{T}+3} - \left(-\frac{1}{5}\right) \right) = 10\left(0 + \frac{1}{5}\right) = \boxed{2}$$

⑭ $$\int_{1/4}^\infty \frac{1}{2x^2+x}\, dx = \lim_{T\to\infty} \int_{1/4}^T \frac{1}{2x^2+x}\, dx \longleftarrow \frac{1}{2x^2+x} = \frac{1}{x(2x+1)} = \frac{A}{x} + \frac{B}{2x+1}$$

$1 = A(2x+1) + Bx$

$\underline{x=0}$: $\underline{1 = A}$

$\underline{x=-1/2}$: $1 = -\frac{1}{2}B$ $\underline{B = -2}$

$$= \lim_{T\to\infty} \int_{1/4}^T \left(\frac{1}{x} - \frac{2}{2x+1} \right) dx$$

$$= \lim_{T\to\infty} \Big[\ln x - \ln(2x+1) \Big]_{1/4}^T = \lim_{T\to\infty} \left[\ln\left(\frac{x}{2x+1} \right) \right]_{1/4}^T$$

$$= \lim_{T\to\infty} \left(\ln\frac{T}{2T+1} - \ln\frac{1/4}{6/4} \right) = \ln\frac{1}{2} - \ln\frac{1}{6} = \ln\left(\frac{1/2}{1/6} \right) = \boxed{\ln 3}$$

(AS $T \to \infty$, $\frac{T}{2T+1} = \frac{1}{2+\frac{1}{T}} \to \frac{1}{2+0} = \frac{1}{2}$ SO $\ln\left(\frac{T}{2T+1}\right) \to \ln\frac{1}{2}$)