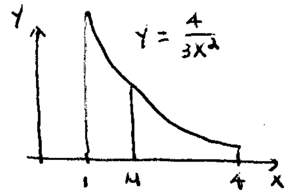
(4) $f(x) = \frac{4}{3x^2}$, $[1,4]$

a) $\mu = E(x) = \int_1^4 x f(x)\,dx = \int_1^4 \cancel{x} \cdot \frac{4}{3x^{\cancel{2}}}\,dx = \frac{4}{3}\int_1^4 \frac{1}{x}\,dx = \frac{4}{3}\Big[\ln x\Big]_1^4 = \frac{4}{3}(\ln 4 - \underset{0}{\cancel{\ln 1}}) = \boxed{\frac{4}{3}\ln 4}$ ≈ 1.85

b) $V(x) = \left(\int_1^4 x^2 f(x)\,dx\right) - \mu^2 = \int_1^4 \cancel{x^2} \cdot \frac{4}{3\cancel{x^2}}\,dx - \left(\frac{4}{3}\ln 4\right)^2 = \frac{4}{3}\int_1^4 1\,dx - \frac{16}{9}(\ln 4)^2$

$= \frac{4}{3}\Big[x\Big]_1^4 - \frac{16}{9}(\ln 4)^2 = \frac{4}{3}(4-1) - \frac{16}{9}(\ln 4)^2 = \boxed{4 - \frac{16}{9}(\ln 4)^2}$

c) $\sigma = \sqrt{V(x)} = \boxed{\sqrt{4 - \frac{16}{9}(\ln 4)^2}} = \sqrt{\frac{4}{9}\left(9 - 4(\ln 4)^2\right)} = \boxed{\frac{2}{3}\sqrt{9 - 4(\ln 4)^2}}$



(10) $f(x) = \frac{1}{18}\sqrt{9-x}$, $[0,9]$

$\mu = \int_0^9 x f(x)\,dx = \int_0^9 x \cdot \frac{1}{18}\sqrt{9-x}\,dx = \frac{1}{18}\int_0^9 x\sqrt{9-x}\,dx$ Let $u = \sqrt{9-x}$, so $x = 9 - u^2$, $dx = -2u\,du$; if $x = 0$, $u = 3$; $x = 9$, $u = 0$

$= \frac{1}{18}\int_3^0 (9-u^2)(u)(-2u)\,du = \frac{1}{9}\int_0^3 u^2(9-u^2)\,du$

$= \frac{1}{9}\int_0^3 (9u^2 - u^4)\,du = \frac{1}{9}\left[3u^3 - \frac{u^5}{5}\right]_0^3 = \frac{1}{9}\left(3\cdot 3^3 - \frac{3^5}{5}\right) = 9 - \frac{27}{5} = \boxed{\frac{18}{5}} = \boxed{3.6}$

[OR let $u = 9 - x$, $x = 9 - u$, $dx = -du$ to get $\mu = \frac{1}{18}\int_9^0 (9-u)\sqrt{u}(-du) = \frac{1}{18}\int_0^9 (9u^{1/2} - u^{3/2})\,du = \boxed{3.6}$]

(27a) $f(t) = \frac{1}{10}$, $0 \le t \le 10$

1) Since T is uniformly distributed, $\mu = \frac{a+b}{2} = \frac{0+10}{2} = \boxed{5}$

2) $V(t) = \int_0^{10} t^2 f(t)\,dt - \mu^2 = \int_0^{10} t^2 \cdot \frac{1}{10}\,dt - 25 = \frac{1}{10}\left[\frac{t^3}{3}\right]_0^{10} - 25 = \frac{1}{10}\cdot\frac{10^3}{3} - 25$

$= \frac{100}{3} - 25 = \frac{25}{3}$, so $\sigma = \sqrt{V(t)} = \sqrt{\frac{25}{3}} = \boxed{\frac{5}{\sqrt{3}}}$

(49b) $P(X > 4) = 1 - P(X \le 4) = 1 - \int_0^4 \underbrace{\tfrac{1}{9}x}\,\underbrace{e^{-x/3}\,dx}$ Let $u = \frac{1}{9}x$, $dv = e^{-x/3}\,dx$; $du = \frac{1}{9}\,dx$, $v = -3e^{-x/3}$

$= 1 - \left[-\frac{1}{3}xe^{-x/3} - \left(-\frac{1}{3}\right)\int e^{-x/3}\,dx\right]_0^4$

$= 1 - \left[-\frac{1}{3}xe^{-x/3} + \frac{1}{3}\left(-3e^{-x/3}\right)\right]_0^4 \longleftarrow$

(OR use

u		dv
$\frac{1}{9}x$	$+$	$e^{-x/3}\,dx$
$\frac{1}{9}$	$-$	$-3e^{-x/3}$
0		$9e^{-x/3}$

)

$= 1 + \left[\frac{1}{3}xe^{-x/3} + e^{-x/3}\right]_0^4$

$= 1 + \left(\frac{4}{3}e^{-4/3} + e^{-4/3}\right) - (0 + 1)$

$= \boxed{\frac{7}{3}e^{-4/3}}$

(5) $f(x) = \frac{5}{2}x^{3/2}$, $[0,1]$

a) $\mu = \int_0^1 x\cdot\frac{5}{2}x^{3/2}\,dx = \frac{5}{2}\int_0^1 x^{5/2}\,dx = \frac{5}{2}\left[\frac{2}{7}x^{7/2}\right]_0^1 = \frac{5}{7}(1-0) = \boxed{\frac{5}{7}}$

b) $V(x) = \int_0^1 x^2\cdot\frac{5}{2}x^{3/2}\,dx - \mu^2 = \frac{5}{2}\int_0^1 x^{7/2}\,dx - \left(\frac{5}{7}\right)^2 = \frac{5}{2}\left[\frac{2}{9}x^{9/2}\right]_0^1 - \frac{25}{49} = \frac{5}{9}(1-0) - \frac{25}{49}$

$= \frac{5}{9} - \frac{25}{49} = \boxed{\frac{20}{441}}$

c) $\sigma = \sqrt{V(x)} = \sqrt{\frac{20}{441}} = \boxed{\frac{2\sqrt{5}}{21}}$

(9) $f(x) = \frac{4}{3(x+1)^2}$, $[0,3]$

$\mu = \int_0^3 x\cdot\frac{4}{3(x+1)^2}\,dx = \frac{4}{3}\int_0^3 \frac{x}{(x+1)^2}\,dx$ LET $u = x+1$, so $x = u-1$, $dx = du$ IF $x=0$, $u=1$; $x=3$, $u=4$

$= \frac{4}{3}\int_1^4 \frac{u-1}{u^2}\,du = \frac{4}{3}\int_1^4\left(\frac{u}{u^2} - \frac{1}{u^2}\right)du = \frac{4}{3}\int_1^4\left(\frac{1}{u} - \frac{1}{u^2}\right)du = \frac{4}{3}\left[\ln u + \frac{1}{u}\right]_1^4$

$= \frac{4}{3}\left(\left(\ln 4 + \frac{1}{4}\right) - (\underbrace{\ln 1}_{0} + 1)\right) = \frac{4}{3}\left(\ln 4 - \frac{3}{4}\right) = \boxed{\frac{4}{3}\ln 4 - 1}$

(OR use PARTIAL FRACTIONS, OR INTEGRATION BY PARTS)

(29) a) $f(t) = \frac{1}{2}e^{-t/2}$, $[0,\infty)$

$u = -t/2$, $du = -\frac{1}{2}\,dt$

b) $P(t<1) = \int_0^1 \frac{1}{2}e^{-t/2}\,dt = -\int_0^1 e^{-t/2}\left(-\frac{1}{2}\right)dt = -\left[e^{-t/2}\right]_0^1 = -(e^{-1/2}-1) = \boxed{1-e^{-1/2}}$

$= \boxed{1 - \frac{1}{\sqrt{e}}}$

(43) $f(x) = 4(1-2x)$, $[0, 1/2]$

a) $\mu = \int_0^{1/2} x\cdot 4(1-2x)\,dx = 4\int_0^{1/2}(x-2x^2)\,dx = 4\left[\frac{x^2}{2} - \frac{2x^3}{3}\right]_0^{1/2} = 4\left(\frac{1/4}{2} - \frac{2/8}{3}\right) = 4\left(\frac{1}{4}\right)\left(\frac{1}{2}-\frac{1}{3}\right) = \boxed{\frac{1}{6}}$

b) $\int_0^m 4(1-2x)\,dx = \frac{1}{2}$ $4\left[x - x^2\right]_0^m = \frac{1}{2}$ $4(m-m^2) = \frac{1}{2}$ $m - m^2 = \frac{1}{8}$ $8m - 8m^2 = 1$

$8m^2 - 8m + 1 = 0$ $m = \frac{8 \pm \sqrt{32}}{16} = \frac{8 \pm 4\sqrt{2}}{16} = \boxed{\frac{2-\sqrt{2}}{4}}$ (USING − SIGN, SINCE m IS IN $[0, 1/2]$)

CH. 9 RE – (31) $f(t) = \frac{1}{4\sqrt{t-4}}$, $[5,13]$

$P(t>8) = \int_8^{13}\frac{1}{4\sqrt{t-4}}\,dt$ LET $u = t-4$, $du = dt$ IF $t=8$, $u=4$; $t=13$, $u=9$

$= \frac{1}{4}\int_4^9 \frac{1}{\sqrt{u}}\,du = \frac{1}{4}\int_4^9 u^{-1/2}\,du = \frac{1}{4}\left[2u^{1/2}\right]_4^9 = \frac{1}{2}(3-2) = \boxed{\frac{1}{2}}$

OR $P(t>8) = \int_8^{13}\frac{1}{4\sqrt{t-4}}\,dt$ LET $u = \sqrt{t-4}$, so $t = u^2+4$, $dt = 2u\,du$ IF $t=8$, $u=2$; $t=13$, $u=3$

$= \frac{1}{4}\int_2^3 \frac{1}{\cancel{u}}\cdot 2\cancel{u}\,du = \frac{1}{4}\int_2^3 2\,du = \frac{1}{4}\left[2u\right]_2^3 = \frac{1}{2}(3-2) = \boxed{\frac{1}{2}}$

9.3 - (10) $f(x) = \frac{1}{18}\sqrt{9-x}$, $[0,9]$

$\mu = \int_0^9 x \cdot \frac{1}{18}\sqrt{9-x}\,dx = \frac{1}{18}\int_0^9 x\sqrt{9-x}\,dx$ LET $u = \sqrt{9-x}$, $x = 9-u^2$, $dx = -2u\,du$ IF $x=0, u=3$; $x=9, u=0$

$= \frac{1}{18}\int_3^0 (9-u^2)\,u\,(-2u)\,du = \frac{1}{9}\int_0^3 u^2(9-u^2)\,du = \frac{1}{9}\int_0^3 (9u^2-u^4)\,du$

$= \frac{1}{9}\left[3u^3 - \frac{u^5}{5}\right]_0^3 = \frac{1}{9}\left(3\cdot 3^3 - \frac{1}{5}\cdot 3^5\right) = 9 - \frac{3^3}{5} = 9 - \frac{27}{5} = \boxed{\frac{18}{5}} = \boxed{3.6}$

(OR LET $u = 9-x$, $x = 9-u$, $dx = -du$ TO GET $\frac{1}{18}\int_9^0 (9-u)\sqrt{u}\,(-1)\,du = \frac{1}{18}\int_0^9 (9u^{1/2} - u^{3/2})\,du$)

(40) $f(T) = \frac{1}{2\sqrt{T-2}}$, $[3,6]$

a) $P(T>4) = \int_4^6 \frac{1}{2\sqrt{T-2}}\,dT$ LET $u = T-2$, $du = dT$ IF $T=4, u=2$; $T=6, u=4$

$= \frac{1}{2}\int_2^4 \frac{1}{\sqrt{u}}\,du = \frac{1}{2}\int_2^4 u^{-1/2}\,du = \frac{1}{\cancel{2}}\left[\cancel{2}u^{1/2}\right]_2^4 = \boxed{2-\sqrt{2}}$

(OR LET $u = \sqrt{T-2}$, $T = u^2+2$, $dT = 2u\,du$ TO GET $\frac{1}{\cancel{2}}\int_{\sqrt{2}}^2 \frac{1}{\cancel{u}}\cdot \cancel{2}\cancel{u}\,du = \int_{\sqrt{2}}^2 1\,du = [u]_{\sqrt{2}}^2 = \boxed{2-\sqrt{2}}$)

b) $\mu = \int_3^6 T\cdot\frac{1}{2\sqrt{T-2}}\,dT = \frac{1}{2}\int_3^6 \frac{T}{\sqrt{T-2}}\,dT$ LET $u = \sqrt{T-2}$, $T = u^2+2$, $dT = 2u\,du$ IF $T=3, u=1$; $T=6, u=2$

$= \frac{1}{\cancel{2}}\int_1^2 \frac{u^2+2}{\cancel{u}}\cdot\cancel{2}\cancel{u}\,du = \int_1^2 (u^2+2)\,du = \left[\frac{u^3}{3}+2u\right]_1^2 = \left(\frac{8}{3}+4\right) - \left(\frac{1}{3}+2\right) = \boxed{\frac{13}{3}\text{ DAYS}}$

(OR LET $u = T-2$, $T = u+2$, $dT = du$ TO GET $\frac{1}{2}\int_1^4 \frac{u+2}{\sqrt{u}}\,du = \frac{1}{2}\int_1^4 (u^{1/2} + 2u^{-1/2})\,du$)

CH. 9 RE - (34) $f(x) = \frac{8-x}{32}$, $[0,8]$

$\mu = \int_0^8 x\cdot\frac{8-x}{32}\,dx = \frac{1}{32}\int_0^8 (8x - x^2)\,dx = \frac{1}{32}\left[4x^2 - \frac{x^3}{3}\right]_0^8 = \frac{1}{32}\left(4\cdot 64 - \frac{8\cdot 64}{3}\right)$

$= \frac{64}{32}\left(4 - \frac{8}{3}\right) = 2\left(\frac{4}{3}\right) = \boxed{\frac{8}{3}}$

(43) $f(x) = \frac{1}{4}e^{-x/4}$, $[0,\infty)$

$\int_0^m \frac{1}{4}e^{-x/4}\,dx = \frac{1}{2}$ $\quad \frac{1}{4}\left[-4e^{-x/4}\right]_0^m = \frac{1}{2}$ $\quad -\left(e^{-m/4} - 1\right) = \frac{1}{2}$

$e^{-m/4} - 1 = -\frac{1}{2}$ $\quad e^{-m/4} = \frac{1}{2}$ $\quad \ln e^{-m/4} = \ln\frac{1}{2}$ $\quad -\frac{m}{4} = \ln\frac{1}{2}$

$m = \boxed{-4\ln\frac{1}{2}} = \boxed{4\ln 2}$ (SINCE $\ln\frac{1}{2} = -\ln 2$)