MATH 10B

SOLUTIONS – SEC. 5.6 AND 6.5

5.6 – 7) $f(x) = 2x^2$, $[1, 3]$

$1 \quad \tfrac{5}{4} \quad \tfrac{3}{2} \quad \tfrac{7}{4} \quad 2 \quad \tfrac{9}{4} \quad \tfrac{5}{2} \quad \tfrac{11}{4} \quad 3$

$\Delta x = \frac{3-1}{4} = \frac{1}{2}$

$$\int_1^3 2x^2\,dx \approx \boxed{2\left(\tfrac{5}{4}\right)^2\cdot\tfrac{1}{2} + 2\left(\tfrac{7}{4}\right)^2\cdot\tfrac{1}{2} + 2\left(\tfrac{9}{4}\right)^2\cdot\tfrac{1}{2} + 2\left(\tfrac{11}{4}\right)^2\cdot\tfrac{1}{2}} = \frac{276}{16} = \boxed{17.25}$$

(MIDPOINT RULE, $n=4$)

EXACT VALUE $= 2\left[\frac{x^3}{3}\right]_1^3 = \frac{2}{3}(27-1) = \boxed{\frac{52}{3}} = 17.333\ldots$

23) $\int_0^2 \frac{1}{x+1}\,dx$

$0 \quad \tfrac{1}{2} \quad 1 \quad \tfrac{3}{2} \quad 2$

$\Delta x = \frac{2-0}{4} = \frac{1}{2}$

$$\int_0^2 \frac{1}{x+1}\,dx \approx \boxed{\frac{1/2}{2}\left[1 + 2\cdot\frac{1}{3/2} + 2\cdot\frac{1}{2} + 2\cdot\frac{1}{5/2} + \frac{1}{3}\right]}$$

(TRAPEZOIDAL RULE, $n=4$)

31)

T	0	5	10	15	20
V	0	29.3	51.3	66	73.3

$$S = \int_0^{20} |v(t)|\,dt \approx \boxed{\frac{5}{2}\left[0 + 2(29.3) + 2(51.3) + 2(66) + 73.3\right] \text{ FT}} = \boxed{916.25 \text{ FT}}$$

(TRAPEZOIDAL RULE, $n=4$)

33) A)

$$\int_0^1 \frac{4}{1+x^2}\,dx \approx \boxed{\frac{4}{1+\left(\frac{1}{8}\right)^2}\cdot\frac{1}{4} + \frac{4}{1+\left(\frac{3}{8}\right)^2}\cdot\frac{1}{4} + \frac{4}{1+\left(\frac{5}{8}\right)^2}\cdot\frac{1}{4} + \frac{4}{1+\left(\frac{7}{8}\right)^2}\cdot\frac{1}{4}}$$

$0 \quad \tfrac{1}{8} \quad \tfrac{1}{4} \quad \tfrac{3}{8} \quad \tfrac{1}{2} \quad \tfrac{5}{8} \quad \tfrac{3}{4} \quad \tfrac{7}{8} \quad 1$

$$= \frac{64}{65} + \frac{64}{73} + \frac{64}{89} + \frac{64}{113} \approx 3.1468$$

(MIDPOINT RULE, $n=4$)

B)

$$\int_0^1 \frac{4}{1+x^2}\,dx \approx \boxed{\frac{1/4}{2}\left[4 + 2\cdot\frac{4}{1+\left(\frac{1}{4}\right)^2} + 2\cdot\frac{4}{1+\left(\frac{1}{2}\right)^2} + 2\cdot\frac{4}{1+\left(\frac{3}{4}\right)^2} + \frac{4}{2}\right]}$$

$$= \frac{1}{8}\left[4 + \frac{16\cdot 8}{17} + \frac{32}{5} + \frac{16\cdot 8}{25} + 2\right] \approx 3.1312$$

(TRAPEZOIDAL RULE, $n=4$)

ACTUAL VALUE $= \pi = 3.14159\ldots$

6.5 – 13b)

$$\int_0^1 \frac{1}{1+x^2}\,dx \approx \boxed{\frac{1/4}{3}\left[1 + 4\cdot\frac{1}{1+(1/4)^2} + 2\cdot\frac{1}{1+(1/2)^2} + 4\cdot\frac{1}{1+(3/4)^2} + \frac{1}{2}\right]}$$

(SIMPSON'S RULE, $n=4$)

15) a)

$$\int_0^1 \sqrt{1-x^2}\,dx \approx \boxed{\frac{1/4}{2}\left[1 + 2\sqrt{15/16} + 2\sqrt{3/4} + 2\sqrt{7/16} + 0\right]}^{*}$$

(TRAPEZOIDAL RULE, $n=4$)

$0 \quad \tfrac{1}{4} \quad \tfrac{1}{2} \quad \tfrac{3}{4} \quad 1$

b)

$$\int_0^1 \sqrt{1-x^2}\,dx \approx \boxed{\frac{1/4}{3}\left[1 + 4\cdot\frac{\sqrt{15}}{4} + 2\cdot\frac{\sqrt{3}}{2} + 4\cdot\frac{\sqrt{7}}{4} + 0\right]}^{*}$$

(SIMPSON'S RULE, $n=4$)

17) a)

$$\int_0^2 e^{-x^2}\,dx \approx \boxed{\frac{1}{2}\left[1 + 2e^{-1} + e^{-4}\right]} \approx .8770$$

(TRAPEZOIDAL RULE, $n=2$)

$0 \quad 1 \quad 2$

b)

$$\int_0^2 e^{-x^2}\,dx \approx \boxed{\frac{1}{3}\left[1 + 4e^{-1} + e^{-4}\right]} \approx .8299$$

(SIMPSON'S RULE, $n=2$)

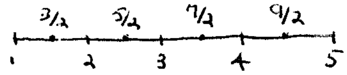
* a) $\approx .7489$

b) $\approx .7709$

MATH 16B

SOLUTIONS – SEC. 5.6 AND 6.5 AND CH. 6 RE

5.6 – (2) $f(x) = \frac{1}{x}$, $[1, 5]$, $n = 4$



$\Delta x = \frac{5-1}{4} = 1$

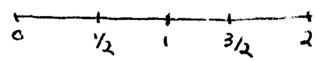
$$\int_1^5 \frac{1}{x}\,dx \approx f(\tfrac{3}{2})\cdot 1 + f(\tfrac{5}{2})\cdot 1 + f(\tfrac{7}{2})\cdot 1 + f(\tfrac{9}{2})\cdot 1 = \boxed{\tfrac{2}{3} + \tfrac{2}{5} + \tfrac{2}{7} + \tfrac{2}{9}} \approx \underline{1.5746}$$

$$\text{EXACT AREA} = \int_1^5 \frac{1}{x}\,dx = \Big[\ln x\Big]_1^5 = \ln 5 - \ln 1 = \boxed{\ln 5} \approx \underline{1.6094}$$

(32b)

$$A \approx \frac{20}{2}\Big[0 + 2.50 + 2.54 + 2.82 + 2.82 + 2.73 + 2.75 + 2.80 + 0\Big] = \boxed{9920 \text{ FT}^2}$$

6.5 – (18) $\int_0^2 e^{-x^2}\,dx$, $n = 4$



$\Delta x = \frac{2-0}{4} = \frac{1}{2}$

a) TRAPEZOIDAL RULE

$$\int_0^2 e^{-x^2}\,dx \approx \frac{1/2}{2}\Big[1 + 2e^{-1/4} + 2e^{-1} + 2e^{-9/4} + e^{-4}\Big] \approx \boxed{.881}$$

(ROUNDED TO 3 DECIMAL PLACES)

b) SIMPSON'S RULE

$$\int_0^2 e^{-x^2}\,dx \approx \frac{1/2}{3}\Big[1 + 4e^{-1/4} + 2e^{-1} + 4e^{-9/4} + e^{-4}\Big] \approx \boxed{.882}$$

(30)

$$A \approx \frac{10}{3}\Big[75 + 4.81 + 2.84 + 4.76 + 2.67 + 4.68 + 2.69 + 4.72 + 2.68 + 4.56 + 2.42 + 4.23 + 0\Big]$$

$$\approx \boxed{7463.3 \text{ FT}^2}$$

CH. 6 RE – (46) $\displaystyle\int \frac{4x^2 - x - 5}{x^2(x+5)}\,dx$

$$\frac{4x^2 - x - 5}{x^2(x+5)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+5}$$

$$4x^2 - x - 5 = Ax(x+5) + B(x+5) + Cx^2$$

$x = 0$: $-5 = 5B$ $\quad \underline{B = -1}$

$x = -5$: $100 = 25C$ $\quad \underline{C = 4}$

COEFF. OF x^2: $4 = A + C$ so $4 = A + 4$ $\quad \underline{A = 0}$

$$= \int \left(-\frac{1}{x^2} + \frac{4}{x+5}\right)dx$$

$$= \boxed{\frac{1}{x} + 4\ln|x+5| + C}$$

(47)

$$\int \frac{x^2}{x^2 + 2x - 15}\,dx = \int \left(1 + \frac{-2x + 15}{x^2 + 2x - 15}\right)dx$$

Long division: $x^2 + 2x - 15 \,\big)\, x^2$ gives quotient 1; $x^2 - (x^2 + 2x - 15) = -2x + 15$

$$\frac{-2x + 15}{(x+5)(x-3)} = \frac{A}{x+5} + \frac{B}{x-3}$$

$$-2x + 15 = A(x-3) + B(x+5)$$

$x = 3$: $9 = 8B$ $\quad \underline{B = 9/8}$

$x = -5$: $25 = -8A$ $\quad \underline{A = -25/8}$

$$= \int \left(1 + \frac{-25/8}{x+5} + \frac{9/8}{x-3}\right)dx$$

$$= \boxed{x - \frac{25}{8}\ln|x+5| + \frac{9}{8}\ln|x-3| + C}$$