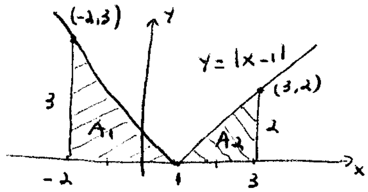
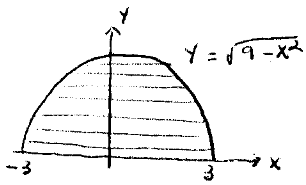
Sec. 5.4 – Solutions

(5) $\displaystyle\int_{-2}^{3} |x-1|\,dx$



$A = A_1 + A_2 = \frac{1}{2}(3)(3) + \frac{1}{2}(2)(2) = \boxed{\frac{13}{2}}$

(7) $\displaystyle\int_{-3}^{3} \sqrt{9-x^2}\,dx$



$A = \frac{1}{2}(\pi r^2) = \frac{1}{2}(\pi \cdot 3^2) = \boxed{\frac{9\pi}{2}}$

(17) $\displaystyle A = \int_1^4 \frac{x^2+4}{x}\,dx = \int_1^4 \left(\frac{x^2}{x} + \frac{4}{x}\right)dx = \int_1^4 \left(x + 4\cdot\frac{1}{x}\right)dx = \left[\frac{x^2}{2} + 4\ln x\right]_1^4$ (since $\ln 4 = \ln 2^2 = 2\ln 2$)

$= (8 + 4\ln 4) - \left(\frac{1}{2} + 4\underbrace{\cancel{\ln 1}}_{0}\right) = \boxed{\frac{15}{2} + 4\ln 4} = \boxed{\frac{15}{2} + 8\ln 2}$

(29) $\displaystyle\int_1^4 \left(\frac{2u-1}{\sqrt{u}}\right)du = \int_1^4 (2u-1)u^{-1/2}\,du = \int_1^4 \left(2u^{1/2} - u^{-1/2}\right)du = \left[\frac{4}{3}u^{3/2} - 2u^{1/2}\right]_1^4$

$= \left(\frac{4}{3}\cdot 4^{3/2} - 2\cdot 4^{1/2}\right) - \left(\frac{4}{3} - 2\right) = \left(\frac{4}{3}\cdot 8 - 2\cdot 2\right) - \left(\frac{4}{3} - 2\right) = \frac{28}{3} - 2 = \boxed{\frac{22}{3}}$

(33) $\displaystyle\int_0^4 \frac{1}{\sqrt{2x+1}}\,dx$ Let $u = 2x+1$, $du = 2\,dx$. If $x = 0$, $u = 1$; $x = 4$, $u = 9$

$\displaystyle = \frac{1}{2}\int_0^4 \frac{1}{\sqrt{2x+1}}\cdot 2\,dx = \frac{1}{2}\int_1^9 \frac{1}{\sqrt{u}}\,du = \frac{1}{2}\int_1^9 u^{-1/2}\,du = \frac{1}{\cancel{2}}\left[\cancel{2}u^{1/2}\right]_1^9 = \sqrt{9} - \sqrt{1} = \boxed{2}$

[or Let $u = \sqrt{2x+1}$, $x = \frac{1}{2}(u^2-1)$, $dx = u\,du$ to get $\displaystyle\int_1^3 \frac{1}{\cancel{u}}\cdot\cancel{u}\,du = \int_1^3 1\,du = [u]_1^3 = \boxed{2}$]

(34) $\displaystyle\int_0^2 \frac{x}{\sqrt{1+2x^2}}\,dx$ Let $u = 1+2x^2$, $du = 4x\,dx$. If $x = 0$, $u = 1$; $x = 2$, $u = 9$

$\displaystyle = \frac{1}{4}\int_0^2 \frac{1}{\sqrt{1+2x^2}}\cdot 4x\,dx = \frac{1}{4}\int_1^9 \frac{1}{\sqrt{u}}\,du = \frac{1}{4}\int_1^9 u^{-1/2}\,du = \frac{1}{4}\left[2u^{1/2}\right]_1^9 = \frac{1}{2}\left(\sqrt{9} - \sqrt{1}\right) = \frac{1}{2}(3-1) = \boxed{1}$

(37) $\displaystyle\int_1^3 \frac{e^{3/x}}{x^2}\,dx$ Let $u = 3/x = 3x^{-1}$, $du = -3x^{-2}dx = -\frac{3}{x^2}dx$. If $x = 1$, $u = 3$; $x = 3$, $u = 1$

$\displaystyle = \left(-\frac{1}{3}\right)\int_1^3 e^{3/x}\cdot\frac{(-3)}{x^2}\,dx = -\frac{1}{3}\int_3^1 e^u\,du = \frac{1}{3}\int_1^3 e^u\,du = \frac{1}{3}\left[e^u\right]_1^3 = \boxed{\frac{1}{3}(e^3 - e)}$

OR Let $u = 3/x$, $du = -3/x^2\,dx$ to get

$\displaystyle \left(-\frac{1}{3}\right)\int_1^3 e^{3/x}\cdot\frac{(-3)}{x^2}\,dx = -\frac{1}{3}\left[e^{3/x}\right]_1^3 = -\frac{1}{3}\left(e^1 - e^3\right) = \boxed{\frac{1}{3}(e^3 - e)}$

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Sec. 5.4 – SOLUTIONS

(41) $\displaystyle\int_0^2 \frac{x}{1+4x^2}\,dx$ LET $u = 1+4x^2$, $du = 8x\,dx$

$$= \frac{1}{8}\int_0^2 \frac{8x}{1+4x^2}\,dx = \frac{1}{8}\Big[\ln(1+4x^2)\Big]_0^2 = \frac{1}{8}\big(\ln 17 - \underbrace{\cancel{\ln 1}}_{0}\big) = \boxed{\frac{1}{8}\ln 17}$$

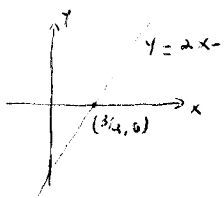
(OR CHANGE THE LIMITS TO GET $\displaystyle\frac{1}{8}\int_1^{17}\frac{1}{u}\,du = \frac{1}{8}\Big[\ln u\Big]_1^{17} = \frac{1}{8}(\ln 17 - \ln 1) =$)

(44) $$\int_0^3 |2x-3|\,dx = \int_0^{3/2}|2x-3|\,dx + \int_{3/2}^3 |2x-3|\,dx = \int_0^{3/2} -(2x-3)\,dx + \int_{3/2}^3 (2x-3)\,dx$$

$$= -\Big[x^2-3x\Big]_0^{3/2} + \Big[x^2-3x\Big]_{3/2}^3$$

$$= -\left(\frac{9}{4}-\frac{9}{2}-0\right) + (9-9) - \left(\frac{9}{4}-\frac{9}{2}\right)$$

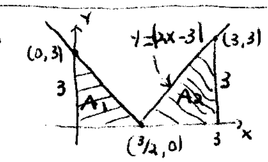
$$= \frac{9}{4} + 0 + \frac{9}{4} = \boxed{\frac{9}{2}}$$



$2x-3 \le 0$ IF $x \le \frac{3}{2}$

$2x-3 \ge 0$ IF $x \ge \frac{3}{2}$

OR



$$A = A_1 + A_2 = \frac{1}{2}\cdot\frac{3}{2}\cdot 3 + \frac{1}{2}\cdot\frac{3}{2}\cdot 3 = \frac{9}{4} + \frac{9}{4} = \boxed{\frac{9}{2}}$$

(45) $$\int_0^4 (2-|x-2|)\,dx = \int_0^2 (2-|x-2|)\,dx + \int_2^4 (2-|x-2|)\,dx$$

IF $x \le 2$, $x-2 \le 0$

IF $x \ge 2$, $x-2 \ge 0$

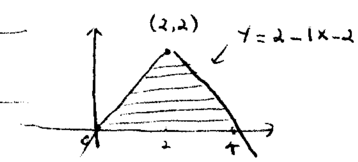
$$= \int_0^2 (2-(-(x-2)))\,dx + \int_2^4 (2-(x-2))\,dx = \int_0^2 x\,dx + \int_2^4 (4-x)\,dx$$

$$= \Big[\frac{x^2}{2}\Big]_0^2 + \Big[4x-\frac{x^2}{2}\Big]_2^4 = \left(\frac{4}{2}-0\right) + \left(16-\frac{16}{2}\right) - \left(8-\frac{4}{2}\right) = 2+8-6 = \boxed{4}$$

OR use $\displaystyle\int_0^4 (2-|x-2|)\,dx = \int_0^4 2\,dx - \int_0^4 |x-2|\,dx = 8 - \int_0^4 |x-2|\,dx$,

OR FIND THE AREA OF THE REGION UNDER THE GRAPH OF $y = 2-|x-2|$:

$A = \frac{1}{2}\cdot 4\cdot 2 = \boxed{4}$



(64) $$\bar{f} = \frac{1}{4-0}\int_0^4 2e^{x/4}\,dx = \frac{2}{4}\int_0^4 e^{\frac{1}{4}x}\,dx = \frac{1}{2}\Big[4e^{\frac{1}{4}x}\Big]_0^4 = \boxed{2(e-1)}$$

(65) $\displaystyle\bar{f} = \frac{1}{2-0}\int_0^2 x\sqrt{4-x^2}\,dx$ LET $u = 4-x^2$, $du = -2x\,dx$; IF $x=0$, $u=4$; $x=2$, $u=0$

$$= \frac{1}{2}\cdot\left(-\frac{1}{2}\right)\int_0^2 \sqrt{4-x^2}\,(-2)x\,dx = -\frac{1}{4}\int_4^0 \sqrt{u}\,du = \frac{1}{4}\int_0^4 u^{1/2}\,du = \frac{1}{4}\Big[\frac{2}{3}u^{3/2}\Big]_0^4 = \frac{1}{6}(8-0) = \boxed{\frac{4}{3}}$$

(91) $A(t) = Pe^{rt} = 2250\,e^{.12t}$, so

$$\bar{A} = \frac{1}{5-0}\int_0^5 2250\,e^{.12t}\,dt = \frac{2250}{5}\int_0^5 e^{.12t}\,dt = 450\Big[\frac{1}{.12}e^{.12t}\Big]_0^5$$

$$= \frac{450}{.12}(e^{.6}-1) = \frac{\overset{150}{\cancel{450}}\,(\overset{25}{\cancel{100}})}{\underset{3}{\cancel{12}}}(e^{.6}-1) = (150)(25)(e^{.6}-1) = \boxed{\$3750(e^{.6}-1)}$$

$\approx$ \$3082.95