MATH 16B

SOLUTIONS – SEC. 5.1 AND 5.2

5.1 – (12) $\frac{dP}{dx} = -40x + 250$, $P(5) = 650$

$\underline{P(x)} = \int (-40x + 250)\,dx = \underline{-20x^2 + 250x + C}$

since $P(5) = -20(25) + 250(5) + C = -500 + 1250 + C = 650$, $\underline{C = -100}$

and $\boxed{P(x) = -20x^2 + 250x - 100}$

(17) 1) $\underline{v(t)} = \int a(t)\,dt = \int -32\,dt = \underline{-32t + C}$ where $C = v(0)$ is the initial velocity

2) $\underline{s(t)} = \int v(t)\,dt = \int (-32t + c)\,dt = -16t^2 + Ct + D = \underline{-16t^2 + Ct}$

since $D = s(0) = 0$.

It reaches max. height when $\underline{v(t) = 0}$, so $-32t + C = 0$, $C = 32t$, $\underline{t = \frac{C}{32}}$

and the max. height is given by

$s\left(\frac{C}{32}\right) = -16\left(\frac{C}{32}\right)^2 + C\left(\frac{C}{32}\right) = -\cancel{16}\cdot\frac{C^2}{\underset{2}{\cancel{32}}\cdot 32} + \frac{C^2}{32} = \frac{C^2}{32} - \frac{C^2}{64} = \frac{2C^2}{64} - \frac{C^2}{64} = \underline{\frac{C^2}{64}}$

Then $\frac{C^2}{64} = 550$ gives $C^2 = 64(550) = 64(25)(22)$, so $C = 8\cdot 5\sqrt{22} = \boxed{40\sqrt{22}\ \text{FT/SEC}}$

5.2 – (19) $\int \frac{x+1}{(x^2+2x-3)^2}\,dx$ Let $\underline{u = x^2 + 2x - 3}$

$du = (2x+2)\,dx = 2(x+1)\,dx$

$= \frac{1}{2}\int \frac{1}{(x^2+2x-3)^2}\cdot 2(x+1)\,dx = \frac{1}{2}\int \frac{1}{u^2}\,du = \frac{1}{2}\int u^{-2}\,du = \frac{1}{2}\left(-u^{-1}\right) + C = \boxed{-\frac{1}{2}(x^2+2x-3)^{-1} + C}$

(20) $\int \frac{6x}{(1+x^2)^3}\,dx$ Let $\underline{u = 1 + x^2}$

$du = 2x\,dx$

$= \frac{6}{2}\int \frac{1}{(1+x^2)^3}\cdot 2x\,dx = 3\int \frac{1}{u^3}\,du = 3\int u^{-3}\,du = 3\left(\frac{u^{-2}}{-2}\right) + C = \boxed{-\frac{3}{2}(1+x^2)^{-2} + C}$

(22) $\int \frac{4x+6}{(x^2+3x+7)^3}\,dx$ Let $\underline{u = x^2 + 3x + 7}$

$du = (2x+3)\,dx$

$= \int \frac{2(2x+3)}{(x^2+3x+7)^3}\,dx = 2\int \frac{1}{(x^2+3x+7)^3}\cdot(2x+3)\,dx = 2\int \frac{1}{u^3}\,du = 2\int u^{-3}\,du = 2\left(\frac{u^{-2}}{-2}\right) + C$

$= \boxed{-(x^2+3x+7)^{-2} + C}$

(24) $\int u^3\sqrt{u^4+2}\,du$ Let $\underline{t = u^4 + 2}$

$dt = 4u^3\,du$

$= \frac{1}{4}\int \sqrt{u^4+2}\cdot 4u^3\,du = \frac{1}{4}\int \sqrt{t}\,dt = \frac{1}{4}\int t^{1/2}\,dt = \frac{1}{4}\left(\frac{2}{3}t^{3/2}\right) + C = \boxed{\frac{1}{6}(u^4+2)^{3/2} + C}$

(26) $\int \frac{x^2}{\sqrt{1-x^3}}\,dx$ Let $\underline{u = 1 - x^3}$

$du = -3x^2\,dx$

$= \left(-\frac{1}{3}\right)\int \frac{1}{\sqrt{1-x^3}}(-3)x^2\,dx = -\frac{1}{3}\int \frac{1}{\sqrt{u}}\,du = -\frac{1}{3}\int u^{-1/2}\,du = -\frac{1}{3}\left(2u^{1/2}\right) + C$

$= \boxed{-\frac{2}{3}(1-x^3)^{1/2} + C}$

(28) $\displaystyle\int \frac{t+2t^2}{\sqrt{t}}\,dt = \int\left(\frac{t}{\sqrt{t}} + \frac{2t^2}{\sqrt{t}}\right)dt = \int\left(t^{1/2} + 2t^{3/2}\right)dt = \boxed{\frac{2}{3}t^{3/2} + \frac{4}{5}t^{5/2} + C}$

[OR use $\displaystyle\int \frac{t+2t^2}{\sqrt{t}}\,dt = \int (t+2t^2)\,t^{-1/2}\,dt =$, OR SUBSTITUTE $u=\sqrt{t}$, $du = \frac{1}{2\sqrt{t}}\,dt$]

(37) $\displaystyle\int x^2(2-3x^3)^{3/2}\,dx$ LET $u = 2-3x^3$

$du = -9x^2\,dx$

$\displaystyle = -\frac{1}{9}\int (2-3x^3)^{3/2}(-9)\,x^2\,dx = -\frac{1}{9}\int u^{3/2}\,du = -\frac{1}{9}\left(\frac{2}{5}u^{5/2}\right) + C = \boxed{-\frac{2}{45}(2-3x^3)^{5/2} + C}$

(42) $\displaystyle\int \sqrt{x}\,(4-x^{3/2})^2\,dx$ LET $u = 4 - x^{3/2}$

$du = -\frac{3}{2}x^{1/2}\,dx$

$\displaystyle = \left(-\frac{2}{3}\right)\int (4-x^{3/2})^2\cdot\left(-\frac{3}{2}\right)\sqrt{x}\,dx = -\frac{2}{3}\int u^2\,du = -\frac{2}{3}\left(\frac{u^3}{3}\right) + C = \boxed{-\frac{2}{9}(4-x^{3/2})^3 + C}$

(47) $f'(x) = x\sqrt{1-x^2}$, GRAPH PASSES THROUGH $(0, 4/3)$

$\displaystyle f(x) = \int x\sqrt{1-x^2}\,dx$ LET $u = 1-x^2$

$du = -2x\,dx$

$\displaystyle = \left(-\frac{1}{2}\right)\int \sqrt{1-x^2}\,(-2)\,x\,dx = -\frac{1}{2}\int\sqrt{u}\,du = -\frac{1}{2}\int u^{1/2}\,du = -\frac{1}{2}\left(\frac{2}{3}u^{3/2}\right) + C$

$\displaystyle = -\frac{1}{3}(1-x^2)^{3/2} + C$ THEN $f(0) = -\frac{1}{3}\cdot 1 + C = \frac{4}{3}$, SO $C = \frac{5}{3}$

$\displaystyle\boxed{f(x) = -\frac{1}{3}(1-x^2)^{3/2} + \frac{5}{3}}$

(55) $\displaystyle\frac{dh}{dt} = \frac{17.6t}{\sqrt{17.6t^2+1}}$, $h(0) = 6$

a) $\displaystyle h(t) = \int \frac{17.6t}{\sqrt{17.6t^2+1}}\,dt$ LET $u = 17.6t^2+1$

$du = 2(17.6t)\,dt$

$\displaystyle = \frac{1}{2}\int \frac{1}{\sqrt{17.6t^2+1}}\cdot 2(17.6)t\,dt = \frac{1}{2}\int\frac{1}{\sqrt{u}}\,du = \frac{1}{2}\int u^{-1/2}\,du = \frac{1}{\cancel{2}}\left(\cancel{2}u^{1/2}\right) + C$

$= (17.6t^2+1)^{1/2} + C$

SINCE $h(0) = 1 + C = 6$, $C = 5$ AND

$\boxed{h(t) = (17.6t^2+1)^{1/2} + 5}$ (INCHES)

b) AFTER 5 YEARS,

$h(5) = \sqrt{17.6(25)+1} + 5 = \sqrt{441} + 5 = 21 + 5 = \boxed{26 \text{ IN}}$

(17) $\displaystyle\int \frac{2}{3x+5}\,dx$ LET $u = 3x+5$, $du = 3\,dx$

$$= \frac{2}{3}\int \frac{3}{3x+5}\,dx = \boxed{\frac{2}{3}\ln|3x+5| + C}$$

(19) $\displaystyle\int \frac{x}{x^2+1}\,dx$ LET $u = x^2+1$, $du = 2x\,dx$

$$= \frac{1}{2}\int \frac{2x}{x^2+1}\,dx = \boxed{\frac{1}{2}\ln|x^2+1| + C} = \boxed{\frac{1}{2}\ln(x^2+1) + C}$$

(CAN DROP THE ABSOLUTE VALUES, SINCE $x^2+1 > 0$ FOR ALL x)

(21) $\displaystyle\int \frac{x^2}{x^3+1}\,dx$ LET $u = x^3+1$, $du = 3x^2\,dx$

$$= \frac{1}{3}\int \frac{3x^2}{x^3+1}\,dx = \boxed{\frac{1}{3}\ln|x^3+1| + C}$$

(23) $\displaystyle\int \frac{x+3}{x^2+6x+7}\,dx$ LET $u = x^2+6x+7$, $du = (2x+6)\,dx = 2(x+3)\,dx$

$$= \frac{1}{2}\int \frac{2x+6}{x^2+6x+7}\,dx = \boxed{\frac{1}{2}\ln|x^2+6x+7| + C}$$

(25) $\displaystyle\int \frac{1}{x\ln x}\,dx$ LET $u = \ln x$, $du = \frac{1}{x}\,dx$

$$= \int \frac{1}{\ln x}\cdot\frac{1}{x}\,dx = \int \frac{1}{u}\,du = \ln|u| + C = \boxed{\ln|\ln x| + C}$$

(37) $\displaystyle\int \frac{4e^{2x}}{5-e^{2x}}\,dx$ LET $u = 5-e^{2x}$, $du = -2e^{2x}\,dx$

$$= \frac{4}{-2}\int \frac{-2e^{2x}}{5-e^{2x}}\,dx = \boxed{-2\ln|5-e^{2x}| + C}$$

(47) $$\int \frac{x^3-8x}{2x^2}\,dx = \int\left(\frac{x^3}{2x^2} - \frac{8x}{2x^2}\right)dx = \int\left(\frac{x}{2} - \frac{4}{x}\right)dx$$

$$= \boxed{\frac{x^2}{4} - 4\ln|x| + C}$$