MATH 16B SEC. 8.5 AND 5.3 – SOLUTIONS

8.5 – (16) $\int \sqrt{\cot x}\,\csc^2 x\,dx$ LET $u = \cot x$, $du = -\csc^2 x\,dx$

$= \ominus \int \sqrt{\cot x}\,(\ominus \csc^2 x)\,dx = -\int \sqrt{u}\,du = -\int u^{1/2}\,du = -\frac{2}{3}u^{3/2} + C = \boxed{-\frac{2}{3}(\cot x)^{3/2} + C}$

(22) $\int \frac{\sin x}{\cos^2 x}\,dx = \int \frac{1}{\cos x}\cdot\frac{\sin x}{\cos x}\,dx = \int \sec x \tan x\,dx = \boxed{\sec x + C}$

OR $\int \frac{\sin x}{\cos^2 x}\,dx$ LET $u = \cos x$, $du = -\sin x\,dx$

$= \ominus \int \frac{1}{\cos^2 x}(\ominus \sin x)\,dx = -\int \frac{1}{u^2}\,du = -\int u^{-2}\,du = -\left(-\frac{1}{u}\right) + C = \frac{1}{\cos x} + C = \boxed{\sec x + C}$

(27) $\int \frac{\csc^2 x}{\cot^3 x}\,dx$ LET $u = \cot x$, $du = -\csc^2 x\,dx$

$= \ominus \int \frac{1}{\cot^3 x}(\ominus \csc^2 x)\,dx = -\int \frac{1}{u^3}\,du = -\int u^{-3}\,du = -\left(\frac{u^{-2}}{-2}\right) + C = \frac{1}{2}(\cot x)^{-2} + C$

$= \frac{1}{2}\cdot\frac{1}{(\cot x)^2} + C = \boxed{\frac{1}{2}(\tan x)^2 + C}$

OR $\int \frac{\csc^2 x}{\cot^3 x}\,dx = \int \frac{\frac{1}{\sin^2 x}}{\frac{\cos^3 x}{\sin^3 x}}\,dx = \int \frac{1}{\sin^2 x}\cdot\frac{\sin^3 x}{\cos^3 x}\,dx = \int \frac{\sin x}{\cos^3 x}\,dx$

$= \int \frac{\sin x}{\cos x}\cdot\frac{1}{\cos^2 x}\,dx = \int \tan x \sec^2 x\,dx$ LET $u = \tan x$, $du = \sec^2 x\,dx$

$= \int u\,du = \frac{1}{2}u^2 + C = \boxed{\frac{1}{2}\tan^2 x + C}$ (OR SUB. $u = \cos x$, $du = -\sin x\,dx$ HERE)

5.3 – (51) $\int \frac{x^2 + 2x + 5}{x - 1}\,dx$

$$\begin{array}{r|l} & x + 3 \\ x - 1 & x^2 + 2x + 5 \\ & x^2 - x \\ & 3x + 5 \\ & 3x - 3 \\ & 8 \end{array}$$

$= \int \left(x + 3 + \frac{8}{x - 1}\right)dx$

$= \frac{x^2}{2} + 3x + 8\int \frac{1}{x - 1}\,dx$ $u = x - 1$, $du = dx$

$= \boxed{\frac{x^2}{2} + 3x + 8\ln|x - 1| + C}$

(OR SUB. $u = x - 1$, $x = u + 1$, $dx = du$

TO GET

$\int \frac{(u+1)^2 + 2(u+1) + 5}{u}\,du$

$= \int \frac{u^2 + 4u + 8}{u}\,du = \int \left(u + 4 + \frac{8}{u}\right)du$)

(53) $\int \frac{1 + e^{-x}}{1 + xe^{-x}}\,dx = \int \frac{1 + e^{-x}}{1 + xe^{-x}}\cdot\frac{e^x}{e^x}\,dx = \int \frac{e^x + 1}{e^x + x}\,dx$ LET $u = e^x + x$, $du = (e^x + 1)\,dx$

$= \boxed{\ln|e^x + x| + C}$

(57a) $P = \int \frac{3000}{1 + .25t}\,dt$ LET $u = 1 + .25t$, $du = \frac{1}{4}\,dt$

$= 3000\cdot 4 \int \frac{1/4}{1 + .25t}\,dt = 12{,}000 \ln(1 + .25t) + C$

$P(0) = 12{,}000 \ln 1 + C = 0 + C = C = 1{,}000$ (SINCE $\ln 1 = 0$),

SO $\boxed{P(t) = 12{,}000 \ln(1 + .25t) + 1{,}000 = 1{,}000\,[12 \ln(1 + .25t) + 1]}$

MATH 16B — SOLUTIONS – SEC. 8.5 AND 5.3

8.5 - (15) $\int \tan^3 x \sec^2 x \, dx$ LET $u = \tan x$, $du = \sec^2 x \, dx$

$$= \int u^3 \, du = \frac{1}{4} u^4 + C = \boxed{\frac{1}{4}\tan^4 x + C}$$

(23) $$\int \frac{\sec x \tan x}{\sec x - 1}\, dx = \boxed{\ln|\sec x - 1| + C}$$ (LET $u = \sec x - 1$, $du = \sec x \tan x \, dx$)

(45) $$\int \frac{\sin x}{1 + \cos x}\, dx = -\int \frac{-\sin x}{1+\cos x}\, dx$$ (LET $u = 1 + \cos x$, $du = -\sin x \, dx$)

$$= \boxed{-\ln|1 + \cos x| + C} = \boxed{-\ln(1 + \cos x) + C}$$ (SINCE $1 + \cos x \geq 0$ FOR ALL x)

5.3 - (5) $$\int 9x e^{-x^2} dx = \frac{9}{-2}\int e^{-x^2}(-2)x \, dx = -\frac{9}{2}\int e^u \, du = -\frac{9}{2} e^u + C = \boxed{-\frac{9}{2} e^{-x^2} + C}$$

LET $u = -x^2$, $du = -2x \, dx$

(7) $$\int 5x^2 e^{x^3} dx = \frac{5}{3}\int e^{x^3} \cdot 3x^2 \, dx = \frac{5}{3}\int e^u \, du = \frac{5}{3} e^u + C = \boxed{\frac{5}{3} e^{x^3} + C}$$

LET $u = x^3$, $du = 3x^2 \, dx$

(29) $\int \frac{1}{x^2} e^{2/x} \, dx$ LET $u = \frac{2}{x}$, $du = -\frac{2}{x^2}\, dx$

$$= -\frac{1}{2}\int e^{2/x}\left(-\frac{2}{x^2}\right) dx = -\frac{1}{2}\int e^u \, du = -\frac{1}{2} e^u + C = \boxed{-\frac{1}{2} e^{2/x} + C}$$

(31) $\int \frac{1}{\sqrt{x}} e^{\sqrt{x}} \, dx$ LET $u = \sqrt{x}$, $du = \frac{1}{2\sqrt{x}}\, dx$

$$= 2\int e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}\, dx = 2\int e^u \, du = 2e^u + C = \boxed{2e^{\sqrt{x}} + C}$$

(33) $$\int (e^x - 2)^2 dx = \int (e^{2x} - 4e^x + 4)\, dx = \boxed{\frac{1}{2} e^{2x} - 4e^x + 4x + C}$$

REMARK NOTICE THAT $(e^x)^2 = e^{2x}$.

(35) $$\int \frac{e^{-x}}{1 + e^{-x}}\, dx = -\int \frac{-e^{-x}}{1 + e^{-x}}\, dx$$ (LET $u = 1 + e^{-x}$, $du = -e^{-x}\, dx$)

$$= \boxed{-\ln|1 + e^{-x}| + C} = \boxed{-\ln(1 + e^{-x}) + C}$$ (SINCE $1 + e^{-x} > 0$ FOR ALL x)

(39) $$\int \frac{e^{2x} + 2e^x + 1}{e^x}\, dx = \int \left(e^x + 2 + \frac{1}{e^x}\right) dx = \int (e^x + 2 + e^{-x})\, dx$$

$$= \boxed{e^x + 2x - e^{-x} + C}$$

(41) $\int e^x \sqrt{1 - e^x}\, dx$ LET $u = 1 - e^x$, $du = -e^x \, dx$

$$= -\int \sqrt{1 - e^x}\,(-e^x)\, dx = -\int \sqrt{u}\, du = -\int u^{1/2} du = -\frac{2}{3} u^{3/2} + C = \boxed{-\frac{2}{3}(1 - e^x)^{3/2} + C}$$

MATH 16B

Sec. 5.3 – SOLUTIONS

(34) $\int (e^x - e^{-x})^2\,dx = \int (e^{2x} - 2 + e^{-2x})\,dx = \boxed{\frac{1}{2}e^{2x} - 2x - \frac{1}{2}e^{-2x} + C}$

(42) $\displaystyle\int \frac{2(e^x - e^{-x})}{(e^x + e^{-x})^2}\,dx$ LET $u = e^x + e^{-x}$, $du = (e^x - e^{-x})\,dx$

$$= 2\int \frac{1}{(e^x+e^{-x})^2}\cdot(e^x - e^{-x})\,dx = 2\int \frac{1}{u^2}\,du = 2\int u^{-2}\,du = 2(-u^{-1}) + C$$

$$= \boxed{-2\,(e^x + e^{-x})^{-1} + C}$$

(50) $\displaystyle\int \frac{3}{1+e^{-3x}}\,dx = \int \frac{3}{1+e^{-3x}}\cdot\frac{e^{3x}}{e^{3x}}\,dx = \int \frac{3e^{3x}}{e^{3x}+1}\,dx$ LET $u = e^{3x}+1$, $du = e^{3x}\cdot 3\,dx$

$$= \boxed{\ln|e^{3x}+1| + C} = \boxed{\ln(e^{3x}+1) + C}$$ (SINCE $e^{3x}+1 > 0$ FOR ALL x)

OR $\displaystyle\int \frac{3}{1+e^{-3x}}\,dx = 3\int \frac{1}{1+e^{-3x}}\,dx = 3\int \frac{1+e^{-3x}-e^{-3x}}{1+e^{-3x}}\,dx$

$$= 3\int\left(\frac{1+e^{-3x}}{1+e^{-3x}} - \frac{e^{-3x}}{1+e^{-3x}}\right)dx = 3\int\left(1 - \frac{e^{-3x}}{1+e^{-3x}}\right)dx = 3\left[x - \int \frac{e^{-3x}}{1+e^{-3x}}\,dx\right]$$

$$= 3\left[x - \left(-\frac{1}{3}\right)\int \frac{(-3)e^{-3x}}{1+e^{-3x}}\,dx\right]$$ LET $u = 1+e^{-3x}$, $du = e^{-3x}(-3)\,dx$

$$= \boxed{3x + \ln|1+e^{-3x}| + C} = \boxed{3x + \ln(1+e^{-3x}) + C}$$ (SINCE $1+e^{-3x} > 0$ FOR ALL x)

(56) $f'(x) = \dfrac{x^3 - 4x^2 + 3}{x-3}$; $(4,-1)$

$$\begin{array}{r|l} & x^2 - x - 3 \\ \hline x-3 & x^3 - 4x^2 \qquad + 3 \\ & x^3 - 3x^2 \\ \hline & -x^2 \\ & -x^2 + 3x \\ \hline & -3x + 3 \\ & -3x + 9 \\ \hline & -6 \end{array}$$

$$f(x) = \int \frac{x^3 - 4x^2 + 3}{x-3}\,dx$$

$$= \int \left(x^2 - x - 3 + \frac{-6}{x-3}\right)dx$$

$$= \frac{x^3}{3} - \frac{x^2}{2} - 3x - 6\int \frac{1}{x-3}\,dx \qquad u = x-3,\ du = dx$$

$$= \frac{x^3}{3} - \frac{x^2}{2} - 3x - 6\ln|x-3| + C$$

$$f(4) = \frac{64}{3} - 8 - 12 - 6\underbrace{\ln 1}_{0} + C = \frac{4}{3} + C = -1 \quad \text{so} \quad C = -\frac{7}{3}$$

$$f(x) = \boxed{\frac{x^3}{3} - \frac{x^2}{2} - 3x - 6\ln|x-3| - \frac{7}{3}}$$