MATH 17B SOLUTIONS – GOLD SHEET

⑥ $y' + y = 6e^x$, $(0,3)$

2) $\underline{u(x)} = e^{\int 1\,dx} = \underline{e^x}$

3) $e^x\left[y' + y\right] = e^x\left[6e^x\right]$ so $(e^x y)' = 6e^{2x}$

4) $e^x y = \int 6e^{2x}\,dx = 3e^{2x} + C$ so $\underline{y = 3e^x + Ce^{-x}}$

IF $x=0$, $y=3$ so $3 = 3 + C$ AND $\underline{C=0}$: $\boxed{y = 3e^x}$

⑦ $y' + 3x^2 y = 3x^2$, $(0,6)$

2) $\underline{u(x)} = e^{\int 3x^2\,dx} = \underline{e^{x^3}}$

3) $e^{x^3}\left[y' + 3x^2 y\right] = e^{x^3}\left[3x^2\right]$ so $(e^{x^3} y)' = e^{x^3}\cdot 3x^2$

4) $e^{x^3} y = \int e^{x^3}\cdot 3x^2\,dx = e^{x^3} + C$ so $\underline{y = 1 + Ce^{-x^3}}$

WHEN $x=0$, $y=6$ so $6 = 1 + C$ AND $\underline{C=5}$: $\boxed{y = 1 + 5e^{-x^3}}$

ER $\frac{dy}{dx} = 3x^2(1-y)$, so $\int \frac{1}{1-y}\,dy = \int 3x^2\,dx$, $-\int \frac{-1}{1-y}\,dy = x^3 + C$,

$-\ln(1-y) = x^3 + C$, $\ln(1-y) = -x^3 + D$, $1-y = e^{-x^3+D} = e^D e^{-x^3} = Ae^{-x^3}$,

$\underline{y = 1 - Ae^{-x^3}}$ WHEN $x=0$, $y=6$ so $6 = 1 - A$ AND $\underline{A=-5}$: $\boxed{y = 1 + 5e^{-x^3}}$

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⑧ $xy' - 2y = -x^2$, $(1,5)$

1) $y' - \frac{2}{x}y = -x$

2) $\underline{u(x)} = e^{\int -\frac{2}{x}\,dx} = e^{-2\ln x} = (e^{\ln x})^{-2} = \underline{x^{-2}}$

3) $x^{-2}\left[y' - \frac{2}{x}y\right] = x^{-2}\left[-x\right]$ so $(x^{-2}y)' = -x^{-1}$

4) $x^{-2}y = \int -x^{-1}\,dx = -\ln x + C$ so $\underline{y = -x^2\ln x + Cx^2}$

WHEN $x=1$, $y=5$ so $5 = 0 + C$ AND $\underline{C=5}$: $\boxed{y = -x^2\ln x + 5x^2}$

⑨ $x^2y' - 4xy = 10$, $(1,10)$

1) $y' - \frac{4}{x}y = \frac{10}{x^2}$

2) $\underline{u(x)} = e^{\int -\frac{4}{x}\,dx} = e^{-4\ln x} = (e^{\ln x})^{-4} = \underline{x^{-4}}$

3) $x^{-4}\left[y' - \frac{4}{x}y\right] = x^{-4}\left[\frac{10}{x^2}\right]$ so $(x^{-4}y)' = 10x^{-6}$

4) $x^{-4}y = \int 10x^{-6}\,dx = -2x^{-5} + C$ so $\underline{y = -2x^{-1} + Cx^4}$

WHEN $x=1$, $y=10$ so $10 = -2 + C$ so $\underline{C=12}$: $\boxed{y = -2x^{-1} + 12x^4}$

(10) $y' - 6xy = 36xe^{5x^2}, \quad (0,5)$

2) $\underline{u(x)} = e^{\int -6x\,dx} = \underline{e^{-3x^2}}$

3) $e^{-3x^2}[y' - 6xy] = e^{-3x^2}[36xe^{5x^2}] = 36xe^{2x^2}$ so $(e^{-3x^2}y)' = 36xe^{2x^2}$

4) $e^{-3x^2}y = \int 36xe^{2x^2}\,dx$ LET $\underline{u = 2x^2}$, $du = 4x\,dx$

$= 9\int e^{2x^2}\cdot 4x\,dx = 9\int e^u\,du = 9[e^u] + C = 9e^{2x^2} + C$

so $\underline{y = 9e^{5x^2} + Ce^{3x^2}}$ WHEN $x = 0$, $y = 5$ so $5 = 9 + C$ AND $\underline{C = -4}$:

$$\boxed{y = 9e^{5x^2} - 4e^{3x^2}}$$

(11) $y' = x\cos\frac{x}{2} + \frac{y}{x}, \quad (\pi, 6\pi)$

1) $y' - \frac{1}{x}y = x\cos\frac{x}{2}$

2) $\underline{u(x)} = e^{\int -\frac{1}{x}dx} = e^{-\ln x} = (e^{\ln x})^{-1} = \underline{x^{-1}}$

3) $x^{-1}[y' - \frac{1}{x}y] = x^{-1}[x\cos\frac{x}{2}]$, so $(x^{-1}y)' = \cos\frac{x}{2}$

4) $x^{-1}y = \int\cos\frac{x}{2}\,dx = 2\sin\frac{x}{2} + C$ so $\underline{y = 2x\sin\frac{x}{2} + Cx}$

WHEN $x = \pi$, $y = 6\pi$ so $6\pi = 2\pi\cdot 1 + C\pi$ AND $\underline{C = 4}$:

$$\boxed{y = 2x\sin\frac{x}{2} + 4x}$$

(12) $\underline{\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)}$

a) LET $N = \frac{1}{y}$, so $\frac{dN}{dt} = -\frac{1}{y^2}\cdot\frac{dy}{dt}$ AND $-\frac{1}{y^2}\cdot\frac{dy}{dt} = r\cdot\frac{1}{y}\left(1 - \frac{\frac{1}{y}}{K}\right)$,

$\underline{\frac{dy}{dt}} = -y^2\left[r\cdot\frac{1}{y}\left(1 - \frac{1}{yK}\right)\right] = -r\left(y - \frac{1}{K}\right) = \underline{-ry + \frac{r}{K}}$

b) 1) $\frac{dy}{dt} + ry = \frac{r}{K}$ 2) $\underline{u(t)} = e^{\int r\,dt} = \underline{e^{rt}}$

3) $e^{rt}\left[\frac{dy}{dt} + ry\right] = e^{rt}\left[\frac{r}{K}\right]$ so $(e^{rt}y)' = \frac{1}{K}e^{rt}\cdot r$

4) $e^{rt}y = \int\frac{1}{K}\cdot e^{rt}r\,dt = \frac{1}{K}\int e^{rt}\cdot r\,dt = \frac{1}{K}e^{rt} + C$

so $\underline{y = \frac{1}{K} + Ce^{-rt}}$

$\frac{1}{N} = \frac{1}{K} + Ce^{-rt} = \frac{1 + ae^{-rt}}{K}$

so $$\boxed{N(t) = \frac{K}{1 + ae^{-rt}}}$$