(15) $\int x\sec^2 x\,dx$ LET $u = x,\ dv = \sec^2 x\,dx$; $du = dx,\ v = \tan x$

$= x\tan x - \int \tan x\,dx = x\tan x - (-\ln|\cos x|) + C = \boxed{x\tan x + \ln|\cos x| + C} = \boxed{x\tan x - \ln|\sec x| + C}$

(17) $\int_0^{\pi/3} x\sin x\,dx$ LET $u = x,\ dv = \sin x\,dx$; $du = dx,\ v = -\cos x$

$= \left[-x\cos x - \int -\cos x\,dx\right]_0^{\pi/3} = \left[-x\cos x + \sin x\right]_0^{\pi/3} = \left(-\frac{\pi}{3}\cdot\frac{1}{2} + \frac{\sqrt{3}}{2}\right) - (0\cdot 1 + 0) = \boxed{\frac{\sqrt{3}}{2} - \frac{\pi}{6}} = \boxed{\frac{3\sqrt{3}-\pi}{6}}$

(19) $\int_1^2 \ln x\,dx$ LET $u = \ln x,\ dv = dx$; $du = \frac{1}{x}dx,\ v = x$

$= \left[x\ln x - \int 1\,dx\right]_1^2 = \left[x\ln x - x\right]_1^2 = (2\ln 2 - 2) - (1\cdot 0 - 1) = \boxed{2\ln 2 - 1} = \boxed{\ln 4 - 1}$

(22) $\int_1^4 \sqrt{x}\ln\sqrt{x}\,dx$ LET $t = \sqrt{x}$, $x = t^2$, $dx = 2t\,dt$ IF $x = 1,\ t = 1$; $x = 4,\ t = 2$

$= \int_1^2 t\ln t\cdot 2t\,dt = 2\int_1^2 t^2\ln t\,dt$ LET $u = \ln t,\ dv = t^2\,dt$; $du = \frac{1}{t}dt,\ v = \frac{t^3}{3}$

$= 2\left[\frac{t^3}{3}\ln t - \frac{1}{3}\int t^2\,dt\right]_1^2 = 2\left[\frac{t^3}{3}\ln t - \frac{1}{3}\cdot\frac{t^3}{3}\right]_1^2 = 2\left(\left(\frac{8}{3}\ln 2 - \frac{8}{9}\right) - \left(\frac{1}{3}\cdot 0 - \frac{1}{9}\right)\right) = \boxed{\frac{16}{3}\ln 2 - \frac{14}{9}}$

OR $\int_1^4 \sqrt{x}\ln\sqrt{x}\,dx = \int_1^4 \sqrt{x}\cdot\frac{1}{2}\ln x\,dx = \frac{1}{2}\int_1^4 \sqrt{x}\ln x\,dx$ LET $u = \ln x,\ dv = \sqrt{x}\,dx$; $du = \frac{1}{x}dx,\ v = \frac{2}{3}x^{3/2}$

$= \frac{1}{2}\left[\frac{2}{3}x^{3/2}\ln x - \frac{2}{3}\int x^{1/2}\,dx\right]_1^4 = \frac{1}{2}\left[\frac{2}{3}x^{3/2}\ln x - \frac{2}{3}\left(\frac{2}{3}x^{3/2}\right)\right]_1^4$

$= \left[\frac{1}{3}x^{3/2}\ln x - \frac{2}{9}x^{3/2}\right]_1^4 = \left(\frac{8}{3}\ln 4 - \frac{16}{9}\right) - \left(0 - \frac{2}{9}\right) = \boxed{\frac{8}{3}\ln 4 - \frac{14}{9}} = \boxed{\frac{2}{9}(12\ln 4 - 7)}$

(24) $\int_0^3 x^2 e^{-x}\,dx = \left[-x^2e^{-x} - 2xe^{-x} - 2e^{-x}\right]_0^3 \longleftarrow$

$= (-9e^{-3} - 6e^{-3} - 2e^{-3}) - (0 - 0 - 2\cdot 1)$

$= \boxed{2 - 17e^{-3}}$

u		dv
x^2	$+$	e^{-x}
$2x$	$-$	$-e^{-x}$
2	$+$	e^{-x}
0		$-e^{-x}$

(25) $\int_0^{\pi/3} e^x\sin x\,dx$

u		dv
e^x	$+$	$\sin x\,dx$
e^x	$-$	$-\cos x$
e^x	$+$	$-\sin x$

$\int e^x\sin x\,dx = -e^x\cos x + e^x\sin x - \int e^x\sin x\,dx$

$2\int e^x\sin x\,dx = -e^x\cos x + e^x\sin x + C$

$\int e^x\sin x\,dx = \frac{1}{2}\left[-e^x\cos x + e^x\sin x\right] + C$

so $\int_0^{\pi/3} e^x\sin x\,dx = \frac{1}{2}\left[-e^x\cos x + e^x\sin x\right]_0^{\pi/3} = \frac{1}{2}\left(\left(-e^{\pi/3}\cdot\frac{1}{2} + e^{\pi/3}\cdot\left(\frac{\sqrt{3}}{2}\right)\right) - (-1\cdot 1 + 1\cdot 0)\right)$

$= \boxed{\frac{e^{\pi/3}}{4}(\sqrt{3} - 1) + \frac{1}{2}}$

OR use

u		dv
$\sin x$	$+$	$e^x\,dx$
$\cos x$	$-$	e^x
$-\sin x$	$+$	e^x

(28) $\int e^{-2x} \sin \frac{x}{2}\, dx$

$$\begin{array}{ccc} u & & dv \\ e^{-2x} & + & \sin \frac{x}{2}\, dx \\ -2e^{-2x} & - & -2\cos\frac{x}{2} \\ 4e^{-2x} & + & -4\sin\frac{x}{2} \end{array}$$

(28) use

$$\begin{array}{ccc} u & & dv \\ \sin\frac{x}{2} & + & e^{-2x}dx \\ \frac{1}{2}\cos\frac{x}{2} & - & -\frac{1}{2}e^{-2x} \\ -\frac{1}{4}\sin\frac{x}{2} & + & \frac{1}{4}e^{-2x} \end{array}$$

$$\int e^{-2x}\sin\frac{x}{2}\,dx = -2e^{-2x}\cos\frac{x}{2} - 8e^{-2x}\sin\frac{x}{2} - 16\int e^{-2x}\sin\frac{x}{2}\,dx$$

$$17\int e^{-2x}\sin\frac{x}{2}\,dx = -2e^{-2x}\cos\frac{x}{2} - 8e^{-2x}\sin\frac{x}{2} + C$$

$$\int e^{-2x}\sin\frac{x}{2}\,dx = \boxed{-\frac{2}{17}e^{-2x}\cos\frac{x}{2} - \frac{8}{17}e^{-2x}\sin\frac{x}{2} + C} = \boxed{-\frac{2}{17}\left[e^{-2x}\cos\frac{x}{2} + 4e^{-2x}\sin\frac{x}{2}\right] + C}$$

(33) $\int \arcsin x\, dx$ Let $u = \arcsin x$, $dv = dx$

$du = \frac{1}{\sqrt{1-x^2}}dx$, $v = x$

$$= x\arcsin x - \int \frac{x}{\sqrt{1-x^2}}dx \qquad \text{Let } t = 1-x^2,\quad dt = -2x\,dx$$

$$= x\arcsin x - \left(-\frac{1}{2}\right)\int \frac{1}{\sqrt{1-x^2}}(-2)x\,dx = x\arcsin x + \frac{1}{2}\int\frac{1}{\sqrt{t}}dt = x\arcsin x + \frac{1}{2}\int t^{-1/2}dt$$

$$= x\arcsin x + \frac{1}{\cancel{2}}(\cancel{2}t^{1/2}) + C = \boxed{x\arcsin x + \sqrt{1-x^2} + C}$$

(38) a) $\int (\ln x)^n dx$ Let $u = (\ln x)^n$, $dv = dx$

$du = n(\ln x)^{n-1}\cdot\frac{1}{x}dx$, $v = x$

$$= x(\ln x)^n - n\int(\ln x)^{n-1}dx$$

b) Using part a),

$$\int(\ln x)^3dx = x(\ln x)^3 - 3\int(\ln x)^2dx$$
$$= x(\ln x)^3 - 3\left[x(\ln x)^2 - 2\int \ln x\,dx\right]$$
$$= \boxed{x(\ln x)^3 - 3x(\ln x)^2 + 6(x\ln x - x) + C}$$
$$= \boxed{x(\ln x)^3 - 3x(\ln x)^2 + 6x\ln x - 6x + C}$$

REMARK

We could also find this integral by letting $t = \ln x$, so $x = e^t$, $dx = e^t dt$, to get $\int(\ln x)^3dx = \int t^3\cdot e^t dt$ and then using tabular integration.

(39) $\int \cos\sqrt{x}\, dx$ Let $t = \sqrt{x}$, so $x = t^2$, $dx = 2t\,dt$

$$= \int \cos t\cdot 2t\,dt = 2\int t\cos t\,dt \qquad \text{Let } u = t,\ dv = \cos t\,dt$$

$du = dt$, $v = \sin t$

$$= 2\left[t\sin t - \int \sin t\,dt\right] = 2\left[t\sin t - (-\cos t)\right] + C = \boxed{2\left[\sqrt{x}\sin\sqrt{x} + \cos\sqrt{x}\right] + C}$$

(42) $\int x^5 e^{x^2}dx$ Let $t = x^2$, $dt = 2x\,dx$

$$= \frac{1}{2}\int x^4 e^{x^2}\cdot 2x\,dx = \frac{1}{2}\int t^2e^t\,dt$$

$$= \frac{1}{2}\left[t^2e^t - 2te^t + 2e^t\right] + C$$

$$= \boxed{\frac{1}{2}x^4e^{x^2} - x^2e^{x^2} + e^{x^2} + C}$$

$$\begin{array}{ccc} u & & dv \\ t^2 & + & e^t dt \\ 2t & - & e^t \\ 2 & + & e^t \\ 0 & & e^t \end{array}$$

MATH 17B

SOLUTIONS – SEC. 7.2

(49) $\int x e^{-2x}\,dx$ LET $u = x,\ dv = e^{-2x}\,dx$; $du = dx,\ v = -\frac{1}{2}e^{-2x}$

$= -\frac{1}{2}xe^{-2x} - \left(-\frac{1}{2}\right)\int e^{-2x}\,dx = \boxed{-\frac{1}{2}xe^{-2x} + \frac{1}{2}\left(-\frac{1}{2}e^{-2x}\right) + C} = \boxed{-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + C}$

OR use

	u		dV
	x	$+$	$e^{-2x}dx$
	1	$-$	$-\frac{1}{2}e^{-2x}$
	0		$\frac{1}{4}e^{-2x}$

(50) $\int x e^{-2x^2}\,dx$ LET $u = -2x^2$, $du = -4x\,dx$

$= -\frac{1}{4}\int e^{-2x^2}(-4x)\,dx = -\frac{1}{4}\int e^u\,du = -\frac{1}{4}e^u + C$

$= \boxed{-\frac{1}{4}e^{-2x^2} + C}$

(53) $\int 2x\sin(x^2)\,dx$ LET $u = x^2$, $du = 2x\,dx$

$= \int \sin(x^2)\cdot 2x\,dx = \int \sin u\,du = -\cos u + C = \boxed{-\cos(x^2) + C}$

(54) $\int 2x^2\sin x\,dx = 2\int x^2\sin x\,dx$

$= \boxed{2\left[-x^2\cos x + 2x\sin x + 2\cos x\right] + C}$

$= \boxed{-2x^2\cos x + 4x\sin x + 4\cos x + C}$

u		dV
x^2	$+$	$\sin x\,dx$
$2x$	$-$	$-\cos x$
2	$+$	$-\sin x$
0		$\cos x$

(65) $\int_1^4 e^{\sqrt{x}}\,dx$ LET $\tau = \sqrt{x}$, $x = \tau^2$, $dx = 2\tau\,d\tau$ IF $x = 1,\ \tau = 1$; $x = 4,\ \tau = 2$

$= \int_1^2 e^{\tau}\cdot 2\tau\,d\tau = 2\int_1^2 \tau e^{\tau}\,d\tau$ $u = \tau,\ dv = e^{\tau}d\tau$; $du = d\tau,\ v = e^{\tau}$

$= 2\left[\tau e^{\tau} - \int e^{\tau}\,d\tau\right]_1^2 = 2\left[\tau e^{\tau} - e^{\tau}\right]_1^2 = 2\left((2e^2 - e^2) - (e - e)\right) = \boxed{2e^2}$

(42) (ALTERNATE SOLUTION)

$\int x^5 e^{x^2}\,dx = \frac{1}{2}\int x^4\cdot e^{x^2}\cdot 2x\,dx$ LET $u = x^4,\ dv = e^{x^2}\cdot 2x\,dx$; $du = 4x^3\,dx,\ v = e^{x^2}$

$= \frac{1}{2}\left[x^4e^{x^2} - 4\int x^3e^{x^2}\,dx\right]$

$= \frac{1}{2}x^4e^{x^2} - 2\cdot\frac{1}{2}\int x^2\cdot e^{x^2}\cdot 2x\,dx$ LET $u = x^2,\ dV = e^{x^2}\cdot 2x\,dx$; $du = 2x\,dx,\ V = e^{x^2}$

$= \frac{1}{2}x^4e^{x^2} - \left[x^2e^{x^2} - \int e^{x^2}\cdot 2x\,dx\right]$

$= \boxed{\frac{1}{2}x^4e^{x^2} - x^2e^{x^2} + e^{x^2} + C}$