MATH 12 SOLUTIONS – SEC. 1.1, 1.3, 1.5

1.1 – 66 a) $|\sqrt{5}-5| = -(\sqrt{5}-5) = \boxed{-\sqrt{5}+5} = \boxed{5-\sqrt{5}}$ SINCE $\sqrt{5}-5<0$ (BECAUSE $\sqrt{5}<5$)

b) $|10-\pi| = \boxed{10-\pi}$ SINCE $10-\pi>0$ (BECAUSE $10>\pi$)

68 a) $|2-|-12|| = |2-12| = |-10| = \boxed{10}$

1.3 – 89 $x^{5/2} - x^{1/2} = x^{1/2}(x^2-1) = \boxed{x^{1/2}(x-1)(x+1)}$

113 $8x^3-125 = (2x)^3-5^3 = (2x-5)((2x)^2+(2x)(5)+5^2)$
$= \boxed{(2x-5)(4x^2+10x+25)}$

125 $5(x^2+4)^4(2x)(x-2)^4 + (x^2+4)^5(4)(x-2)^3$
$= 2(x^2+4)^4(x-2)^3\left[5x(x-2)+(x^2+4)(2)\right]$
$= 2(x^2+4)^4(x-2)^3\left[5x^2-10x+2x^2+8\right]$
$= \boxed{2(x^2+4)^4(x-2)^3\,(7x^2-10x+8)}$ ← (NOTICE THAT $b^2-4ac = 100-28(8)<0$, SO $7x^2-10x+8$ CANNOT BE FACTORED OVER THE REAL NUMBERS)

1.5 – 56 $x^2-4x+2=0$ $\quad x^2-4x+\underline{4} = -2+\underline{4}$
$(x-2)^2 = 2$ $\quad x-2 = \pm\sqrt{2}$ $\quad \boxed{x = 2\pm\sqrt{2}}$

57 $x^2-6x-11=0$ $\quad x^2-6x+\underline{9} = 11+\underline{9}$ $\quad (x-3)^2 = 20$
$x-3 = \pm\sqrt{20} = \pm 2\sqrt{5}$ $\quad \boxed{x = 3\pm 2\sqrt{5}}$

83 $4x^2+5x+\frac{13}{8} = 0$ $\quad D = b^2-4ac = 25-4(4)\left(\frac{13}{8}\right) = 25-26<0$,
SO THERE ARE $\boxed{\text{NO REAL SOLUTIONS}}$.

89 $\dfrac{x+5}{x-2} = \dfrac{5}{x+2} + \dfrac{28}{x^2-4}$ MULTIPLYING BY $x^2-4 = (x-2)(x+2)$ GIVES

$(x+5)(x+2) = 5(x-2)+28$, $\quad x^2+7x+10 = 5x-10+28$,
$x^2+2x-8=0$, $\quad (x+4)(x-2)=0$, $\quad \boxed{x=-4}$ OR $\cancel{x=2}$ ← (DOESN'T CHECK)

91 $\sqrt{2x+1}+1 = x$, $\quad \sqrt{2x+1} = x-1$, $\quad (\sqrt{2x+1})^2 = (x-1)^2$
$2x+1 = x^2-2x+1$, $\quad 0 = x^2-4x$, $\quad x(x-4)=0$, $\quad \cancel{x=0}$ OR $\boxed{x=4}$
(DOESN'T CHECK)

96 $x^4-5x^2+4=0$ $\quad (x^2-4)(x^2-1)=0$ $\quad (x-2)(x+2)(x-1)(x+1)=0$
$\boxed{x=\pm 2}$ OR $\boxed{x=\pm 1}$

(OR SUBSTITUTE $\underline{t=x^2}$ FIRST TO GET
$t^2-5t+4=0$, $\quad (t-4)(t-1)=0$, $\quad \underline{t=4}$ OR $\underline{t=1}$
SO $x^2=4$ OR $x^2=1$
$\boxed{x=\pm 2}$ OR $\boxed{x=\pm 1}$)

MATH 12 SOLUTIONS – SEC. 1.5, 1.8

1.5 – (104) $x - 5\sqrt{x} + 6 = 0$ $\quad (\sqrt{x})^2 - 5\sqrt{x} + 6 = 0$ $\quad (\sqrt{x} - 3)(\sqrt{x} - 2) = 0$ so

$\sqrt{x} = 3$ or $\sqrt{x} = 2$ and $\boxed{x = 9}$ or $\boxed{x = 4}$

(or substitute $T = \sqrt{x}$ first to get $T^2 - 5T + 6 = 0$)

(119) $\dfrac{\sqrt{d}}{4} + \dfrac{d}{1090} = 3$, so multiplying by $2(1090)$ gives

$545\sqrt{d} + 2d = 6540$ so $2d + 545\sqrt{d} - 6540 = 0$

Letting $T = \sqrt{d}$ gives $2T^2 + 545T - 6540 = 0$, so

(using +, since $T = \sqrt{d}$ must be positive)

$$T = \frac{-545 \pm \sqrt{(545)^2 - 4(2)(-6540)}}{2(2)} = \frac{-545 + \sqrt{349,345}}{4} \approx 11.5135$$

so $d = T^2 \approx \boxed{132.6 \text{ FT}}$

1.8 – (40) $A(6,-7)$, $B(11,-3)$, $C(2,-2)$

$|AB| = d(A,B) = \sqrt{(6-11)^2 + (-7-(-3))^2} = \sqrt{25+16} = \sqrt{41}$

$|AC| = d(A,C) = \sqrt{(6-2)^2 + (-7-(-2))^2} = \sqrt{16+25} = \sqrt{41}$

$|BC| = d(B,C) = \sqrt{(11-2)^2 + (-3-(-2))^2} = \sqrt{81+1} = \sqrt{82}$

Since $|AB|^2 + |AC|^2 = 41 + 41 = 82 = |BC|^2$, the triangle is a right triangle.

$A = \frac{1}{2}bh = \frac{1}{2}(\sqrt{41})(\sqrt{41}) = \boxed{\frac{41}{2}}$

(43) Let $P(0,y)$ be the point, and let $Q = (5,-5)$ and $R = (1,1)$.

Then $d(P,Q) = \sqrt{(0-5)^2 + (y-(-5))^2} = \sqrt{25 + y^2 + 10y + 25} = \sqrt{y^2 + 10y + 50}$

and $d(P,R) = \sqrt{(0-1)^2 + (y-1)^2} = \sqrt{1 + y^2 - 2y + 1} = \sqrt{y^2 - 2y + 2}$.

Then $\sqrt{y^2 + 10y + 50} = \sqrt{y^2 - 2y + 2}$ gives $y^2 + 10y + 50 = y^2 - 2y + 2$,

so $12y = -48$, $y = -4$, and $\boxed{P = (0,-4)}$

(46) $A = (2,3)$ and $M = (6,8)$, so let $B = (x,y)$.

Then $M = \left(\dfrac{x+2}{2}, \dfrac{y+3}{2}\right) = (6,8)$, so $\dfrac{x+2}{2} = 6$ and $\dfrac{y+3}{2} = 8$,

$x + 2 = 12$ and $y + 3 = 16$, $\boxed{x = 10 \text{ and } y = 13}$ so $B = \boxed{(10,13)}$

(96) Center: $(-1,5)$ passes through $(-4,-6)$

The circle has equation $(x+1)^2 + (y-5)^2 = r^2$

Since it passes through $(-4,-6)$, $(-4+1)^2 + (-6-5)^2 = r^2$ so

$9 + 121 = r^2$, $r^2 = 130$, $\boxed{(x+1)^2 + (y-5)^2 = 130}$

(or use $r = d((-1,5),(-4,-6)) = \sqrt{(-1-(-4))^2 + (5-(-6))^2} = \sqrt{9+121} = \sqrt{130}$)

MATH 12 SOLUTIONS - SEC. 1.8, 1.10

1.8 - (97) $P(-1,1)$ AND $Q(5,9)$ ARE ENDPOINTS OF A DIAMETER,

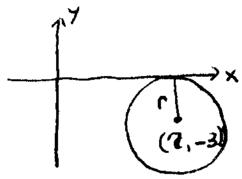
SO $C = M = \left(\frac{-1+5}{2}, \frac{1+9}{2}\right) = (2,5)$ AND THE CIRCLE HAS EQUATION

$(x-2)^2 + (y-5)^2 = r^2$ SUBSTITUTING THE COORDINATES OF Q GIVES

$(5-2)^2 + (9-5)^2 = r^2$, SO $r^2 = 9+16 = 25$ AND $\boxed{(x-2)^2 + (y-5)^2 = 25}$

(OR USE $r = d(C,P) = \sqrt{(2-(-1))^2 + (5-1)^2} = \sqrt{9+16} = 5$)

(99)



$(x-7)^2 + (y+3)^2 = r^2$ WHERE $r = 3$, SO

$\boxed{(x-7)^2 + (y+3)^2 = 9}$

(103) $x^2 + y^2 - 4x + 10y + 13 = 0$ $\quad (x^2 - 4x + \underline{4}) + (y^2 + 10y + \underline{25}) = -13 + \underline{4} + \underline{25}$

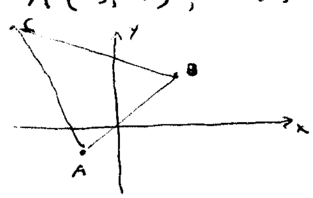
$(x-2)^2 + (y+5)^2 = 16$ $\quad$ CENTER: $\boxed{(2,-5)}$ $\quad$ RADIUS: $\boxed{4}$

(108) $3x^2 + 3y^2 + 6x - y = 0$ $\quad x^2 + y^2 + 2x - \frac{1}{3}y = 0$

$(x^2 + 2x + \underline{1}) + (y^2 - \frac{1}{3}y + \underline{\frac{1}{36}}) = \underline{1} + \frac{1}{36}$

$(x+1)^2 + (y - \frac{1}{6})^2 = \frac{37}{36}$ $\quad$ CENTER: $\boxed{(-1, \frac{1}{6})}$ $\quad$ RADIUS: $\boxed{\frac{\sqrt{37}}{6}}$

1.10 - (58) $A(-3,-1)$, $B(3,3)$, $C(-9,8)$



$m_{AB} = \frac{3-(-1)}{3-(-3)} = \frac{4}{6} = \frac{2}{3}$

$m_{AC} = \frac{8-(-1)}{-9-(-3)} = \frac{9}{-6} = -\frac{3}{2}$

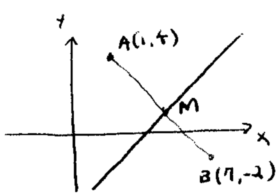
SINCE $m_{AB}\, m_{AC} = \left(\frac{2}{3}\right)\left(-\frac{3}{2}\right) = -1$, $\overline{AB}$ AND $\overline{AC}$ ARE PERPENDICULAR LINE SEGMENTS.

(60) a) $A(1,1)$, $B(3,9)$, $C(6,21)$

$m_{AB} = \frac{9-1}{3-1} = \frac{8}{2} = \underline{4}$ AND $m_{BC} = \frac{21-9}{6-3} = \frac{12}{3} = \underline{4}$,

SO THE POINTS ARE COLLINEAR.

(61)



$M = \left(\frac{1+7}{2}, \frac{4+(-2)}{2}\right) = (4,1)$ IS A POINT ON THE LINE.

$m_{AB} = \frac{4-(-2)}{1-7} = \frac{6}{-6} = -1$, SO THE LINE HAS SLOPE $m = \frac{-1}{-1} = 1$,

THEN $y - 1 = 1(x-4)$ GIVES $\boxed{y = x-3}$

(OR USE THAT (x,y) IS ON THE LINE IFF $d((x,y),(1,4)) = d((x,y),(7,-2))$,

SO $\sqrt{(x-1)^2 + (y-4)^2} = \sqrt{(x-7)^2 + (y-(-2))^2}$ AND THEREFORE

$\sqrt{x^2 - 2x + 1 + y^2 - 8y + 16} = \sqrt{x^2 - 14x + 49 + y^2 + 4y + 4}$)

(71) a) LET $A = (120,70)$ AND $B = (168,80)$. THEN $m_{AB} = \frac{80-70}{168-120} = \frac{10}{48} = \frac{5}{24}$,

SO $T - 70 = \frac{5}{24}(n-120)$ GIVES $T - 70 = \frac{5}{24}n - 25$

SO $\boxed{T = \frac{5}{24}n + 45}$

b) IF $n = 150$, $T = \frac{5}{24}(150) + 45 = \frac{250}{8} + 45 = \boxed{\frac{305}{4}}^{\circ} \approx \underline{76^{\circ}}$