1) Prove that $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n \cdot (n+1) = n(n+1)(n+2)/3$ for all $n$ in $\mathbb{N}$.

2) Prove that $(1+x)^n \geq 1+nx$ for every $n$ in $\mathbb{N}$ if $x > -1$. (*Bernoulli's inequality*)

3) Let $m$ be in $\mathbb{Z}$. Prove that if $m^3$ is even, then $m$ is even.

4) Prove that if $x$ is an irrational number and $r$ is a rational number, then $x + r$ is irrational.

5) Give a proof by contradiction that $\sqrt{5}$ is irrational.

6) Prove that if $a$, $b$, and $c$ are integers with $a^2 + b^2 = c^2$, then $a$ or $b$ is even.

7) Use the Well-Ordering Principle to show that every integer $n > 1$ has a prime factor.

8) Prove the following statement, or else show that it is false by giving a counterexample:
   If $a$ and $b$ are integers such that $ab$ is divisible by 6, then $a$ or $b$ is divisible by 6.

9) Give a proof by contradiction that $\sqrt{2} + \sqrt[3]{5}$ is irrational.