

③ a)  $Y = Ce^{kT}$   
 when  $T = 6$ ,  $Y = 2C$ ; so  $e^{6k} = 2$ ,  $e^{6k} = 2$ ,  $6k = \ln 2$ ,  $k = \frac{1}{6} \ln 2$

$$Y = C e^{(\frac{1}{6} \ln 2)T} = C (e^{\ln 2})^{T/6} = C (2)^{T/6}$$

when  $T = 30$ ,  $Y = 100,000$ :  $C (2)^5 = 100,000$ , so  $C = \frac{100,000}{2^5} = \frac{100,000}{32} = \boxed{3,125}$

⑪ a)  $Y(0) = 20,000$

b)  $Y = Ce^{kT} = 20,000 e^{kT}$

when  $T = 4$ ,  $Y = 31,000$ ; so  $20,000 e^{4k} = 31,000$ ,  $e^{4k} = \frac{31}{20}$ ,  $4k = \ln \frac{31}{20}$ ,

$k = \frac{1}{4} \ln \frac{31}{20}$  so  $Y = 20,000 e^{(\frac{1}{4} \ln \frac{31}{20})T} = 20,000 (e^{\ln \frac{31}{20}})^{T/4} = \boxed{20,000 \left(\frac{31}{20}\right)^{T/4}}$

⑬  $Y = Ce^{kT} = 8600 e^{kT}$

a) when  $T = 1$ ,  $Y = 10,000$ ; so  $8600 e^k = 10,000$ ,  $e^k = \frac{100}{86} = \frac{50}{43}$

so  $Y = 8600 (e^k)^T = \boxed{8600 \left(\frac{50}{43}\right)^T}$

b) when  $T = 2$ ,  $Y = 8600 \left(\frac{50}{43}\right)^2 \approx \boxed{11,628}$  BACTERIA

c) when  $Y = 2(8600)$ ,  $8600 \left(\frac{50}{43}\right)^T = 2(8600)$ ,  $\left(\frac{50}{43}\right)^T = 2$ ,  $\ln \left(\frac{50}{43}\right)^T = \ln 2$

$T \ln \frac{50}{43} = \ln 2$ ,  $T = \frac{\ln 2}{\ln \frac{50}{43}} \text{ HR} \approx \underline{4.6 \text{ HR}}$

⑰ b)  $Y = Ce^{kT} = 22 e^{kT}$

when  $T = 1600$ ,  $Y = \frac{1}{2}(22)$ ; so  $22 e^{1600k} = \frac{1}{2}(22)$ ,  $e^{1600k} = \frac{1}{2}$ ,

$\ln e^{1600k} = \ln \frac{1}{2}$ ,  $1600k = \ln \frac{1}{2}$ ,  $k = \frac{1}{1600} \ln \frac{1}{2}$

$Y = 22 e^{(\frac{1}{1600} \ln \frac{1}{2})T} = 22 (e^{\ln \frac{1}{2}})^{\frac{T}{1600}} = \boxed{22 \left(\frac{1}{2}\right)^{\frac{T}{1600}}}$

c) when  $T = 4000$ ,  $Y = 22 \left(\frac{1}{2}\right)^{\frac{4000}{1600}} = 22 \left(\frac{1}{2}\right)^{5/2} = \frac{22}{2^{5/2}} = \frac{11}{2^{3/2}} = \frac{11\sqrt{2}}{4} \text{ mg} \approx \underline{3.9 \text{ mg}}$

d) when  $Y = 18$ ,  $22 \left(\frac{1}{2}\right)^{\frac{T}{1600}} = 18$ ,  $\left(\frac{1}{2}\right)^{\frac{T}{1600}} = \frac{18}{22} = \frac{9}{11}$ ,

$\ln \left(\frac{1}{2}\right)^{\frac{T}{1600}} = \ln \frac{9}{11}$ ,  $\frac{T}{1600} \ln \frac{1}{2} = \ln \frac{9}{11}$ ,

$T = \frac{1600 \ln \frac{9}{11}}{\ln \frac{1}{2}} \text{ YR} \approx \underline{463 \text{ YR}}$

$$(19) \quad y = ce^{kT} = 50e^{kT}$$

when  $T = 28$ ,  $y = \frac{1}{2}(50)$ ; so  $50e^{28k} = \frac{1}{2}(50)$ ,  $e^{28k} = \frac{1}{2}$ ,  $\ln e^{28k} = \ln \frac{1}{2}$ ,

$$28k = \ln \frac{1}{2}, \quad k = \frac{1}{28} \ln \frac{1}{2} \quad \text{so}$$

$$y = 50 e^{\left(\frac{1}{28} \ln \frac{1}{2}\right)T} = 50 \left(e^{\ln \frac{1}{2}}\right)^{\frac{T}{28}} = 50 \left(\frac{1}{2}\right)^{\frac{T}{28}}$$

when  $y = 32$ ,  $50 \left(\frac{1}{2}\right)^{\frac{T}{28}} = 32$ ,  $\left(\frac{1}{2}\right)^{\frac{T}{28}} = \frac{32}{50} = \frac{16}{25}$ ,

$$\ln \left(\frac{1}{2}\right)^{\frac{T}{28}} = \ln \frac{16}{25}, \quad \frac{T}{28} \ln \frac{1}{2} = \ln \frac{16}{25}, \quad T = \frac{28 \ln \frac{16}{25}}{\ln \frac{1}{2}} \text{ YR} = \frac{56 \ln \frac{4}{5}}{\ln \frac{1}{2}} \text{ YR} \approx \underline{18 \text{ YR}}$$

(since  $\ln \frac{16}{25} = \ln \left(\frac{4}{5}\right)^2 = 2 \ln \frac{4}{5}$ )

$$(21) \quad y = ce^{kT} = 250e^{kT}$$

when  $T = 48$ ,  $y = 200$ ; so  $250e^{48k} = 200$ ,  $e^{48k} = \frac{200}{250} = \frac{4}{5}$ ,  $48k = \ln \frac{4}{5}$ ,

$$k = \frac{1}{48} \ln \frac{4}{5} \quad \text{and} \quad y = 250 e^{\left(\frac{1}{48} \ln \frac{4}{5}\right)T} = 250 \left(e^{\ln \frac{4}{5}}\right)^{\frac{T}{48}} = 250 \left(\frac{4}{5}\right)^{\frac{T}{48}}$$

when  $y = 125$ ,  $250 \left(\frac{4}{5}\right)^{\frac{T}{48}} = 125$ ,  $\left(\frac{4}{5}\right)^{\frac{T}{48}} = \frac{1}{2}$ ,  $\ln \left(\frac{4}{5}\right)^{\frac{T}{48}} = \ln \frac{1}{2}$

$$\frac{T}{48} \ln \left(\frac{4}{5}\right) = \ln \frac{1}{2}, \quad T = \frac{48 \ln \frac{1}{2}}{\ln \frac{4}{5}} \text{ HR} \approx \underline{149.1 \text{ HR}}$$

$$(23) \quad y = ce^{kT}$$

when  $T = 5730$ ,  $y = \frac{1}{2}c$ ; so  $c e^{5730k} = \frac{1}{2}c$ ,  $e^{5730k} = \frac{1}{2}$ ,

$$5730k = \ln \frac{1}{2}, \quad k = \frac{1}{5730} \ln \frac{1}{2} \quad \text{so}$$

$$y = c e^{\left(\frac{1}{5730} \ln \frac{1}{2}\right)T} = c \left(e^{\ln \frac{1}{2}}\right)^{\frac{T}{5730}} = c \left(\frac{1}{2}\right)^{\frac{T}{5730}}$$

when  $y = .65c$ ,  $c \left(\frac{1}{2}\right)^{\frac{T}{5730}} = .65c$ ,  $\left(\frac{1}{2}\right)^{\frac{T}{5730}} = .65$ ,

$$\ln \left(\frac{1}{2}\right)^{\frac{T}{5730}} = \ln .65, \quad \frac{T}{5730} \ln \frac{1}{2} = \ln .65,$$

$$T = \frac{5730 \ln .65}{\ln .5} \text{ YR} \approx \underline{3561 \text{ YR}}$$