List of important distributions – Probability Theory (235A), Fall 2011

Name	Notation	Formula		$\mathbf{E}(X)$	$\mathbf{V}(X)$	$\mathbf{E}(X^k)$
Discrete uniform	$X \sim U\{1,\dots,n\}$	$\mathbf{P}(X=k)=\frac{1}{n}$	$(1 \le k \le n)$	$\frac{n+1}{2}$	$\frac{n^2-1}{12}$	
Bernoulli	$X \sim \text{Bernoulli}(p)$	$\mathbf{P}(X=0)=1-p,\ \mathbf{P}(X=1)=p$		p	$p(1-p)$	p
Binomial	$X \sim \text{Binomial}(n,p)$	$\mathbf{P}(X=k)=\binom{n}{k}p^k(1-p)^{n-k}$	$(0 \le k \le n)$	np	$np(1-p)$	
Geometric (from 0)	$X \sim \text{Geom}_0(p)$	$\mathbf{P}(X=k)=p(1-p)^k$	$(k \ge 0)$	$\frac{1}{p}-1$	$\frac{1-p}{p^2}$	
Geometric (from 1)	$X \sim \text{Geom}(p)$	$\mathbf{P}(X=k)=p(1-p)^{k-1}$	$(k \ge 1)$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	
Poisson	$X \sim \text{Poisson}(\lambda)$	$\mathbf{P}(X=k)=e^{-\lambda}\frac{\lambda^k}{k!}$	$(k \ge 0)$	λ	λ	Bell numbers (for $\lambda=1$)
Negative binomial	$X \sim \text{NB}(m,p)$	$\mathbf{P}(X=k)=\binom{k+m-1}{m-1}p^m(1-p)^k$	$(k \ge 0)$	$\frac{m(1-p)}{p}$	$\frac{m(1-p)}{p^2}$	
Uniform	$X \sim U(a,b)$	$f_X(x)=\frac{1}{b-a}$	$(a<x<b)$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$\frac{b^{k+1}-a^{k+1}}{(k+1)(b-a)}$
Exponential	$X \sim \text{Exp}(\lambda)$	$f_X(x)=\lambda e^{-\lambda x}$	$(x>0)$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$\lambda^{-k}k!$
Standard normal	$X \sim N(0,1)$	$f_X(x)=\frac{1}{\sqrt{2\pi}}e^{-x^2/2}$	$(x \in \mathbb{R})$	0	1	$\begin{cases}\frac{k!}{(k/2)!2^{k/2}} & k \text{ even}\\ 0 & k \text{ odd}\end{cases}$
Normal	$X \sim N(\mu,\sigma^2)$	$f_X(x)=\frac{1}{\sqrt{2\pi}\sigma}e^{-(x-\mu)^2/2\sigma^2}$	$(x \in \mathbb{R})$	μ	σ^2	
Gamma	$X \sim \text{Gamma}(\alpha,\lambda)$	$f_X(x)=\frac{\lambda^\alpha}{\Gamma(\alpha)}e^{-\lambda x}x^{\alpha-1}$	$(x>0)$	$\frac{\alpha}{\lambda}$	$\frac{\alpha}{\lambda^2}$	$\lambda^{-k}\frac{\Gamma(\alpha+k)}{\Gamma(\alpha)}$
Cauchy	$X \sim \text{Cauchy}$	$f_X(x)=\frac{1}{\pi}\frac{1}{1+x^2}$	$(x \in \mathbb{R})$	N/A	N/A	N/A
Beta	$X \sim \text{Beta}(a,b)$	$f_X(x)=\frac{1}{B(a,b)}x^{a-1}(1-x)^{b-1}$	$(0<x<1)$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	$\frac{B(a+k,b)}{B(a,b)}$
Chi-squared	$X \sim \chi^2_{(n)}$	$f_X(x)=\frac{1}{2^{n/2}\Gamma(n/2)}e^{-x/2}x^{\frac{n}{2}-1}$	$(x>0)$	n	$2n$	

Useful facts: ("$*$" denotes convolution, i.e., sum of independent samples; "$=$" denotes equality of distributions)

$\text{Binomial}(n,p) * \text{Binomial}(m,p) = \text{Binomial}(n+m,p)$

$\text{Poisson}(\lambda) * \text{Poisson}(\mu) = \text{Poisson}(\lambda+\mu)$

$\text{Geom}_0(p) = \text{NB}(1,p)$

$\text{NB}(n,p) * \text{NB}(m,p) = \text{NB}(n+m,p)$

$N(0,1)^2 = \text{Gamma}(1/2,1/2) = \chi^2_{(1)}$

$\text{Gamma}(\alpha,\lambda) * \text{Gamma}(\beta,\lambda) = \text{Gamma}(\alpha+\beta,\lambda)$

$N(\mu_1,\sigma_1^2) * N(\mu_2,\sigma_2^2) = N(\mu_1+\mu_2,\sigma_1^2+\sigma_2^2)$

$\text{Exp}(\lambda) = \text{Gamma}(1,\lambda)$

$\big(\alpha\,\text{Cauchy}\big) * \big((1-\alpha)\,\text{Cauchy}\big) = \text{Cauchy} \quad (0 \le \alpha \le 1)$

$\chi^2_{(n)} = \text{Gamma}(n/2,1/2)$

The Euler gamma and beta functions

Definitions:

$$\Gamma(t) = \int_0^\infty e^{-x} x^{t-1}\, dx \qquad (t > 0)$$

$$B(a,b) = \int_0^1 x^{a-1}(1-x)^{b-1}\, dx \qquad (a, b > 0)$$

Functional equation:

$$\Gamma(t+1) = t\,\Gamma(t)$$

Special values:

$$\Gamma(n+1) = n! \quad (n = 0, 1, 2, \ldots)$$

$$\Gamma(1/2) = \sqrt{\pi}$$

$$B(n,m) = \tfrac{(n-1)!(m-1)!}{(n+m-1)!} \quad (n, m = 0, 1, 2, \ldots)$$

Relation between Γ and B:

$$B(a,b) = \tfrac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$