MAT 167: Applied Linear Algebra Lecture 22: Text Mining

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- 4 Latent Semantic Indexing

1 Introduction

2 Preprocessing the Documents and Queries

3 The Vector Space Model

4 Latent Semantic Indexing

- Text mining = Methods for extracting useful information from large and often unstructured collections of texts.
- It is also closely related to "information retrieval."
- In this context, keywords that carry information about the contents of a document are called terms.
- A list of all the terms in a document is called an index.
- For each term, a list of all the documents that contain that particular term is called an inverted index.
- A typical application is to search databases of scientific papers for given query terms.
- Because of Lecture 2 and HW #1, you should already be familiar with the concept of term-document matrix.
- Each column represents a document while each row represents a term.
- The *ij*th entry of such a matrix normally represents the frequency of occurrence of term *i* in document *j*.
- \bullet In reality, the size of such matrices are huge ($\gtrapprox 10^5 \times 10^6).$
- However, fortunately, most of the times, they are guite sparse.

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- This is a collection of 1500 (= n) articles (mostly in the field of machine learning and computational neuroscience) published in the proceedings of *Conference on Neural Information Processing Systems* (NIPS) over certain periods.
- The total number of terms (words) examined for these articles is $12419 \ (= m)$.
- More precisely, after tokenization (i.e., breaking a stream of text up into words, phrases, symbols, or other meaningful elements called tokens), and removal of stop words (i.e., common words that do not give useful info; more about these in the next section), the vocabulary of unique words was truncated by only keeping words that occurred more than ten times.

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- First 10 words sorted in the alphabetical order: 'a2i', 'aaa', 'aaai', 'aapo', 'aat', 'aazhang', 'abandonment', 'abbott', 'abbreviated', 'abcde'.
- 10 most frequently used words: 'network', 'model', 'learning', 'function', 'input', 'neural', 'set', 'algorithm', 'system', 'data'.

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- Before the index (a list of terms contained in a given document) is made, we need to do the following two preprocessing steps:
 - Elimination of stop words
 - ② Stemming
- Stop words are words that can be found in virtually any document (i.e., most likely useless words to characterize the documents), e.g., 'a', 'able', 'about', 'above', 'according', 'accordingly', 'across', 'actually', 'after', ...
- Stemming is the process of reducing each word that is conjugated or has a suffix to its stem. For example, 'fishing', 'fished', 'fish', 'fisher' stemming 'fish' (the root word).
- There are some public domain stemming software systems; see 'Stemming' page in Wikipedia.
- Note that stemming was not performed in the NIPS dataset, e.g., the terms include 'model', 'modeled', 'modeling', 'modelled', 'modelling'.

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- The main idea of this model is to create a term-document matrix, say, $A = (a_{ij}) \in \mathbb{R}^{m \times n}$, where each document is represented by a column vector \mathbf{a}_j that has nonzero entries in the position that correspond to terms found in that document.
- Consequently, each row represents a term and has nonzero entries in those positions that correspond to the documents where that term can be found, i.e., the inverted index.
- In practice, a text parser (a program) is used to create termdocument matrices, and does stemming and stop words removal too.
- The entry *a_{ij}* is normally set to the term frequency *f_{ij}*, i.e., the number of times term *i* appears in document *j*.
- Can have weights, e.g., a_{ij} = f_{ij} log(n/n_i) where n_i is the number of documents that contain term i. If term i occurs frequently in only a few documents, then the log factor becomes significant. On the other hand, if term i occurs many documents, the log factor makes a_{ij} ≈ 0, i.e., term i is not useful. Stop words removal mitigates this to some extent.

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Usually, the term-document matrix is sparse. For example, in the NIPS dataset, the number of nonzero entries in the term-document matrix of size 12419 \times 1500 is 746,316, which is only 4% of the whole matrix entries.

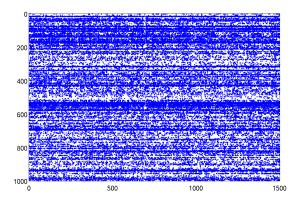


Figure: The first 1000 rows of the NIPS term-document matrix. Each dotrepresents a nonzero entry.

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Query Matching

- Query matching = a process of finding the relevant documents for a given query vector q ∈ ℝ^m.
- We must define a distance or similarity between **q** and each document $\mathbf{a}_j \in \mathbb{R}^m$, j = 1 : n.
- Often the following cosine distance (in fact, it would be better to say similarity rather than distance) is used:

$$\cos(heta(\mathbf{q},\mathbf{a}_j)) = rac{\mathbf{q}^{ extsf{T}}\mathbf{a}_j}{\|\mathbf{q}\|_2\|\mathbf{a}_j\|_2}$$

- If $\theta(\mathbf{q}, \mathbf{a}_j)$ is small enough, then \mathbf{a}_j is deemed relevant.
- More precisely, we set some predefined tolerance and if cos(θ(q, a_j)) > tol, then a_j is deemed relevant.
- The smaller the value of tol is, the more documents are retrieved and *considered* as relevant even if many of them are not really relevant.

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- The smaller the value of tol is, the more documents are retrieved and *considered* as relevant even if many of them are not really relevant.

- Query matching = a process of finding the relevant documents for a given query vector q ∈ ℝ^m.
- We must define a distance or similarity between **q** and each document $\mathbf{a}_j \in \mathbb{R}^m$, j = 1 : n.
- Often the following cosine distance (in fact, it would be better to say similarity rather than distance) is used:

$$\cos(heta(\mathbf{q},\mathbf{a}_j)) = rac{\mathbf{q}^{ op}\mathbf{a}_j}{\|\mathbf{q}\|_2\|\mathbf{a}_j\|_2}.$$

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A Query Matching Example

 Let's consider the NIPS dataset and set up q ∈ ℝ¹²⁴¹⁹ = e₃₅₂₈ + e₆₇₀₀ + e₆₉₃₂, i.e., only three nonzero entries that correspond to the three terms, 'entropy', 'minimum', 'maximum'.
Compute cos(θ(q, a_i)), j = 1 : 1500.

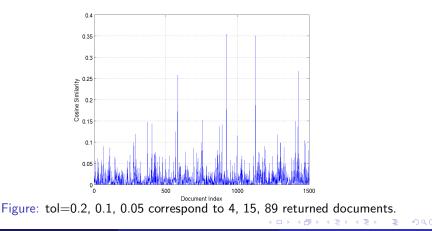
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• Compute $\cos(\theta(\mathbf{q}, \mathbf{a}_j)), j = 1: 1500.$



• Let us define the following quantities:

Precision:
$$P := \frac{D_r}{D_t};$$

Recall: $R := \frac{D_r}{N_r},$

- If we set tol large in the cosine similarity measure, then we expect to have high *P* but low *R*.
- On the other hand, if we set tol small, the situation is the other way around.
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Introduction

2 Preprocessing the Documents and Queries

3 The Vector Space Model

4 Latent Semantic Indexing

- Is an indexing and retrieval method that uses SVD to identify patterns in the relationships between the terms and documents.
- Is based on the principle that words that are used in the same contexts tend to have similar meanings.
- A key feature of LSI: its ability to extract the conceptual content of a body of text by establishing associations between those terms that occur in similar contexts.
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- Let $A \in \mathbb{R}^{m \times n}$ be a term-document matrix, and let $A_k := U_k \Sigma_k V_k^T$ be the rank k approximation of A using the first k singular values and singular vectors. Let $H_k := \Sigma_k V_k^T$, i.e., $A_k = U_k H_k$.
- For an appropriate value of k, A ≈ A_k. Hence, we have a_j ≈ U_kh_j where a_j and h_j are the jth column vectors of A and H_k, respectively.
- This means that **h**_j are the expansion coefficients of the best *k*-term approximation to **a**_j w.r.t. the ONB vectors {**u**₁,...,**u**_k}.
- Previously, for a given query vector **q**, in order to compute the cosine similarities between **q** and **a**_j, j = 1 : n, we had to compute **q**^TA followed by the normalization by ||**q**||₂ and ||**a**_j||.
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- Hence, we can simplify the cosine similarity computation as follows:

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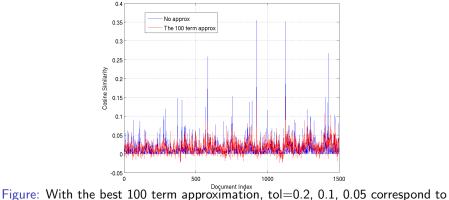


Figure: With the best 100 term approximation, to = 0.2, 0.1, 0.05 correspond to 0, 4, 72 returned documents; Compare with the no approximation case: 4, 15, 89.

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