MAT 167: Applied Linear Algebra
Lecture 23: Text Mining II

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Outline

1. Clustering
2. Nonnegative Matrix Factorization

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- Let $C_k = [\mathbf{c}_1 \ \ldots \ \mathbf{c}_k] \in \mathbb{R}^{m \times k}$ be the k cluster centroids obtained by the k-means algorithm.
- $\mathbf{c}_j$'s are non-orthogonal; hence it is more convenient to obtain a set of orthonormal vectors that spans range(C_k).
- To do so, we can use the reduced QR factorization: $C_k = Q_k R_k$ where $Q_k \in \mathbb{R}^{m \times k}$, and $R_k \in \mathbb{R}^{k \times k}$.
- Now, let's approximate A using Q_k in the sense of the least squares as:

$$\min_{G_k \in \mathbb{R}^{k \times k}} \|A - Q_k G_k\|_F.$$

- Let $G_k = [\mathbf{g}_1 \ \ldots \ \mathbf{g}_k]$. Then the above is equivalent to the following set of the LS problems:

$$\min_{\mathbf{g}_j \in \mathbb{R}^k} \|\mathbf{a}_j - Q_k \mathbf{g}_j\|_2, \ j = 1 : k.$$

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- Since the columns of Q_k are orthonormal, we can get the following LS solution: $\mathbf{g}_j = Q_k^{\mathsf{T}} \mathbf{a}_j$, $j = 1 : k$. Hence $G_k = Q_k^{\mathsf{T}} A$.
- The inner product between the query vector $\mathbf{q}$ and the document vector $\mathbf{a}_j$ can be approximated as:

$$\mathbf{q}^{\mathsf{T}} \mathbf{a}_j \approx \mathbf{q}^{\mathsf{T}} Q_k \mathbf{g}_j = (Q_k^{\mathsf{T}} \mathbf{q})^{\mathsf{T}} \mathbf{g}_j = \mathbf{q}_k^{\mathsf{T}} \mathbf{g}_j, \ \mathbf{q}_k := Q_k^{\mathsf{T}} \mathbf{q}.$$

- Hence, the cosine similarity can be approximated as:

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An Example Trial with the NIPS Data

- $k = 50$; the same query vector ('entropy', 'minimum', 'maximum').
- The approximation error between $Q_k G_k$ and A was $\|A - Q_k G_k\|_F / \|A\|_F \approx 0.7227$, which was worse than that using the top 100 SVD basis.

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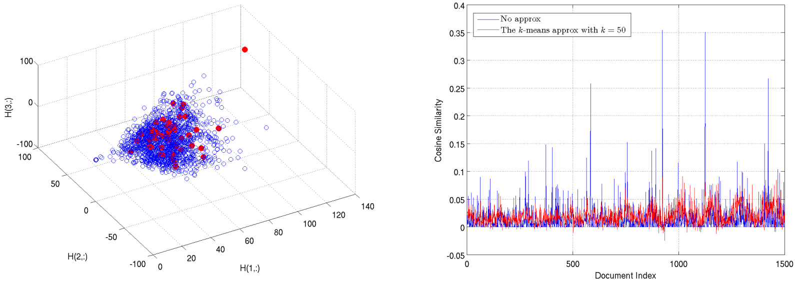

(a) Documents in $U_{100}(:, 1:3)$ (b) Cosine Similarity

Figure: With the 50-means based approximation, tol=0.2, 0.1, 0.05 correspond to 0, 0, 81 returned documents; Compare these with the no approximation case: 4, 15, 89; and with the best 100 approximation using SVD: 0, 4, 72.

My Reaction

- Running the k-means algorithm with large m and n is slow in general.
- If your document set really consists of k different topics (or categories), then this k-means-based approach should work well. Example: *The Science News Dataset* consisting of articles in the area of *Anthropology, Astronomy, Behavioral Sciences, Earth Sciences, Life Sciences, Math & CS, Medicine, Physics*. Which value of k should be used is still a question though.
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Using NNMF for Text Mining

- Consider the NNMF of a term-document matrix $A \approx WH$ where $W \in \mathbb{R}^{m \times k}$, $H \in \mathbb{R}^{k \times n}$, $1 < k \leq \min(m, n)$.
- We want to represent (or approximate) both query vectors and the term-document matrix using the basis vectors $\{\mathbf{w}_1, \ldots, \mathbf{w}_k\}$ and do query task in that basis (or coordinates).
- $\mathbf{a}_j$ is already approximated using $\{\mathbf{w}_1, \ldots, \mathbf{w}_k\}$ with the coordinate vector $\mathbf{h}_j$, $j = 1 : n$, i.e., $\mathbf{a}_j \approx W\mathbf{h}_j$.
- We need to approximate $\mathbf{q}$ in the basis of W. To do so, we seek the LS approximation of $\mathbf{q}$ in range(W), i.e., $\min_{\hat{\mathbf{q}} \in \mathbb{R}^k} \|\mathbf{q} - W\hat{\mathbf{q}}\|_2$.
- Hence we need to solve the normal equation: $W^{\mathsf{T}} W \hat{\mathbf{q}} = W^{\mathsf{T}} \mathbf{q}$.
- To do so, we use the reduced QR factorization of $W = \widehat{Q}\widehat{R}$.
- Then, using the argument of Lecture 10, the normal equation above is equivalent to $\widehat{R}\hat{\mathbf{q}} = \widehat{Q}^{\mathsf{T}}\mathbf{q}$, i.e., $\hat{\mathbf{q}} = \widehat{R}^{-1}\widehat{Q}^{\mathsf{T}}\mathbf{q}$.
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An Example Trial with the NIPS Data

- $k = 100$ was used.
- $\|A - WH\|_F / \|A\|_F \approx 0.6302$, which was *slightly* worse than that using the top 100 SVD basis (0.6074).
- Each $\mathbf{w}_j$ concentrates on one term, and is close to the canonical vector $\mathbf{e}_i \in \mathbb{R}^m$ for some i.
- The peaks of $\mathbf{w}_j$, $j = 1 : 10$, correspond to: 'network', 'model', 'learning', 'function', 'unit', 'algorithm', 'input', 'data', 'neuron', 'cell', which are quite similar to the $\mathbf{u}_1$ vector or the 10 most frequently used terms.
- On the other hand, because $\mathbf{w}_j$'s are localized, the interpretation of the row vectors of H matrix becomes easy.

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- The peaks of $\mathbf{w}_j$, $j = 1 : 10$, correspond to: 'network', 'model', 'learning', 'function', 'unit', 'algorithm', 'input', 'data', 'neuron', 'cell', which are quite similar to the $\mathbf{u}_1$ vector or the 10 most frequently used terms.
- On the other hand, because $\mathbf{w}_j$'s are localized, the interpretation of the row vectors of H matrix becomes easy.

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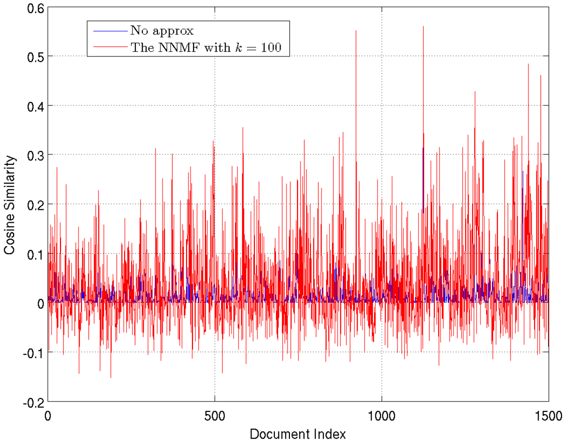

Figure: With the NNMF-based approach using $k = 100$, tol=0.2, 0.1, 0.05 correspond to 101, 312, 535 returned documents; Compare with the no approximation case: 4, 15, 89. Changing the tol=0.4, 0.3, 0.2 with NNMF returns 5, 26, 101 documents.

My Reaction

- Using the LS solution for the query saves computational cost given the NNMF is already obtained because one can avoid the explicit computation and storage of WH.
- If we can compute and store WH, then we could use the following approximation of the original cosine similarity:

$$\frac{\mathbf{q}^{\mathsf{T}}\mathbf{a}_j}{\|\mathbf{q}\|_2\|\mathbf{a}_j\|_2} \approx \frac{\mathbf{q}^{\mathsf{T}}W\mathbf{h}_j}{\|\mathbf{q}\|_2\|W\mathbf{h}_j\|_2}.$$

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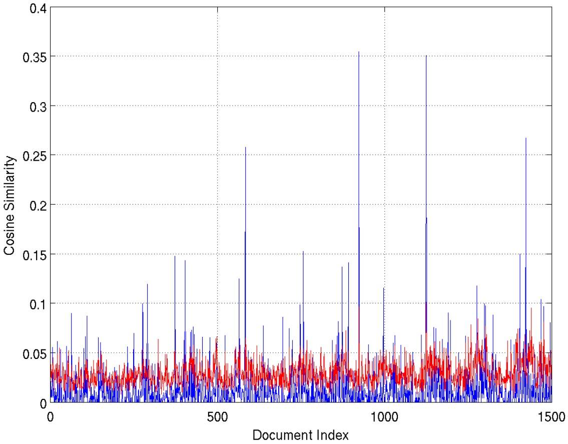

Figure: With the NNMF-based approach using $k = 100$ using the above cosine similarity approximation, tol=0.2, 0.1, 0.05 correspond to 0, 1, 97 returned documents; Compare with the no approximation case: 4, 15, 89. Without using the LS query, some of the relevant documents do not stick out clearly.