229A: Numerical Methods in Linear Algebra
Homework 2: due 2/7/02

Problem 1: Solve Exercise 5.1

Problem 2: Solve Exercise 5.3

Problem 3: Image compression experiments with SVD.

(a) Using `matlab`, read (or load) the image called "`mandrill.mat`", display it on the screen (using the gray scale colormap), and print it.

(b) Compute the SVD of this mandrill image and plot the distribution of its singular values. Print that plot.

(c) Let σ_j, $\boldsymbol{u}_j$, $\boldsymbol{v}_j$ be a singular value, the left and right singular vectors of the mandrill image, respectively. Let us define the rank k approximation of an image $X \in \mathbb{R}^{m \times n}$ as

$$X_k \triangleq \sum_{j=1}^{k} \sigma_j \boldsymbol{u}_j \boldsymbol{v}_j^* \quad k = 1, \ldots, \min(m, n).$$

Then, for $k = 1, 6, 11, 31$, compute X_k of the mandrill, and display the results. Fit these four images in one page by using `subplot` function and print them out.

(d) For $k = 1, 6, 11, 31$, display the residuals, i.e., $X - X_k$. Again print them out as one page.

Problem 4: Solve Exercise 6.1

Problem 5: Solve Exercise 6.4

Problem 6: Solve Exercise 7.1

Problem 7: Solve Exercise 7.4

Problem 8: Solve Exercise 8.2

Problem 9: Write a MATLAB function `[Q,R]=clgs(A)` that computes a reduced QR factorization $A = \hat{Q}\hat{R}$ of an $m \times n$ matrix A with $m \geq n$ using the Classical Gram-Schmidt (unstable) algorithm (Algorithm 7.1 in the textbook). The output variables are a matrix $\hat{Q} \in \mathbb{R}^{m \times n}$ with orthonormal columns and an upper triangular matrix $\hat{R} \in \mathbb{R}^{n \times n}$.

Problem 10: [The Pseudo Inverse Matrices] For any $A \in \mathbb{C}^{m \times n}$, there exists a unique matrix X (often written as $A^\dagger$) satisfying the following *Moore-Penrose conditions*:

1. $AXA = A$,
2. $XAX = X$,
3. $(AX)^* = AX$,
4. $(XA)^* = XA$.

$A^\dagger$ is referred to the *pseudo inverse* of A.

(a) Show that $AA^\dagger$ is an orthogonal projection onto $\mathrm{range}(A)$.

(b) Show that $A^\dagger A$ is an orthogonal projection onto $\mathrm{range}(A^*)$.