229A: Numerical Methods in Linear Algebra
Homework 4 (LAST ONE!): due 3/14/02

Problem 1: Solve Exercise 12.3

Problem 2: Solve Exercise 13.3 [For older print of the text, there is a typo. The range of x should be $1.920, 1.921, \ldots, 2.080$ intead of $-1.920, -1.919, \ldots, 2.080$. I think the newer print of the text corrected this typo already.]

Problem 3: Solve Exercise 14.1

Problem 4: Solve Exercise 15.1

Problem 5: Solve Exercise 16.2

Problem 6: [The Normal Error Model and Weighted Least Squares]

Suppose that you have the following system:

$$\boldsymbol{d} = G\boldsymbol{m} + \boldsymbol{\epsilon},$$

where $\boldsymbol{d} \in \mathbb{R}^m$ is a given data vector (measurement), $\boldsymbol{m} \in \mathbb{R}^n$ is a model you want to infer from the data, and $G \in \mathbb{R}^{m \times n}$ describes this measurement system with $m \geq n$. Suppose the measurement error $\boldsymbol{\epsilon} \in \mathbb{R}^m$ obeys a multivariate normal distribution with mean $\mathbf{0}$ and covariance matrix Σ, i.e., its probability density function (pdf) is written as:

$$p(\boldsymbol{\epsilon}) = \frac{1}{(2\pi)^{m/2}|\Sigma|^{1/2}} \exp\{-\frac{1}{2}\boldsymbol{\epsilon}^*\Sigma^{-1}\boldsymbol{\epsilon}\}.$$

(a) Using the maximum likelihood method, derive the optimal solution $\widehat{\boldsymbol{m}}_{\text{ML}}$. Recall that the maximum likelihood method seeks the maximizer of the likelihood function (= the pdf viewed as a function of the parameters $\boldsymbol{m}$ given data $\boldsymbol{d}$). In this case, the minimizer of the likelihood function is the same as that of the $\log$ of the likelihood function (called log-likelihood function).

(b) Determine the probability distribution of the solution $\widehat{\boldsymbol{m}}_{\text{ML}}$. [Hint: The answer is another multivariate normal distribution. What is the mean and covariance of $\widehat{\boldsymbol{m}}_{\text{ML}}$?]