MAT 280: Harmonic Analysis on Graphs & Networks
Lecture 15: Applications of Dimension Reduction Techniques to Signal Ensemble Classification

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Outline

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Acknowledgment

- Raphy Coifman (Yale)
- Quyen Hyunh (formerly Naval Surface Warfare Center)
- Yosi Keller (Bar-Ilan Univ., Israel)
- Linh Lieu (formerly UC Davis)
- Stéphane Lafon (Google)
- Bradley Marchand (UC Davis $\Longrightarrow$ NSWC, Panama City, FL)
- NSF
- ONR

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Signal Ensemble Classification Problems

- We want to classify *ensembles of signals*, not individual signals.
- Examples include: Underwater object classification using sonar waveforms; Classification of video clips, ...
- Let $X := \bigcup_{i=1}^{N} X^i \subset \mathbb{R}^d$ be a collection of N training ensembles. Each X^i consists of n_i individual signals, i.e., $X^i := \{\boldsymbol{x}_1^i, \ldots, \boldsymbol{x}_{n_i}^i\}$, and has a unique label among C possible labels. Let $n_\star := \sum_{i=1}^{N} n_i$. Let $Y := \bigcup_{j=1}^{M} Y^j \subset \mathbb{R}^d$ be a collection of test (i.e., unlabeled) ensembles where $Y^j := \{\boldsymbol{y}_1^j, \cdots, \boldsymbol{y}_{m_j}^j\}$. Our goal is to classify each Y^j to one of the possible C classes given the training ensembles X. This task is different from classifying each signal $\boldsymbol{y}_k^j \in Y$ individually.

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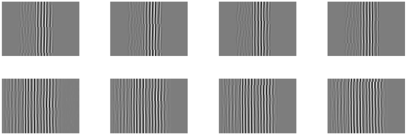
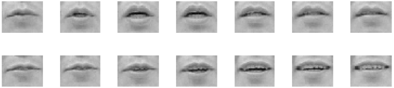


(a) Sonar Waveforms



(b) Video Clips of Digit Speaking Lips

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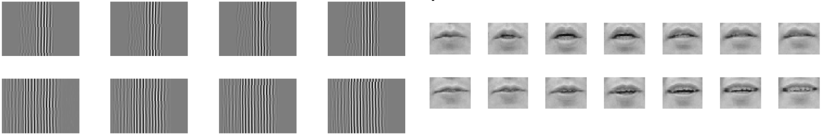

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Our Proposed Algorithm

- Training Stage (X is given)
  1. Preset a large enough initial dimension $1 \le s_0 \ll \min(d, n_\star)$.
  2. Construct a low-dimensional embedding map $\Psi : \mathbb{R}^d \to \mathbb{R}^{s_0}$.
  3. For $i = 1 : N$, construct a *signature* P^i using $\Psi(X^i)$.
  4. Determine the appropriate dimension $1 \le s \le s_0$ and re-adjust each signature P^i in Step 3.
- Test Stage (Now Y is fed)
  1. *Extend* the learned map Ψ to the test ensembles Y to embed them in $\mathbb{R}^s$.
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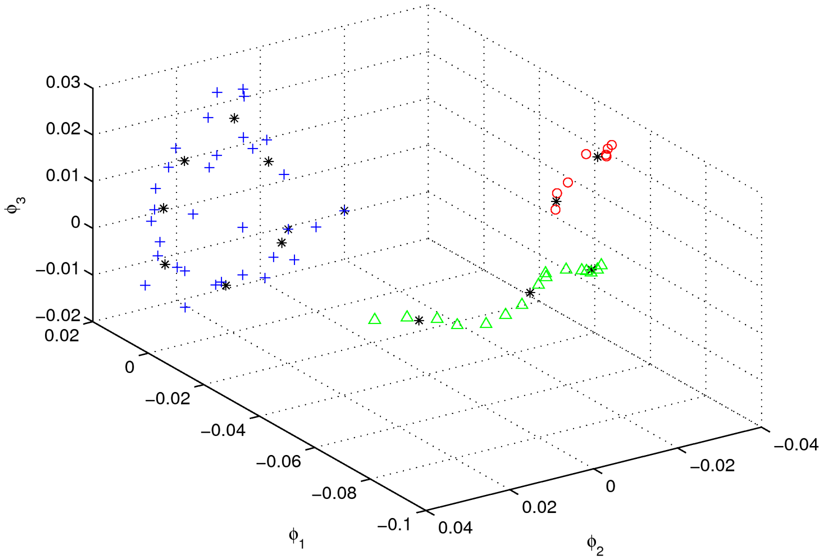
Sonar Waveform Signatures Embedded in $\mathbb{R}^3$



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Earth Mover's Distance (EMD)

- Originated from the *Monge-Kantorovich optimal transport problems*
- Used successfully in image retrieval from large databases, image registration and warping, etc.
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$$d_H(\Psi(X^i), \Psi(Y^j)) := \max\left(\max_{\boldsymbol{y}\in\Psi(Y^j)} \min_{\boldsymbol{x}\in\Psi(X^i)} \|\boldsymbol{x}-\boldsymbol{y}\|, \max_{\boldsymbol{x}\in\Psi(X^i)} \min_{\boldsymbol{y}\in\Psi(Y^j)} \|\boldsymbol{x}-\boldsymbol{y}\| \right).$$

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- Then, the *Earth Mover's Distance* (EMD) is defined by

$$\mathrm{EMD}(P,Q) := \frac{\sum_{i=1}^n \sum_{j=1}^m f_{ij} c_{ij}}{\sum_{i=1}^n \sum_{j=1}^m f_{ij}}.$$

- c_{ij} is the cost of moving one unit mass from the ith cluster in P to the jth cluster in Q. A typical example: $c_{ij} = \frac{1}{2}\|\boldsymbol{x}_i - \boldsymbol{y}_j\|^2$.
- $f_{ij} \geq 0$: the *optimal* flow between two distributions that minimizes the total cost $\sum_{i=1}^n \sum_{j=1}^m f_{ij} c_{ij}$, subject to the following constraints:
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- Linear programming (the *simplex* method) is used to compute EMDs.

- Let $P = \{(\boldsymbol{x}_1, p_1), \dots, (\boldsymbol{x}_n, p_n)\}$ and $Q = \{(\boldsymbol{y}_1, q_1), \dots, (\boldsymbol{y}_m, q_m)\}$ be two *signatures* characterizing two classes or objects of interest. $\boldsymbol{x}_i, \boldsymbol{y}_j \in \mathbb{R}^s$ are cluster centers and p_i, q_j are populations (or mass) of the corresponding clusters.
- Then, the *Earth Mover's Distance* (EMD) is defined by

$$\mathrm{EMD}(P, Q) := \frac{\sum_{i=1}^{n} \sum_{j=1}^{m} f_{ij} c_{ij}}{\sum_{i=1}^{n} \sum_{j=1}^{m} f_{ij}}.$$

- c_{ij} is the cost of moving one unit mass from the ith cluster in P to the jth cluster in Q. A typical example: $c_{ij} = \frac{1}{2} \|\boldsymbol{x}_i - \boldsymbol{y}_j\|^2$.
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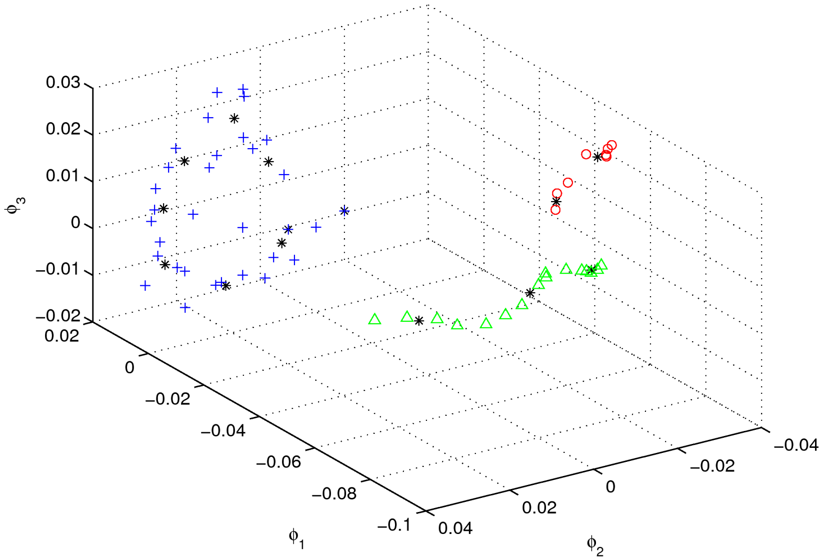
Signatures in the Reduced Embedding Space (again)



Outline

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Underwater Object Classification

- Sonar waveforms in the acoustic scattering experiments were collected in a fresh water pond at Naval Surface Warfare Center (NSWC), Panama City, FL.
- Three experiments on different days were performed. Each time, there were two objects in the pond.
  1. C1: Buried Al cylinder; S1: Fe Sphere filled with air
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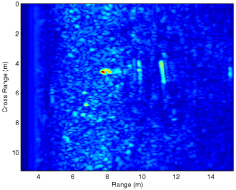

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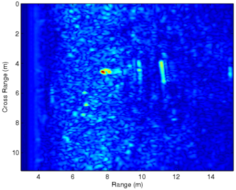

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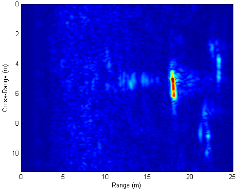

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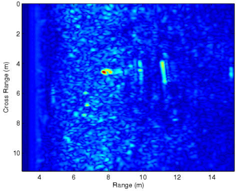

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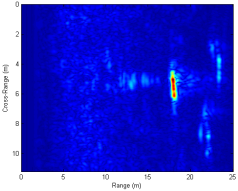
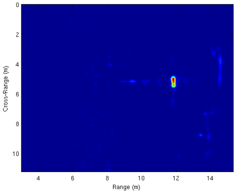


(b) Proud



(c) Short Proud

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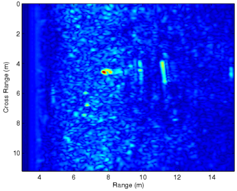

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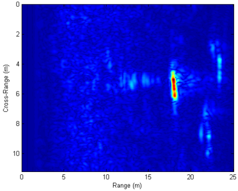
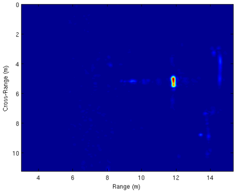


(b) Proud



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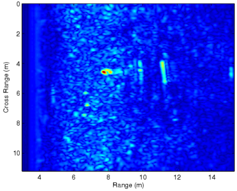

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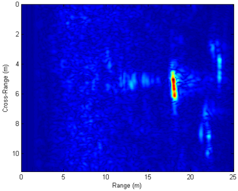

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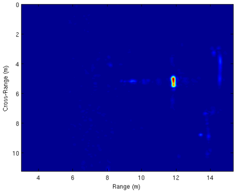

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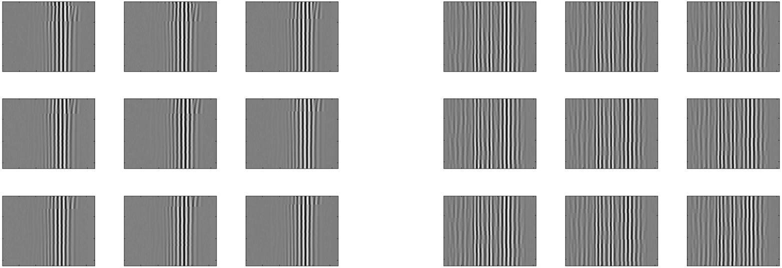
Underwater Object Classification . . .

- Our objective is to classify objects according to their *material compositions independent of shapes, sizes, buried or proud*.
- Each data point is in $\mathbb{R}^{17\times 600}$; The number of data points in C1, C2, C3, S1, S2, S3 are 8, 8, 16, 32, 32, 32, respectively.
- Pick one of these 6 ensembles as a test ensemble $Y = Y^1$ whereas the other 5 ensembles are used as training ensembles $X = \bigcup_{i=1}^{5} X^i$. Then classify Y.
- Repeat this process 5 more times.



(a) C3 waveforms

(b) S3 waveforms

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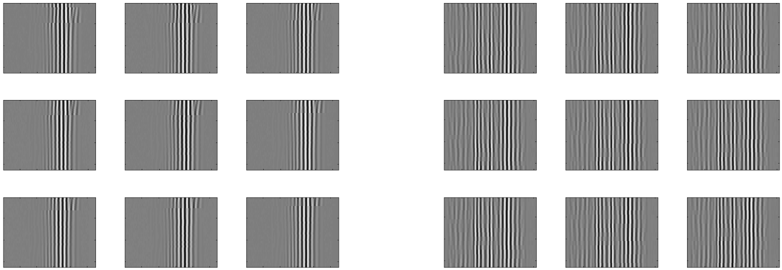

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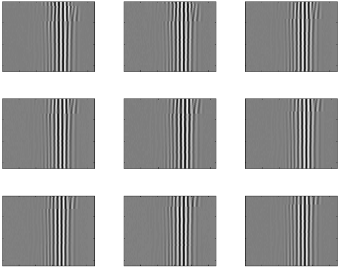
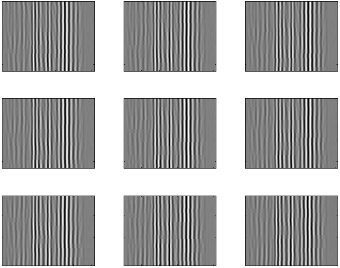


(a) C3 waveforms



(b) S3 waveforms

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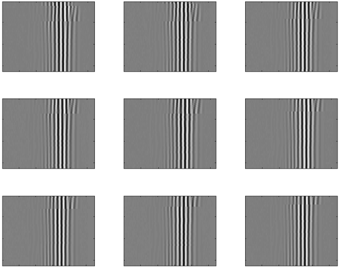

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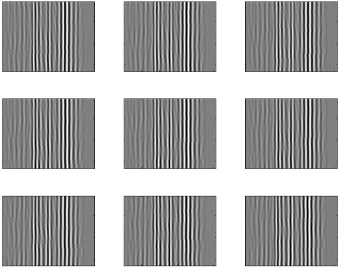

(b) S3 waveforms

Underwater Object Classification: Results

Object		$C1$	$C2$	$C3$	$S1$	$S2$	$S3$
True Label		**Al**	**Al**	**Al**	**IA**	**IS**	**IS**
PCA	EMD	**Al**	**Al**	**Al**	**IS**	**IS**	**IA**
	HD	**Al**	**Al**	**Al**	**IS**	**IS**	**IA**
LE_{rw}	EMD	**Al**	**Al**	**Al**	**Al**	**IS**	**IS**
	HD	**Al**	**Al**	**Al**	**Al**	**Al**	**IS**
LE_{sym}	EMD	**Al**	**Al**	**Al**	**Al**	**IS**	**IS**
	HD	**Al**	**Al**	**Al**	**Al**	**IS**	**IS**
DM	EMD	**Al**	**Al**	**Al**	**Al**	**IS**	**IS**
	HD	**Al**	**Al**	**Al**	**Al**	**IS**	**IS**

Al = Aluminum; IA = Iron-Air; IS = Iron-Silicone Oil

Underwater Object Classification: EMD vs HD

EMD and HD values in the $\mathrm{LE}_{\mathrm{rw}}$ coordinates between $S2$ and all other objects

Object	$C1$	$C2$	$C3$	$S1$	$S3$
EMD	0.0070	0.0064	0.0057	0.0085	0.0053
HD	0.1917	0.2374	0.1237	0.1500	0.1684

Outline

Video Clip Classification: Lip Reading

- Lips speaking five digits, 'one', . . . , 'five' were captured by a camcorder with the rate 60 frames/second.
- Each video frame is cropped to have 55×70 pixels.
- A single speaker spoke each digit 10 times (i.e., totally 50 video clips).
- Each video clip consists of $30 \sim 63$ video frames.
- Split the whole data randomly into the training and test ensembles as $X = \bigcup_{i=1}^{25} X^i$, $Y = \bigcup_{j=1}^{25} Y^j$. Then, do the classification.
- Repeat this process 99 times more.

Lip-Reading total recognition errors (averaged over 100 trials)

PCA EMD	PCA HD	$\mathrm{LE_{rw}}$ EMD	$\mathrm{LE_{rw}}$ HD	$\mathrm{LE_{sym}}$ EMD	$\mathrm{LE_{sym}}$ HD	DM EMD	DM HD
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- Global (PCA) vs Local (LE/DM): Lip-reading video clips involve more *global trajectories* while sonar waveforms involve more *localized clusters*.
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Outline

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The following articles are available at
http://www.math.ucdavis.edu/~saito/publications/

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