Reader's guide

Depending on the setup in which the main questions of Gabor analysis are formulated one may see Gabor analysis as a particular chapter (or circle of problems) within the following fields:

- If questions of convergence of a Gabor series, for certain classes of functions or distributions or continuity properties of the frame operator are discussed, one may regard Gabor analysis as a chapter of *functional analysis* respectively *operator theory*.
- Drawing the attention on the Heisenberg group, which is fundamental to analyse the structure of modulations and translations such as arising in the Gabor system $\{T_{na}M_{mb}g\}$, and exploiting this link to study Gabor theory by group theoretical methods, shows this field as a branch of *abstract harmonic analysis*.
- If the focus is on the development of efficient and stable algorithms, we are dealing with a problem of *numerical mathematics*. For computer implementations we have to deal with a finite model. The general problem of expanding "finite" signals, such as vectors of finite length or digital images of format $N \times M$ is essentially a chapter of *linear algebra*.
- If Gabor expansions are used for certain applications in communication theory or time-frequency signal analysis, it may be seen as a chapter of *signal processing*.
- When we use the properties of the Gabor transform to extract local features or to study textures in images, we are dealing with a classical topic of *pattern recognition* or *image analysis*.
- If we apply Weyl-Heisenberg frames to the analysis of coherent states, (see e.g. a recent paper of Zak), we study a classical field in *quantum physics*.

The chapters of this book reflect this diversity and also the interrelations between these fields. Although all chapters are more or less self-contained, they are ineinander verflochten, by many cross-connections between the chapters. The use of a consistent notation throughout the book erleichtert the reader to discover and follow these cross-connections.

In a nutshell the organization of the chapters may be characterized in the following way. The emphasize in the chapters of the first half of the book is on theoretical questions, mainly concentrating on mathematical problems, whereas in the chapters of the second half the focus is on applications and numerical algorithms. Clearly this classification is really very shallow. For instance, the development of efficient numerical algorithms heavily relies on careful theoretical analysis, and many deep theoretical results are inspired by practical problems. In the sequel we give a brief overview of the chapters.

The first chapter by A.J.E.M. Janssen gives a detailed survey on the condition of duality for Gabor frames for the continuous-time and the discrete-time case. Many results are derived for the general framework of translation invariant systems or filter banks. Besides the formulation of the duality condition in various domains and formulas for the frame bounds, the author derives a representation of the Gabor frame operator as a series of time-frequency shift operators. This representation is intimately related to the "fundamental identity of Gabor analysis", which is obtained by the author by means of classical Fourier methods.

Chapter 2 by John J. Benedetto, Christopher Heil, and David F. Walnut, presents a tutorial to the famous Balian–Low Theorem (BLT) and related topics such as Wilson bases and the uncertainty principle. As a variation of the theme, a so-called Amalgam BLT is derived, the difference to the standard BLT is illustrated by examples. Major tools in this chapter are the Zak transform and the notion of Further the authors give completeness conditions for irregular Gabor systems $\{g_{p,q}\}_{(p,q)\in\Lambda}$, based on the density of Λ, where Λ is a discrete subset (but not necessarily a lattice) of $\mathbb{R} \times \mathbb{R}^d$.

The space $\boldsymbol{S}_0$ can be characterized as the smallest time-frequency homogeneous Banach space of (continuous) functions. Exploiting the properties of $\boldsymbol{S}_0$, Hans G. Feichtinger and Georg Zimmermann demonstrate in Chapter 3 that this space is particularly usefull for Gabor analysis. Among others they present (a) an extended version of the fundamental identity of Gabor analysis; (b) the continuous dependence of the canonical dual Gabor atom on the given Gabor window and on the lattice; (c) continuity of thresholding and masking operators from signal processing and (d) an approximate Balian–Low Theorem stating that for close-to-critical lattices, the dual Gabor atoms progressively loose their time-frequency localization.

In Chapter 4, Richard Rochberg and Kazuya Tachizawa use Gabor frames for the derivation of sufficient conditions on the Weyl symbol, to ensure that the corresponding pseudodifferential operators belong to the Schatten–von Neumann classes $S_{p,q}$. Further they give estimates for the size of eigenfunctions of pseudodifferential operators. This chapter is also a good example how localized systems can be applied to obtain an approximate diagonalization of operators. In particular, the authors use local trigonometric bases to approximately diagonalize elliptic pseudo-differential operators.

Chapter 5 by O.Christensen is devoted to the problem of perturbation of frames, with an emphasize on the consequences for Gabor frames. Based on functional analytic arguments it is explained that both small changes of the lattice points of the time-frequency lattice or of the Gabor atom lead to coherent families which are still frames for $\boldsymbol{L}^2(\mathbb{R})$. Furthermore, the change of frame bounds is described in dependence of the perturbation parameters.

Most results in the chapters described above are derived for functions defined on $\mathbb{R}, \mathbb{R}^d$ and $\mathbb{Z}$. In Chapters 6 and 7 the focus is on the more general setting of (elementary) locally compact abelian groups. This group theoretical setting allows a unifying approach to Gabor analysis, without repaeting proofs for different settings (such as $\mathbb{R}, \mathbb{R}^d, \mathbb{Z}, \mathbb{Z}_N$). In Chapter 6 Karlheinz Gröchenig uses Liebs' inequalities for the short time Fourier transform to express an uncertainty principle for the setting of lca groups. Using the Zak transform, introduced on lca groups already by A. Weil, the author analyzes Gabor frames for the cases of critical sampling and integer oversampling. In this context it turns out that the known version of the Balian–Low theorem does not hold for discrete and compact groups. A notion of density is defined and necessary density conditions for lattices are derived to generate Gabor frames.

In Chapter 7 by Hans G. Feichtinger and Werner Kozek, various new aspects come into play. It is shown that a properly generalized Kohn–Nirenberg symbol is a powerful tool for the study of the Gabor frame operator. The duality conditions are extended to general discrete subgroups Λ (with compact quotient), allowing a unified formulation for continuous-time, discrete-time, multi-dimensional signals, including the case of separable and non-separable lattices and atoms. The Gabor frame operator is interpreted as a time-frequency periodization (with respect to Λ) of the one-dimensional projection onto the vector space generated by the atom g. The case of multi-window Gabor analysis is obtained as a simple modification, replacing the rank 1 operator $K : f \to \langle f, g \rangle g$ by some finite rank operator. In order to derive these results, the concept of Gelfand triples, the spreading function and a generalized Kohn–Nirenberg correspondence turn out to be very useful.

One of the challenging numerical problems in Gabor analysis is the development of efficient methods for the inversion of the Gabor frame operator in order to compute the dual window. In Chapter 8 Thomas Strohmer presents a unifying approach to derive fast numerical algorithms, based on unitary factorizations of the Gabor frame operator. Using classical tools from linear algebra the author shows that different algorithms for the computation of the dual window correspond to different factorizations of the frame operator. Based on simple number theoretic conditions new structural properties of the frame operator for the finite setting are derived. The chapter concludes with a discussion of the conjugate gradient method, demonstrating the efficiency of certain preconditioners by numerical experiments.

Oversampled filter banks offer increased design freedom and noise immunity as compared to critically sampled filter banks (FB). Since these advantages come at the cost of greater computational complexity, those oversampled FBs are of particular interest, which can be realized in practice with little computational effort. In Chapter 9 Helmut Bölcskei and Franz Hlawatsch discuss oversampled DFT FBs and oversampled cosine modulated FBs which allow efficient implementations. Exploiting the relationship of the Gabor frame operator with the filter bank polyphase matrix, the authors provide a frame-theoretic analysis of filter banks. Conditions for perfect reconstruction are presented and the increased design freedom is illustrated by numerical simulations.

In many applications the choice of the analysis window g and the lattice parameters a, b follows traditional rules of thumb. In Chapter 10 Werner Kozek presents a theoretical framework for the adaptation of continuous ("fully oversampled") and discrete Gabor frames to underspread operators in the sense of approximate diagonalization. The author uses underspread operators to classify slowly time-varying processes. The optimization criteria for the adaptation are formulated in terms of the ambiguity function of g and the spreading function of the operator. Numerical experiments demonstrate the practical performance of the proposed method.

Recently, the problem of signal detection based on linear time-frequency representations has received considerable attention. In Chapter 11 Ariela Zeira and Benjamin Friedlander formulate the problem of detecting a Gabor transient as problem of detecting a subspace signal in background noise. Using global ratio likelihood tests, they derive matched subspace detectors and discuss their sensitivity in terms of mismatch of the model parameters. Based on the sensitivity analysis they develop robust matched subspace detectors and analyze their performance.

In Chapter 12 Yehoshua Y. Zeevi, Meir Zibulski and Moshe Porat present a detailed analysis of multi-window Gabor expansions. They extend the

Balian–Low Theorem to the case of multi-windows and discuss also the case of non-separable time-frequency lattices. Applications to image processing and computer vision are presented with regard to texture images, and considered in the context of two typical tasks: image representation by partial information and pattern recognition. In both cases the results indicate that the multi-window approach allows to overcome drawbacks of the single window approach.

The subject of Chapter 13 is the application of Gabor functions to object recognition. Jezekiel Ben-Arie and Zhiqian Wang show that 3-D pose invariant object recognition can be achieved by a specific multi-window configuration of Gabor functions. By matching the extracted local spectral signature against a set of iconic models using multi-dimensional indexing in the frequency domain, the authors are able to identify objects under varying conditions with high reliability, as demonstrated by numerical experiments.

The final Chapter, written by Martin J. Bastiaans, presents applications in the field of optics. Special attention is paid to Gaussian windows, which are related to the classical concept of Gaussian light beams in optics. The propagation of an optical signal in terms of its Gabor coefficients is investigated and the relation of the case of critical sampling to the degrees of freedom of an optical signal is discussed. The author demonstrates how a product of Zak transforms establishes the basis of a coherent-optical setup with which the Gabor transform can be generated.

A comprehensive bibliography completes the book.