The Instability of Critical and Under-dense Friedmann Spacetimes at the Big Bang as an Alternative to Dark Energy

Blake Temple
UC-Davis

Distinguished Lecture
Polytechnic University of Hong Kong
April 27, 2026

Collaborators: Christopher Alexander, Zeke Vogler

(Dedicated to Joel Smoller)

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The Friedmann Spacetimes describe a uniformly expanding 3-space of constant density and curvature evolving in time according to Einstein's equations of General Relativity.

Point of departure is our paper 2017 RSPA…

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Research

Check for updates

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Subject Areas:
applied mathematics, cosmology, mathematical physics

Keywords:
general relativity, anomalous acceleration, dark energy, cosmological constant, instability of the Friedmann space–time

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An instability of the standard model of cosmology creates the anomalous acceleration without dark energy

Joel Smoller[1], Blake Temple[2] and Zeke Vogler[2]

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[2]Department of Mathematics, University of California, Davis, CA 95616, USA

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We identify the condition for smoothness at the centre of spherically symmetric solutions of Einstein's original equations without the cosmological constant or dark energy. We use this to derive a universal phase portrait which describes general, smooth, spherically symmetric solutions near the centre of symmetry when the pressure $p=0$. In this phase portrait, the critical $k=0$ Friedmann space–time appears as a saddle rest point which is unstable to spherical perturbations. This raises the question as to whether the Friedmann space–time is observable by redshift versus luminosity measurements looking outwards from any point. The unstable manifold of the saddle rest point corresponding to Friedmann describes the evolution of local uniformly expanding space–times whose accelerations closely mimic the effects of dark energy. A unique simple wave perturbation from the radiation epoch is shown to trigger the instability, match the accelerations of dark energy up to second order and distinguish the theory from dark energy at third order. In this sense, anomalous accelerations are not only consistent with Einstein's original theory of general relativity, but are a prediction of it without the cosmological constant or dark energy.

1. Introduction

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Doing Without Dark Energy

Mathematicians Propose Alternative Explanation for Cosmic Acceleration

By Andy Fell on December 13, 2017 in Science & Technology

"Dark energy," a mysterious force that counters gravity, has been proposed to explain why the universe is expanding at an accelerating rate. Mathematicians at UC Davis and the University of Michigan, Ann Arbor, argue for an alternative. Galaxy cluster image from the Hubble Space Telescope.

Three mathematicians have a different explanation for the accelerating expansion of the universe that does without theories of "dark energy." Einstein's original equations for General Relativity actually predict cosmic acceleration due to an

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…the resulting stability analysis stands on its own.

…here we complete the stability analysis for case $p = 0$ by incorporating the non-critical $k \neq 0$ Friedmann spacetimes into the theory.

Our paper was just accepted last month by Proceedings of the Royal Society-A (RSPA)…

PROCEEDINGS A

royalsocietypublishing.org/journal/rspa

Research

Check for updates

Article submitted to journal

Subject Areas:

Mathematical Physics, Differential Equations, Relativity

Keywords:

General Relativity, Instability, Cosmology, Big Bang, Dark Energy

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The Instability of the Critical Friedmann Spacetime at the Big Bang as an Alternative to Dark Energy

C. Alexander[1], B. Temple[2] and Z. Vogler[3]

[1]Department of Mathematics, University College London, London, WC1H 0AY, United Kingdom

[2]Department of Mathematics, University of California, Davis, CA 95616, United States

[3]Department of Mathematics, University of California, Davis, CA 95616, United States

We characterize the local instability of pressureless Friedmann spacetimes to radial perturbation at the Big Bang. The analysis is based on a formulation of the Einstein–Euler equations in self-similar variables (t,ξ), with $\xi = r/t$, conceived to realize the critical $(k=0)$ Friedmann spacetime as a stationary solution whose character as an unstable saddle rest point SM is determined via an expansion of smooth solutions in

We received extremely positive referee report (finally!)…

Referee: 1

Comments to the Author(s)
Report on the paper
”The Instability of the Critical Friedmann Spacetime at the Big Bang as an Alternative to Dark Energy”
by C. Alexander, B. Temple and Z. Vogler

The paper is concerned with a careful analysis of perturbations of the Friedmann-Lemaitre-Robertson-Walker (FLRW) spacetime of cosmology. The authors find an instability which provides an alternative explanation of the anomalous expansion of the universe. More precisely, the authors start from the homogeneous and isotropic FLRW spacetime in the flat case (k=0) for a pressureless gas. They reformulate the corresponding Einstein-Euler equations in self-similar variables, in which the metric resembles the Schwarzschild metric, and perturb in powers of the variable r/t. Expanding up to order n, they obtain a system of $2n$ coupled ODEs. The authors perform a careful stability analysis of these systems. They find an unstability to every order n, having the effect that the solutions generically accelerate away from the FLRW spacetime for intermediate times, but decay back to FLRW for large times. A quantitative analysis of the deviation from FLRW for intermediate times reveals that this effect could indeed account for the anomalous expansion of our universe which is usually explained by dark energy. This finding is very interesting and extremely important, because it indicates that dark energy might not needed to explain the anomalous expansion of the universe. Another finding which I find interesting is that, solving the Einstein-Euler system backward in time beyond the radiation-dominated epoch, there is an instabiility which could break the self-similarity of the big bang.

The paper is very well written. The main points are first described clearly in words. Then detailed formulas and theorems make the statements and conclusions mathematically precise. I should note that the paper is a continuation of an earlier paper by Smoller, Temple and Vogler (reference [29] of the present paper), where the equations were derived and analyzed numerically. The analysis in the present paper goes much deeper and comes to extremely important physical implications. To my opinion, this clearly justifies publication. I strongly recommend the paper for publication in Proceedings of the Royal Society A.

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arXiv:2412.00643 (gr-qc)

[Submitted on 1 Dec 2024]

Cosmic Accelerations Characterize the Instability of the Critical Friedmann Spacetime

Christopher Alexander, Blake Temple, Zeke Vogler

Introduction

A Brief History of Friedmann Spacetimes

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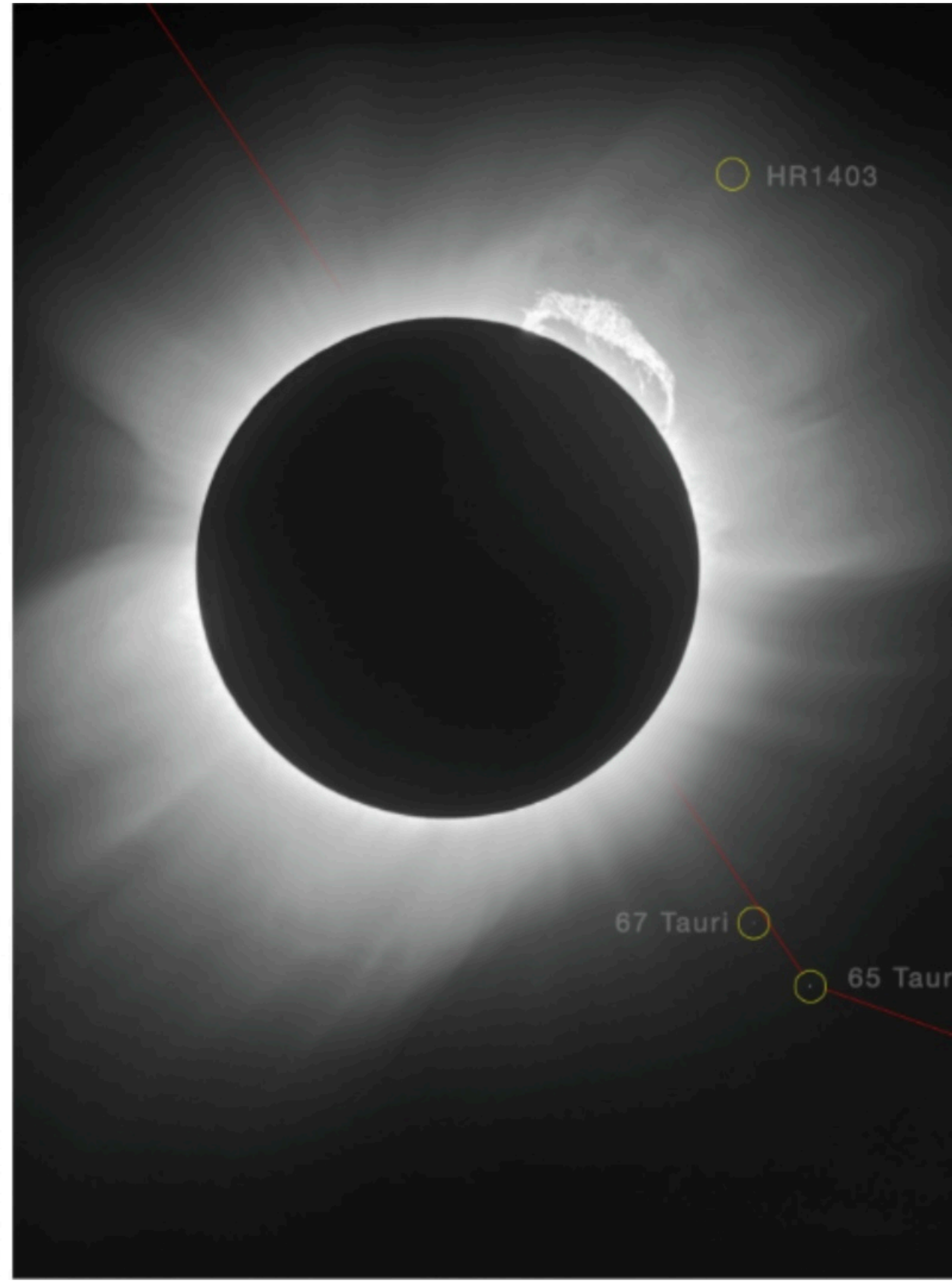
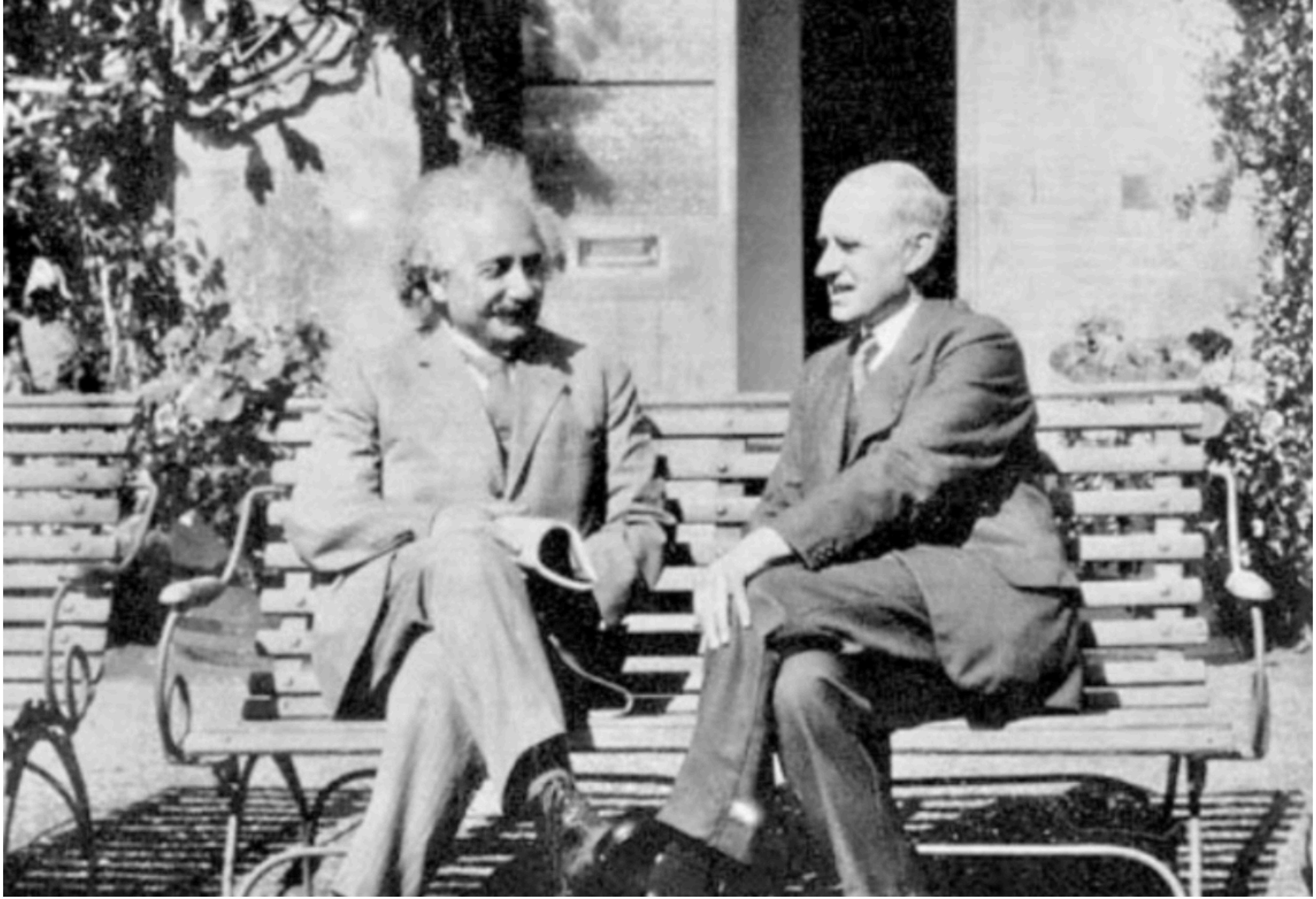
…General Relativity was immediately accepted…

…and Einstein became an international celebrity.



Sir Arthur Eddington-1919



Albert Einstein and Sir Arthur Eddington

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…but accepted Friedmann's work was correct following Friedmann's appeal.

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…that the universe on the largest scale was evolving according to a Friedmann spacetime.

Edwin Hubble (1889-1953)

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1933-Einstein said Lemaitre's theory of the primeval atom was ``the most beautiful and satisfactory explanation of creation to which I have ever listened".

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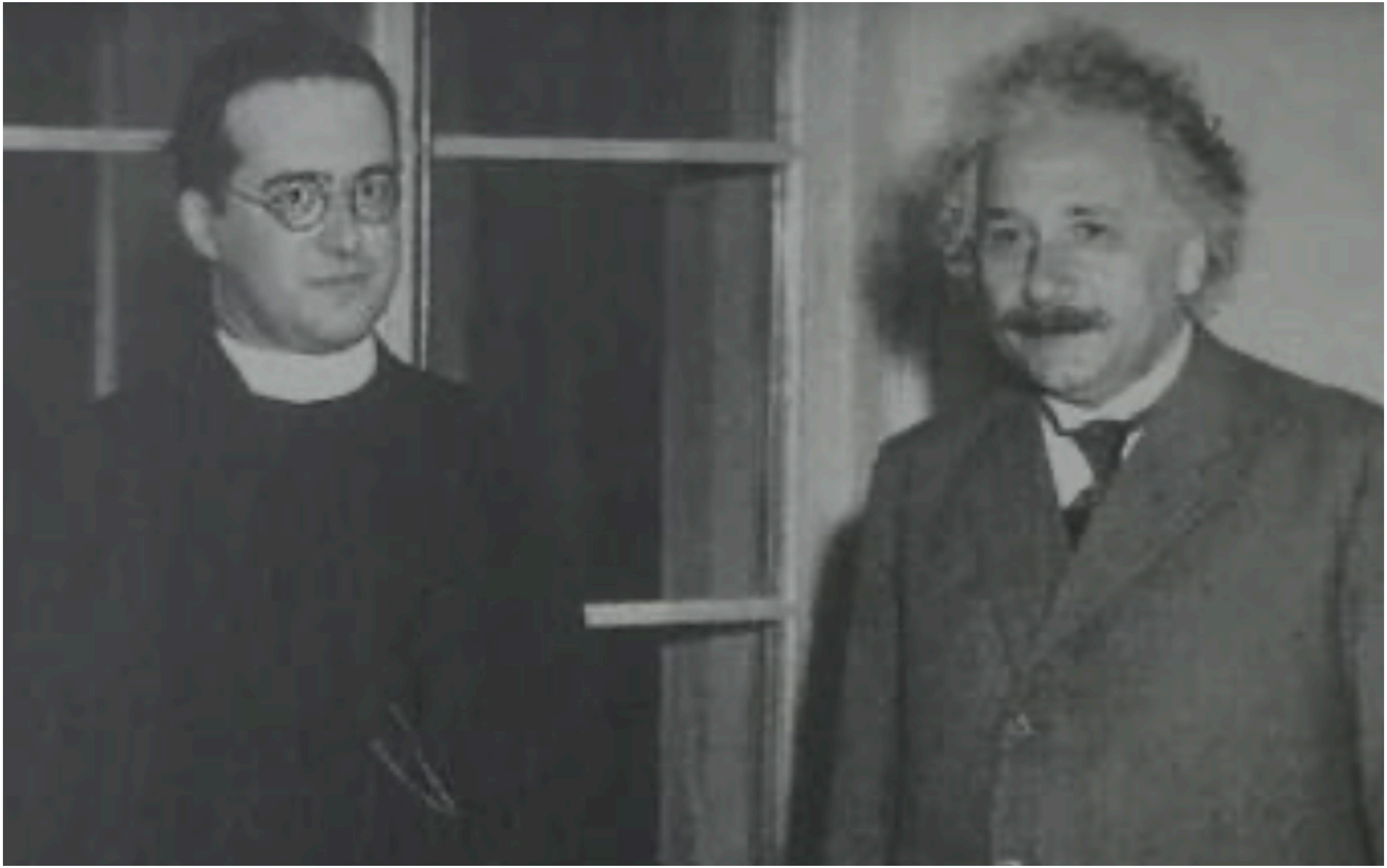
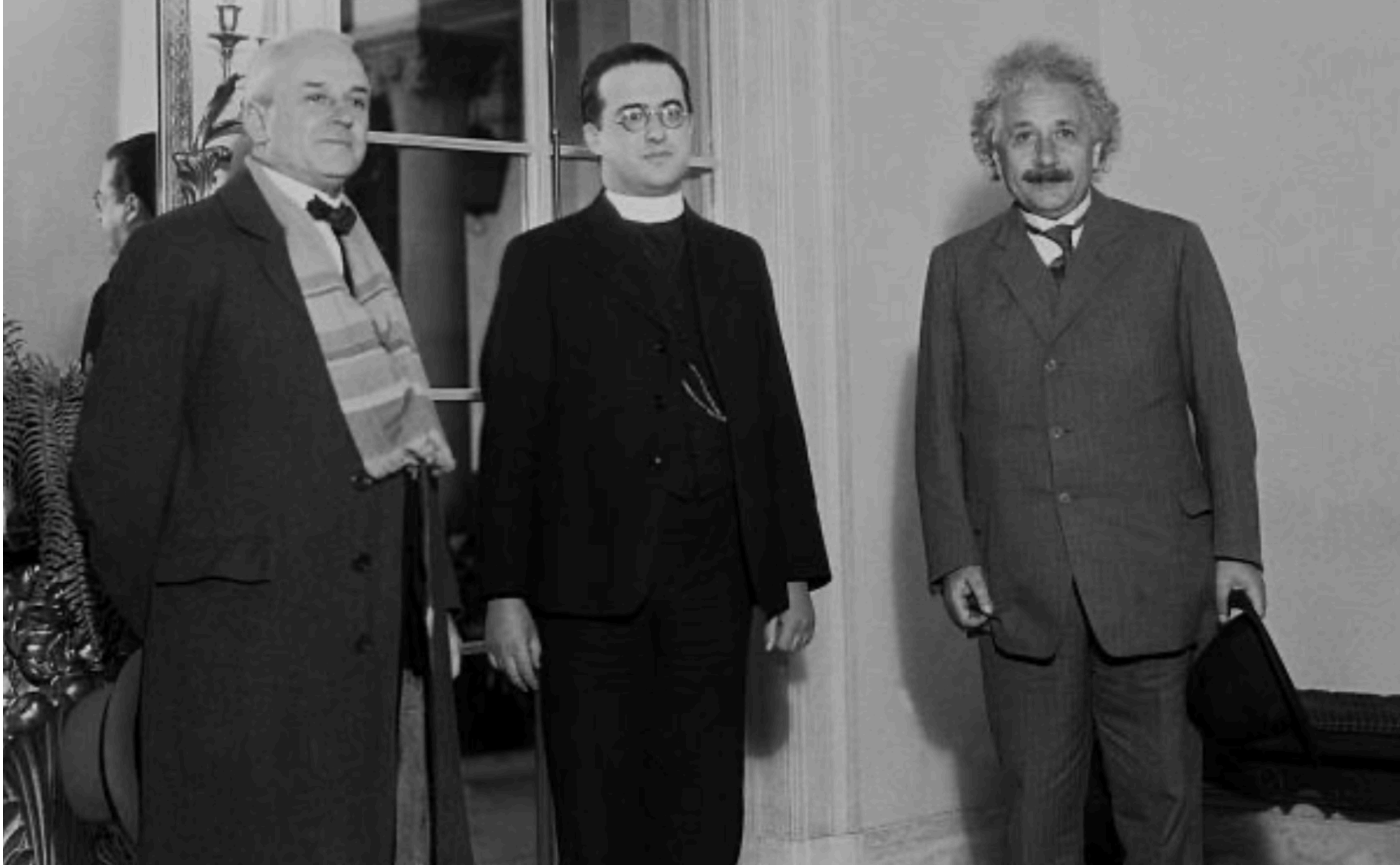


Georges Lemaitre and Albert Einstein

Georges Lemaitre- Mendel Medal (1934)
- Francqui Prize (1934)



Robert Millikan, G. Lemaitre and A. Einstein

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Copernican Principle: If the 3-space at each fixed time is uniform and isotropic about every point, then the spacetime is Friedmann.

This connected Friedmann space-times with pre-scientific notions of earth not being in a special place in the universe, and stuck as a sort of principle of physics more or less accepted by established cosmologists.

Howard Robertson

Arthur Walker

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Often referred to as the FLRW spacetimes.

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$\Lambda \approx .67$	70% Dark Energy
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(Under-dense Friedmann $(k<0)$ implies faster expansion consistent with $\Lambda > 0$.)

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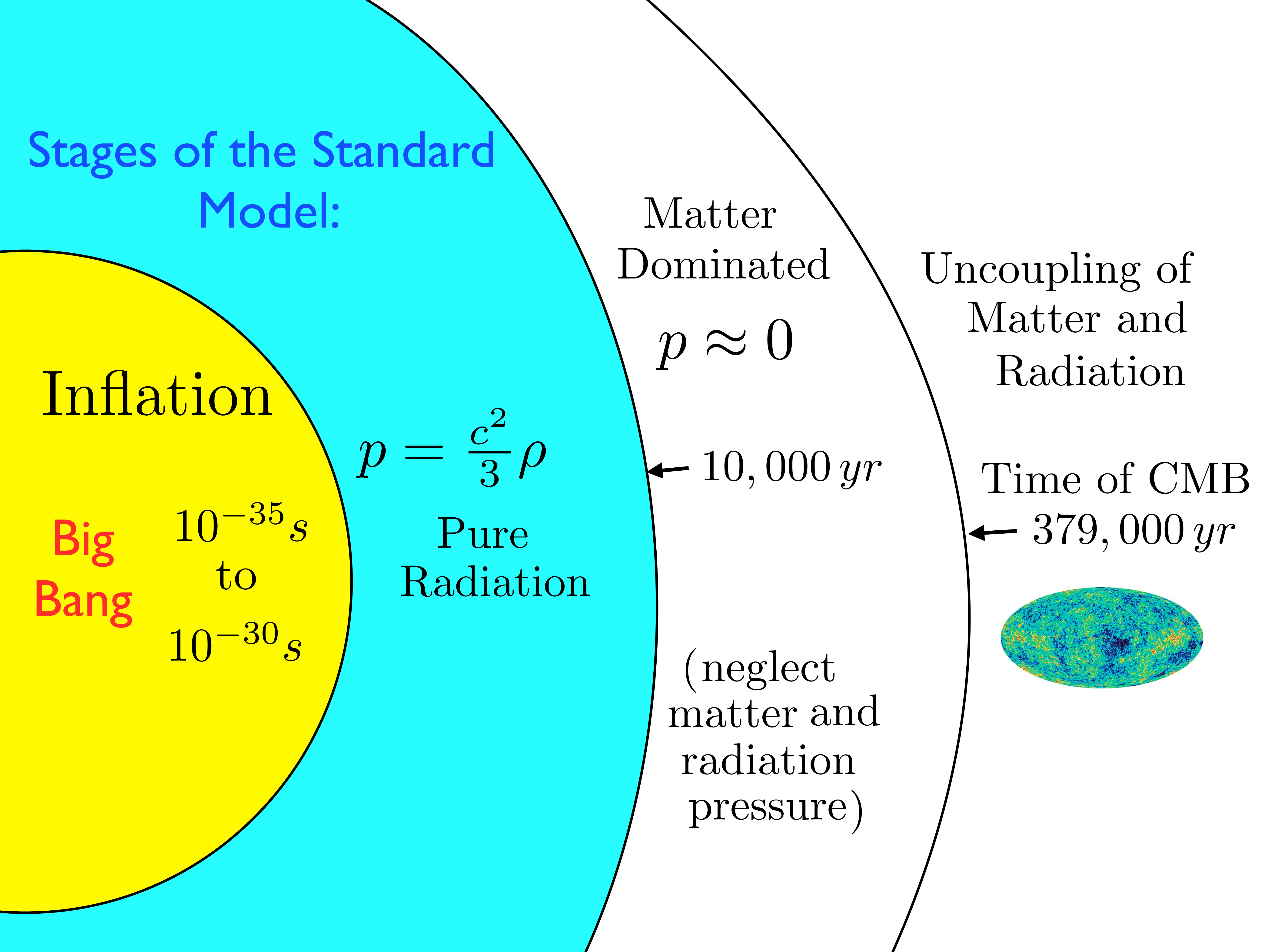
We characterize all under-dense solutions which are smooth at the center of symmetry.



Stages of the Standard Model:
Inflation
Big Bang
$10^{-35}s$
to
$10^{-30}s$
$p = \frac{c^2}{3}\rho$
Pure Radiation
Matter Dominated
$p \approx 0$
$10,000\,yr$
(neglect matter and radiation pressure)
Uncoupling of Matter and Radiation
Time of CMB
$379,000\,yr$

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Problem: Radial perturbations create a center which violates the Copernican Principle.

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One parameter $-\infty < k < \infty$

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The Friedmann spacetimes describe a 3-dimensional space of constant curvature evolving in time.

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Note the gauge freedom $t \to t - t_* \ldots$

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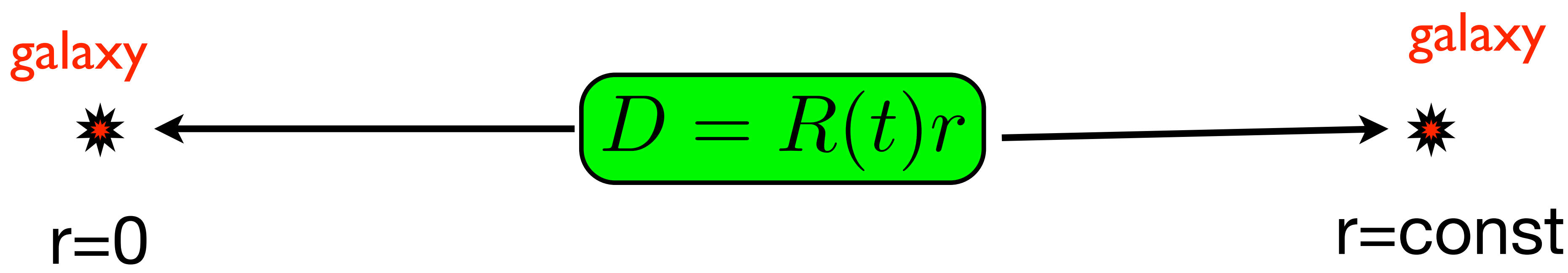

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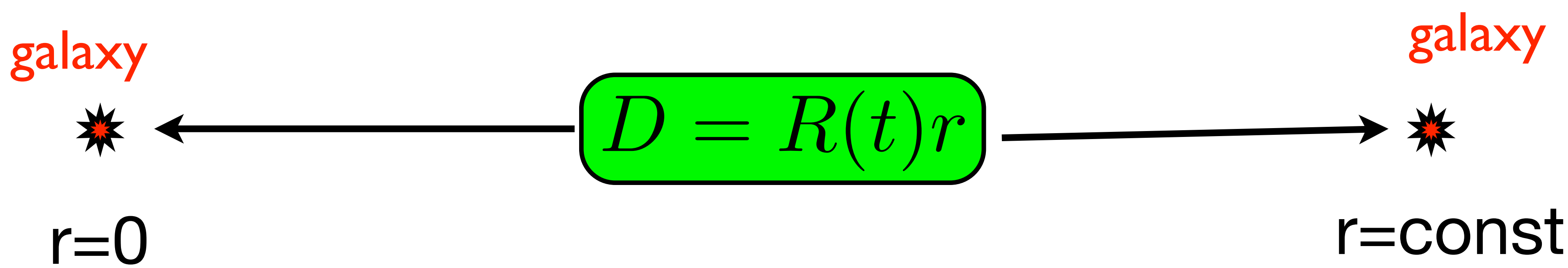

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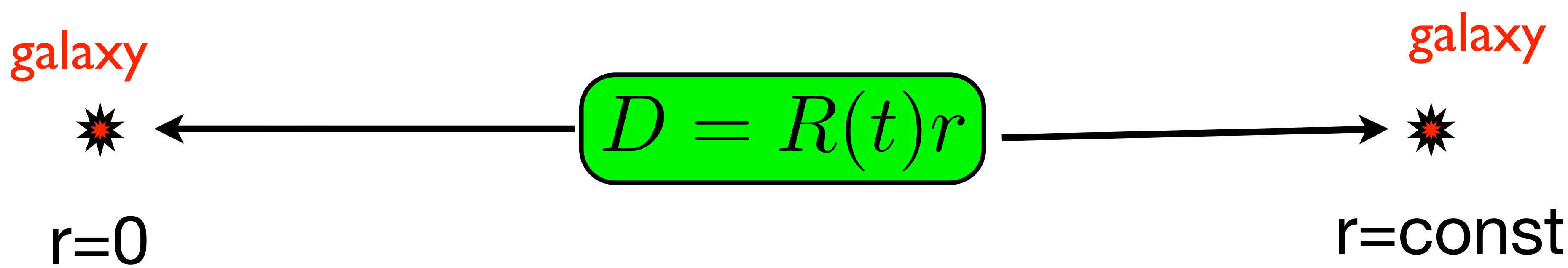

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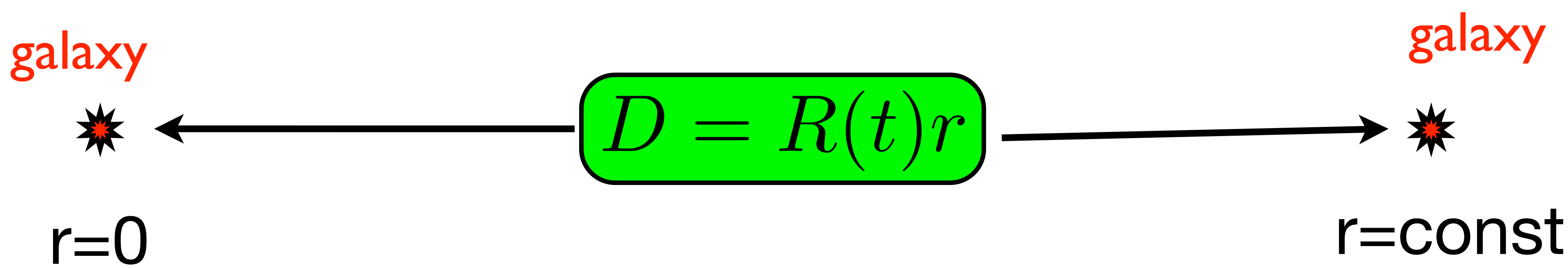

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$$\text{Hubble's Constant} \equiv H \equiv \frac{\dot{R}}{R}$$

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galaxy ← $D = R(t)r$ → galaxy

r=0 r=const

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$$H^2 = \frac{\kappa}{3}\rho - \frac{k}{R^2}$$

$$\dot{\rho} = -3(\rho + p)H$$

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$T_{ij} = (\rho + p)u_i u_j + p g_{ij}$=Stress Energy Tensor (perfect fluid)

Einstein Equations for Friedmann metrics:

$$H^2 = \frac{\kappa}{3}\rho - \frac{k}{R^2}$$

$$\dot{\rho} = -3(\rho + p)H$$

Equations close with equation of state: $p = p(\rho)$

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$k = 0$	"Critical"	$R'(t) > 0$	$0 < t < \infty$
$k < 0$	"Underdense"	$R'(t) > 0$	$0 < t < \infty$
$k > 0$	"Overdense"	$R'(t) > 0$	$0 < t < t_{max}$

Standard Model of Cosmology (ΛCDM)

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Conclude--

``The universe is expanding like a balloon"

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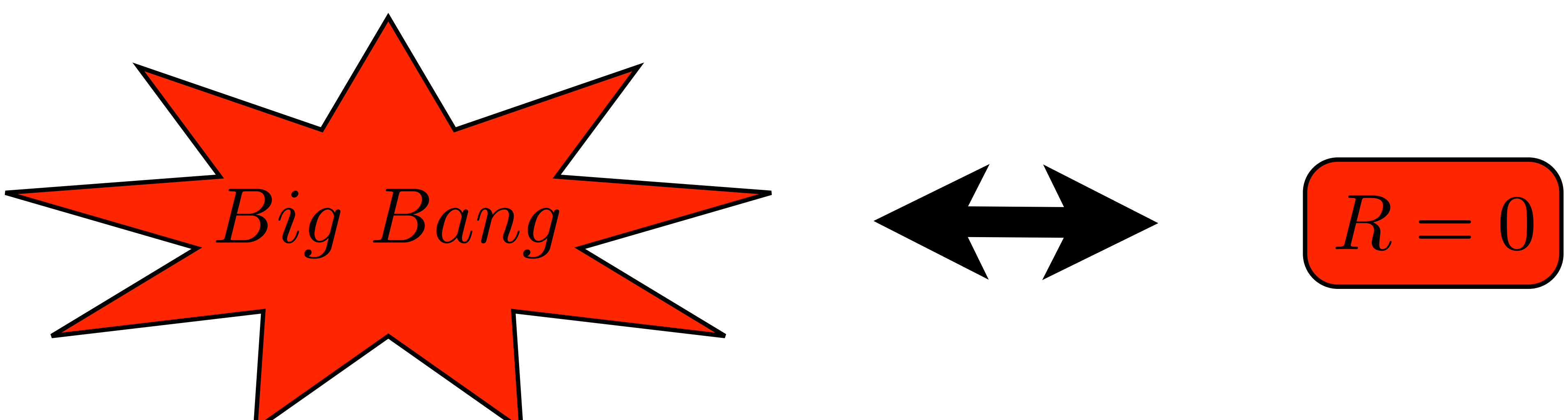

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Assume Einstein equations with a cosmological constant:

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"Equation for the unknown metric" g_{ij}

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Implies: The universe is 70 percent dark energy

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We carry out a stability analysis in Standard Schwarzschild Coordinates (SSC) when p=0…

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We carry out a stability analysis in Standard Schwarzschild Coordinates (SSC) when $p=0$…

SSC is better aligned with the physics than co-moving coordinates.

Standard Schwarzschild Coordinates

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A General Spherically Symmetric metric

$$\mathrm{ds}^2 = -D(t,\bar{r})d\bar{t}^2 + E(\bar{t},\bar{r})d\bar{t}d\bar{r} + F(\bar{t},\bar{r})d\bar{r}^2 + G(\bar{t},\bar{r})d\Omega^2$$

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$$d\Omega^2 = d\theta^2 + \sin^2\theta\, d\phi^2$$

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Transforms to SSC form: $(\bar{t},\bar{r}) \rightarrow (t,r)$

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SSC Gauge Freedom

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(SSC)

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$$t \rightarrow \phi(t)$$

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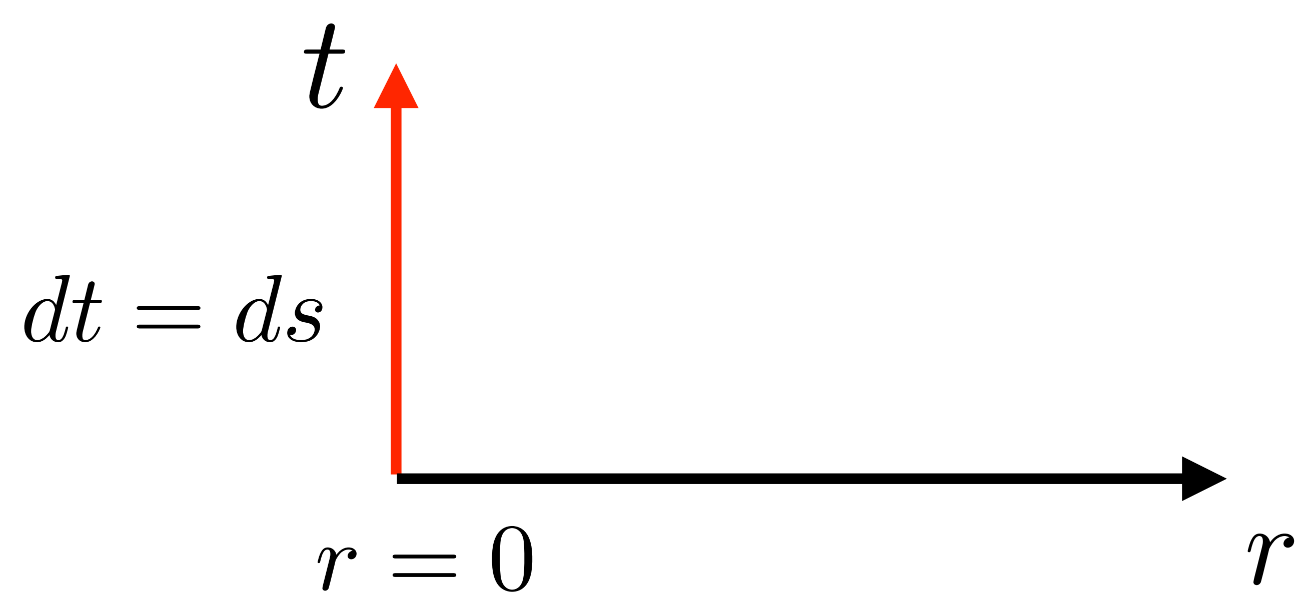

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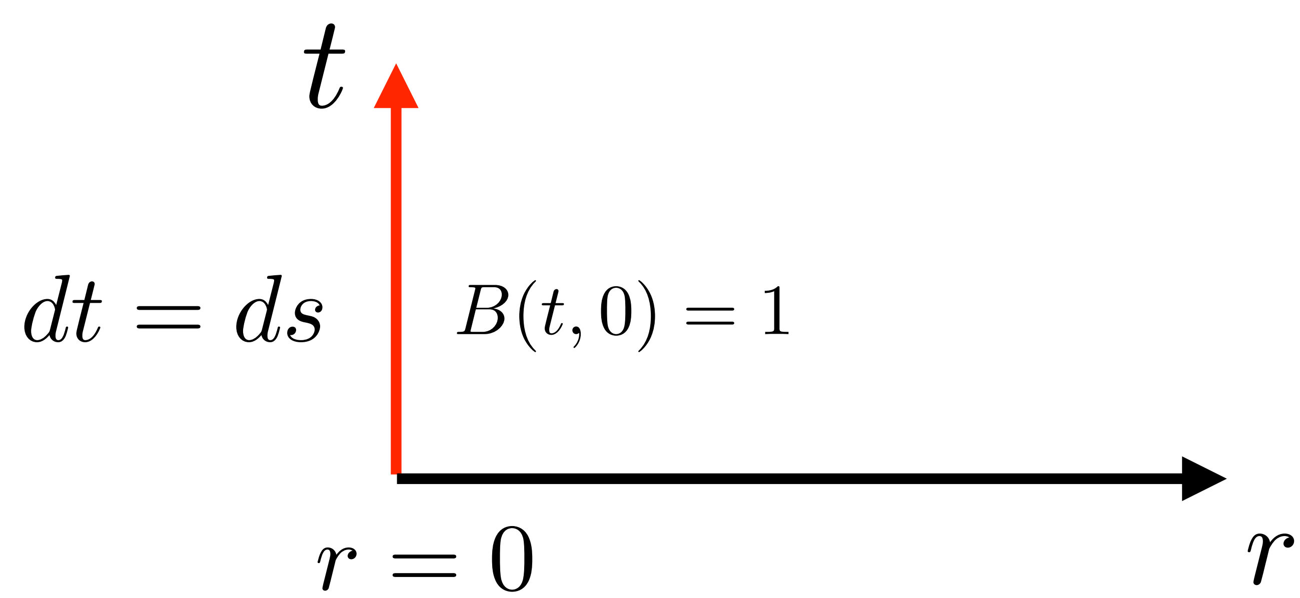

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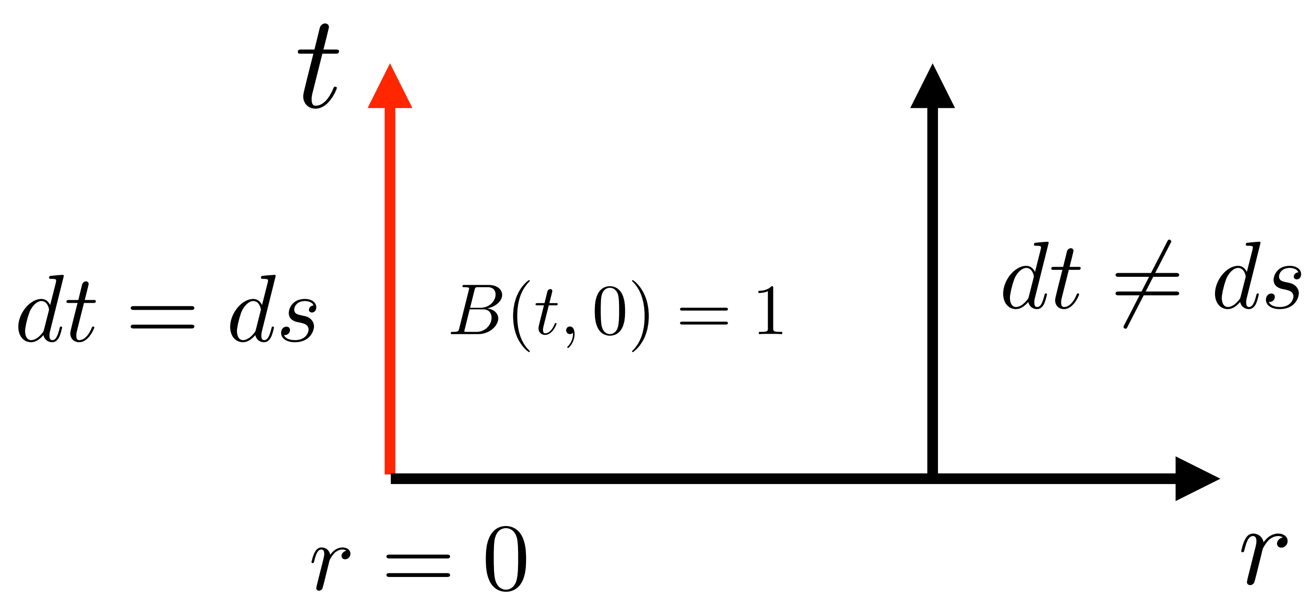

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There's still one gauge freedom left in SSC

$$t \to t - t_*$$

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(Time-translation freedom!)

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This is very important to our analysis!!

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Substituting SSC into the Einstein equations

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yields four equations…

Standard Schwarzschild Coordinates

Four PDE's

$$\left\{-r\frac{A_r}{A}+\frac{1-A}{A}\right\} = \frac{\kappa B}{A}r^2T^{00} \tag{1}$$

$$\frac{A_t}{A} = \frac{\kappa B}{A}rT^{01} \tag{2}$$

$$\left\{r\frac{B_r}{B}-\frac{1-A}{A}\right\} = \frac{\kappa}{A^2}r^2T^{11} \tag{3}$$

$$-\left\{\left(\frac{1}{A}\right)_{tt}-B_{rr}+\Phi\right\} = 2\frac{\kappa B}{A}r^2T^{22}, \tag{4}$$

where

$$\Phi = \frac{B_tA_t}{2A^2B}-\frac{1}{2A}\left(\frac{A_t}{A}\right)^2-\frac{B_r}{r}-\frac{BA_r}{rA} +\frac{B}{2}\left(\frac{B_r}{B}\right)^2-\frac{B}{2}\frac{B_r}{B}\frac{A_r}{A}.$$

(1)+(2)+(3)+(4) ⟺ (1)+(3)+div T=0

(weakly)

Theorem: (Gr-Te) The equations close in the "locally inertial" formulation (1), (2) & Div T=0:

$$\left\{T_M^{00}\right\}_{,0} + \left\{\sqrt{AB}T_M^{01}\right\}_{,1} = -\frac{2}{r}\sqrt{AB}T_M^{01}, \tag{1}$$

$$\left\{T_M^{01}\right\}_{,0} + \left\{\sqrt{AB}T_M^{11}\right\}_{,1} = -\frac{1}{2}\sqrt{AB}\left\{\frac{4}{r}T_M^{11} + \frac{(1-A)}{Ar}(T_M^{00} - T_M^{11}) + \frac{2\kappa r}{A}(T_M^{00}T_M^{11} - (T_M^{01})^2) - 4rT^{22}\right\}, \tag{2}$$

$$rA_r = (1-A) - \kappa r^2 T_M^{00}, \tag{3}$$

$$rB_r = \frac{B(1-A)}{A} + \frac{B}{A}\kappa r^2 T_M^{11}. \tag{4}$$

$$T_M^{00} = \frac{\rho c^2 + p}{1 - \left(\frac{v}{c}\right)^2} \qquad T_M^{01} = \frac{\rho c^2 + p}{1 - \left(\frac{v}{c}\right)^2}\frac{v}{c}$$

$$T_M^{11} = \frac{p + \left(\frac{v}{c}\right)^2}{1 - \left(\frac{v}{c}\right)^2}\rho c^2 \qquad T^{22} = \frac{p}{r^2}$$

$$v = \frac{1}{\sqrt{AB}}\frac{u^1}{u^0}$$

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Theorem: When $t - t_*$ is "time since Big Bang", (k=0) Friedmann in SSC-NG depends only on

$$\xi = r/t$$

(k=0) Friedmann in self-similar coordinates

Lemma: Assume $R(t) = 0$ at $t = 0$ and $p = \sigma\rho$.

Then in co-moving coordinates: $\vec{u} = (1, 0, 0, 0)$

$$R(t) = t^{\frac{2}{3(1+\sigma)}}$$

$$H(t) = \frac{2}{3(1+\sigma)t}$$

$$\rho(t) = \frac{4}{3\kappa(1+\sigma)^2 t^2}$$

(k=0) Friedmann in self-similar coordinates

The following coordinate transformation takes k=0 Friedmann to SSC coordinates $(\bar{t}, \bar{r})$.

$$\Phi : (t, r) \to (\bar{t}, \bar{r})$$

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$$\mathcal{F}(\eta) = \left(1 + \frac{\alpha(2-\alpha)}{4}\eta^2\right)^{\frac{1}{2-\alpha}}, \qquad \alpha = \frac{4}{3(1+\sigma)}$$

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...so η is a function of ξ alone!

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…which depend only on $\quad \xi = \frac{\bar{r}}{\bar{t}} = \frac{\eta}{F(\eta)}$

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$$\kappa\rho_\sigma r^2 = \frac{3}{4}\alpha^2\eta^2 \qquad v_\sigma = \frac{\alpha}{2}\eta$$

…so $z = \kappa\rho r^2$ and $w = \dfrac{v}{\xi}$

are natural density and velocity variables…

…which depend only on $\xi = \dfrac{\bar{r}}{\bar{t}} = \dfrac{\eta}{F(\eta)}$

Conclude: To represent k=0 Friedmann as stationary, transform SSC-NG equations to unknowns (z, w)…

…using $\xi = r/t$ as a spacelike coordinate.

$ct \approx$ distance of light travel since the Big Bang.

$\xi =$ "Fractional distance to Hubble Radius".

$0 \leq \xi < 1$ represents the visible universe.

$(t, r) \to (t, \xi)$ is regular coordinate transformation.

STV-PDE

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$$tw_t + (-1+Dw)\xi w_\xi - w + Dw^2$$
$$-\frac{(1+\sigma^2)\sigma^2}{\xi z}\left(D\frac{(1-v^2)v^2}{(1+\sigma^2 v^2)^2}z\right)_\xi$$
$$+\frac{\sigma^2\xi}{z}\left(D\frac{1-v^2}{1+\sigma^2 v^2}\frac{z}{\xi^2}\right)_\xi = \mathrm{RHS}$$

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$$\mathrm{RHS} = -\frac{1}{\xi^2}\frac{1-v^2}{1+\sigma^2v^2}\frac{D}{2A}\left((1-\sigma^2)(1-A) + 2\sigma^2\frac{1-v^2}{1+\sigma^2v^2}z\right)$$

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Together with…

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(Turns out…. $z = \kappa\rho r^2 + O(\xi^6)$)

STV-PDE

Restricting to the case of zero pressure $\sigma = 0$

…we get the $p = 0$ STV-PDE :

$\sigma = 0$

STV-PDE

$$tz_t + \xi\big((-1 + Dw)z\big)_\xi = -Dwz$$

$$tw_t + \xi(-1 + Dw)w_\xi = w - D\left(w^2 + \frac{1}{2\xi^2}(1 - \xi^2 w^2)\frac{1 - A}{A}\right)$$

$$\xi A_\xi = -z + (1 - A)$$

$$\xi D_\xi = \frac{D}{2A}\left(2(1 - A) - (1 - v^2)z\right)$$

$$z = \frac{\kappa\rho r^2}{1 - v^2} \qquad w = \frac{v}{\xi}$$

$$p = 0$$

$\sigma = 0$

STV-PDE

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$$z = \frac{\kappa\rho r^2}{1-v^2} \qquad w = \frac{v}{\xi}$$

$$p = 0$$

We know k=0 Friedmann is a rest point.

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Note: The STV-PDE are NOT the Einstein Equations in self-similar coordinates $z(t,\xi), w(t,\xi)$

STV-PDE

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…they are the SSC-NG equations expressed in terms of $z(t,\xi), w(t,\xi)$

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Expanding solutions about the center in even powers of ξ the resulting ODE's close at every order…

…even powers iff smooth at the center restricts and simplifies the solution space …

Ansatz:

STV-ODE

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STV-ODE

$$z(t,\xi) = z_2(t)\xi^2 + z_4(t)\xi^4 + \ldots + z_{2n}(t)\xi^{2n} + \ldots$$

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Substituting into STV-PDE and collecting like powers yields the STV-ODE of order n=1,2,3…

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$$\frac{d}{d\tau}\boldsymbol{v}_n = \begin{pmatrix} (2n+1)(1-w_0)-1 & -(2n+1)z_2 \\ -\frac{1}{2(2n+1)} & 2n(1-w_0)-1 \end{pmatrix}\boldsymbol{v}_n + \boldsymbol{q}_n$$

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…nested ODE's… linear at leading order.

STV-ODE

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...autonomous ODE in $\tau = \ln t$

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…autonomous means…

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…autonomous means…

…amenable to phase portrait analysis.

STV-ODE (n=1)

Setting n=1 we obtain:

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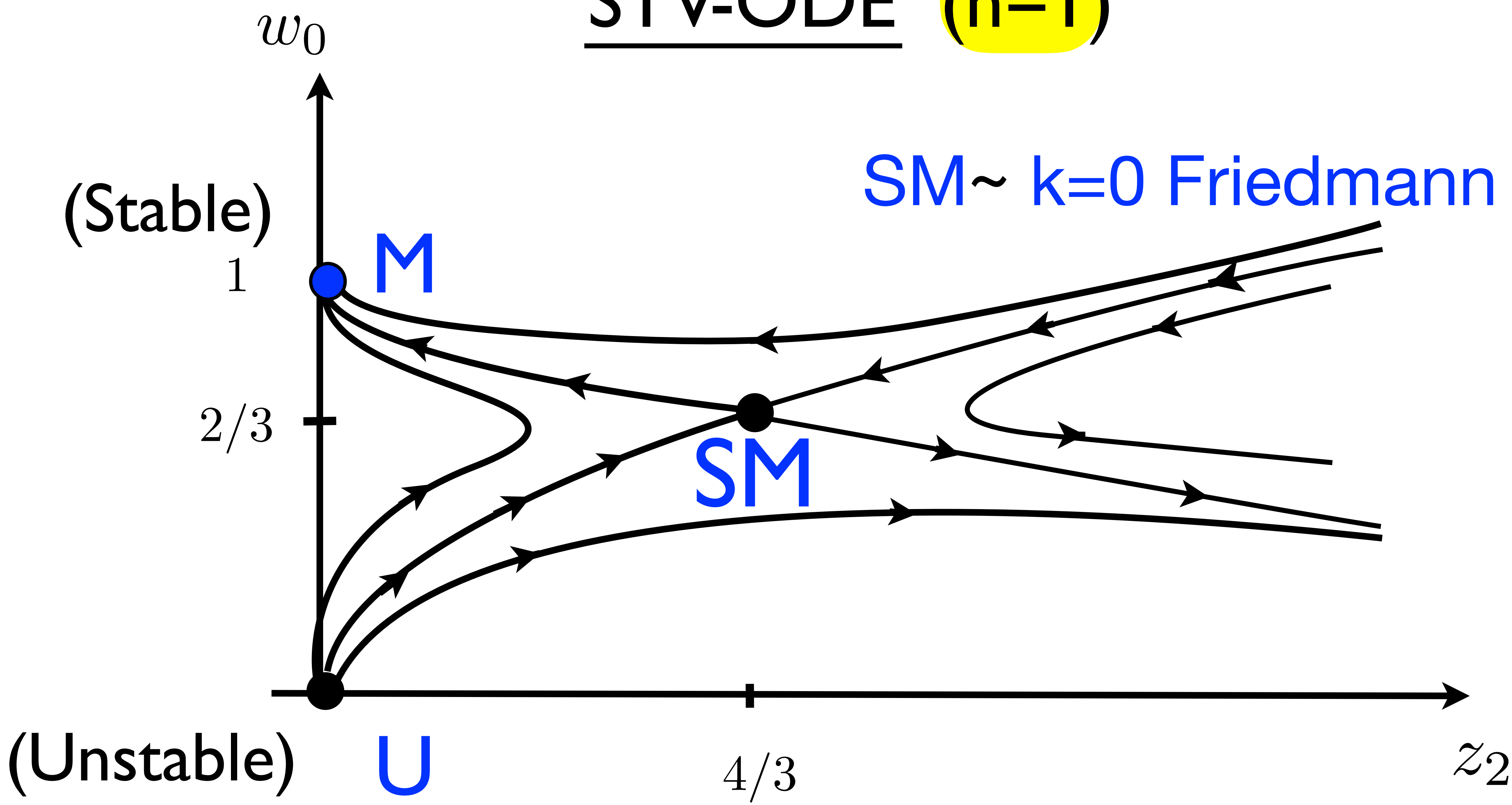
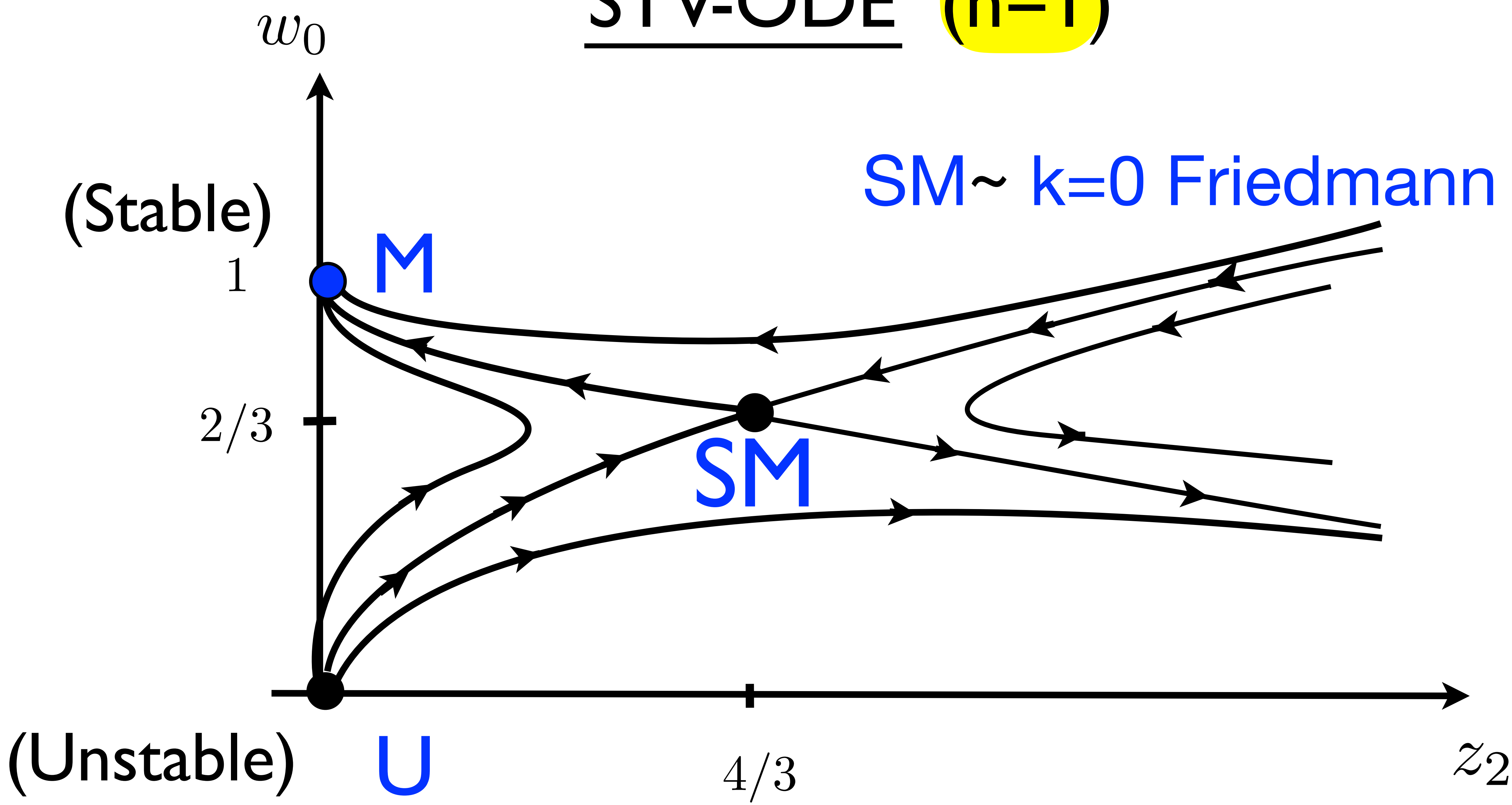
…a 2x2 autonomous system with 3 rest points

STV-ODE (n=1)

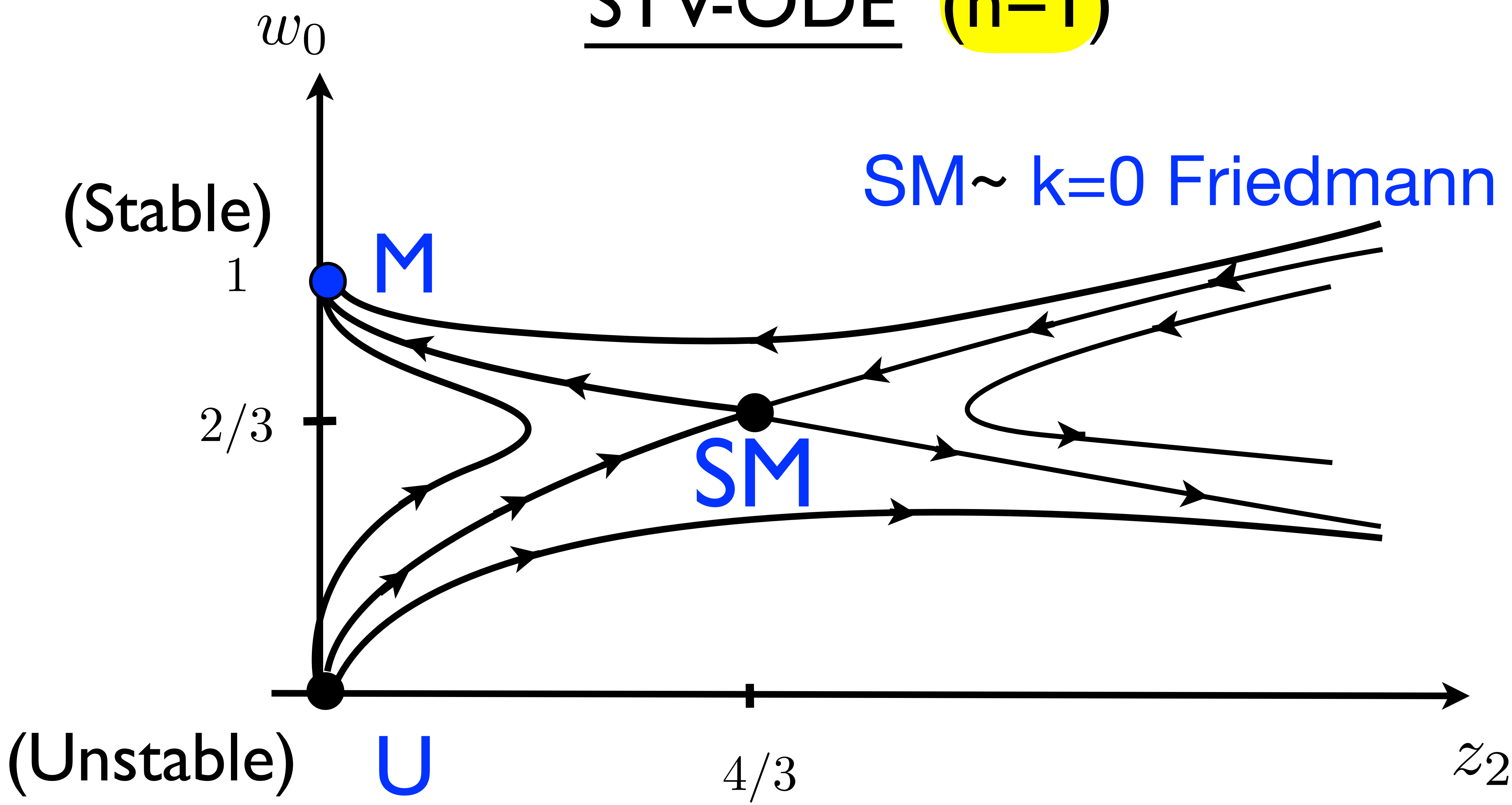


STV-ODE (n=1)



M degenerate stable node: $\lambda_M = -1 \quad \boldsymbol{R}_M = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

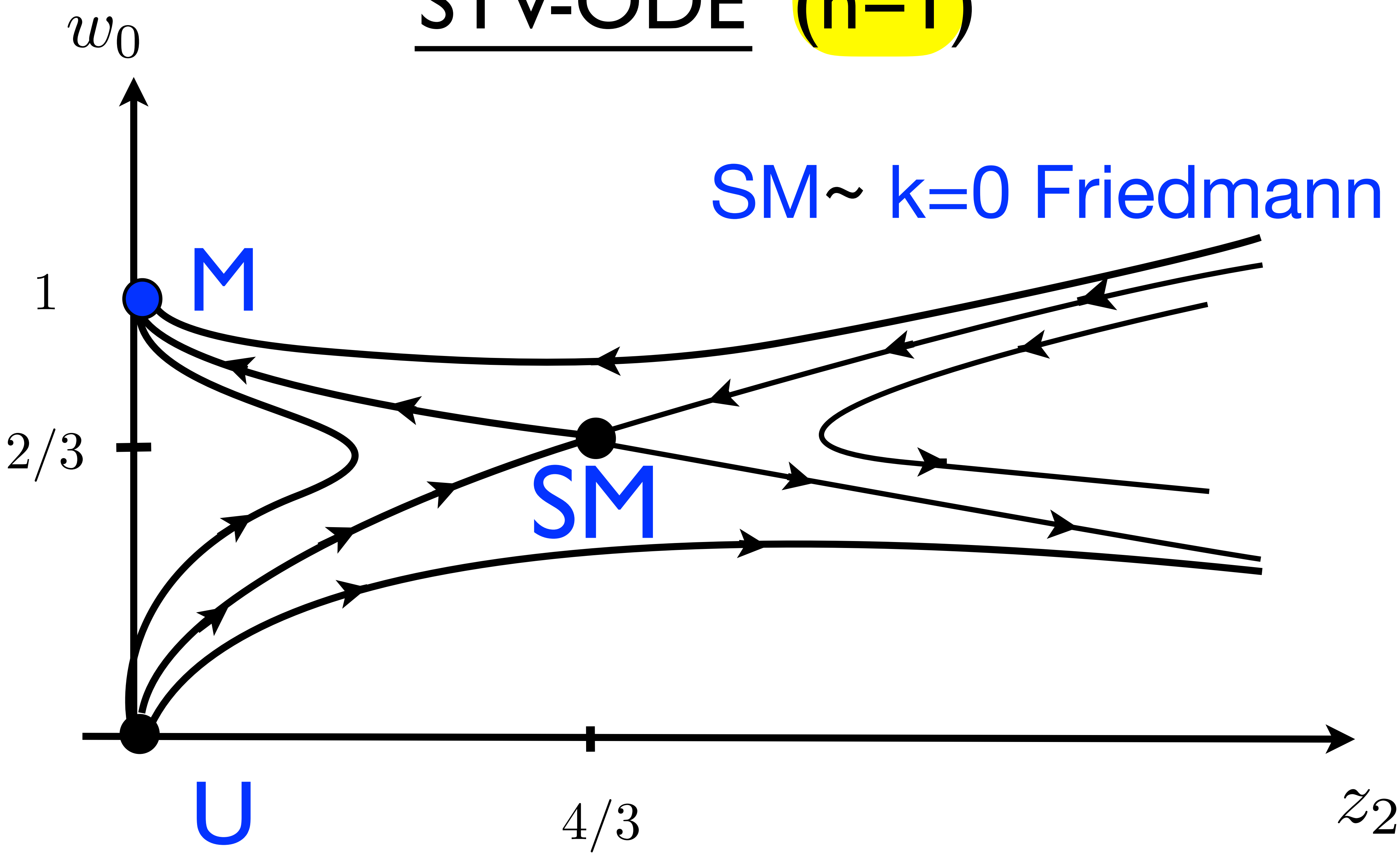
STV-ODE (n=1)



M degenerate stable node: $\lambda_M = -1 \quad \boldsymbol{R}_M = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

U degenerate unstable node: $\lambda_U = 1 \quad \boldsymbol{R}_U = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

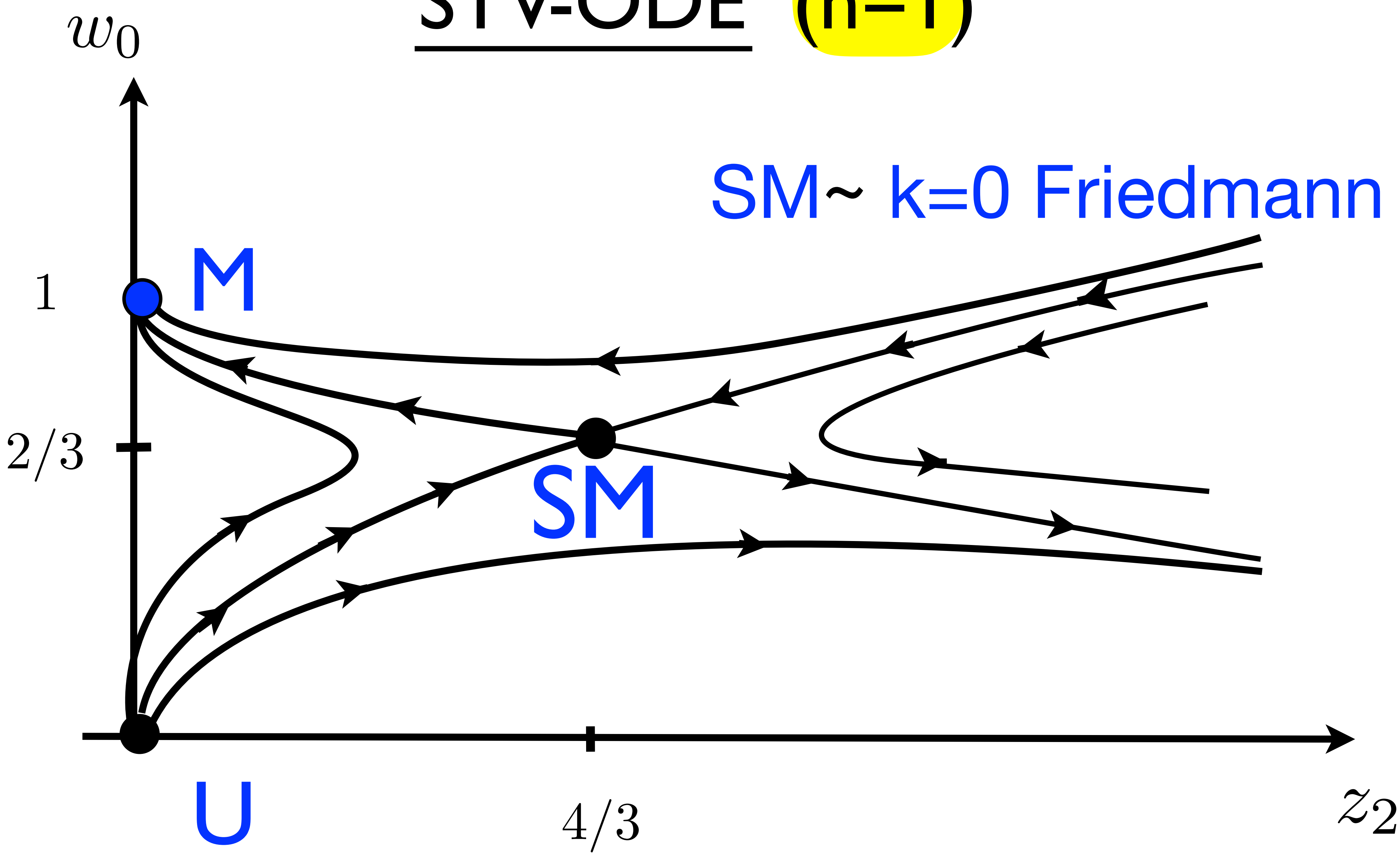
STV-ODE (n=1)



SM $= \left(\frac{4}{3}, \frac{2}{3}\right)$ regular unstable saddle:

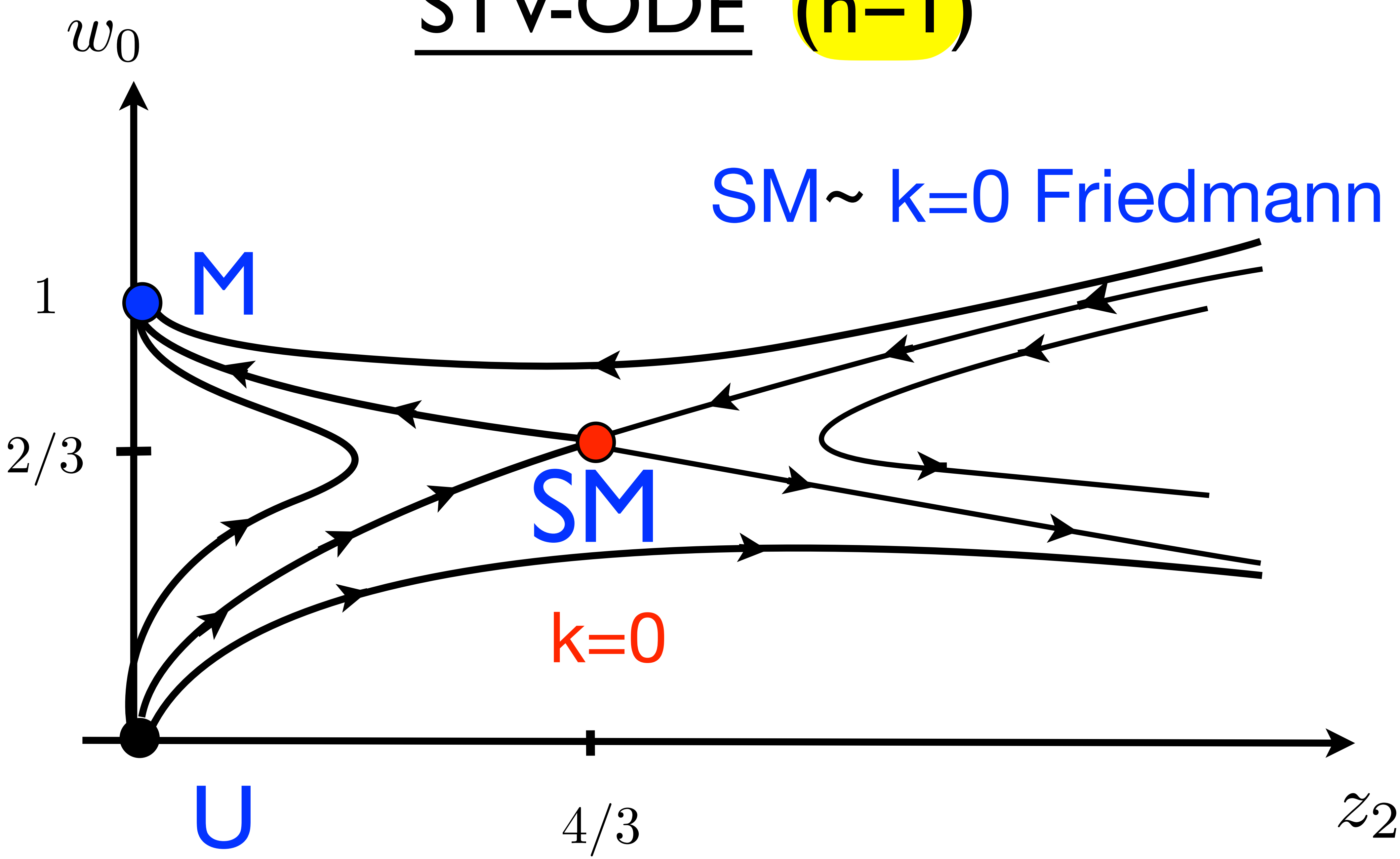
$$\lambda_{A1} = \frac{2}{3} \quad \boldsymbol{R}_{A1} = \begin{pmatrix} -9 \\ \frac{3}{2} \end{pmatrix}, \quad \lambda_{B1} = -1 \quad \boldsymbol{R}_{B1} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

STV-ODE (n=1)



Theorem: The Friedmann spacetimes are SM and unstable manifold of SM.

STV-ODE (n=1)



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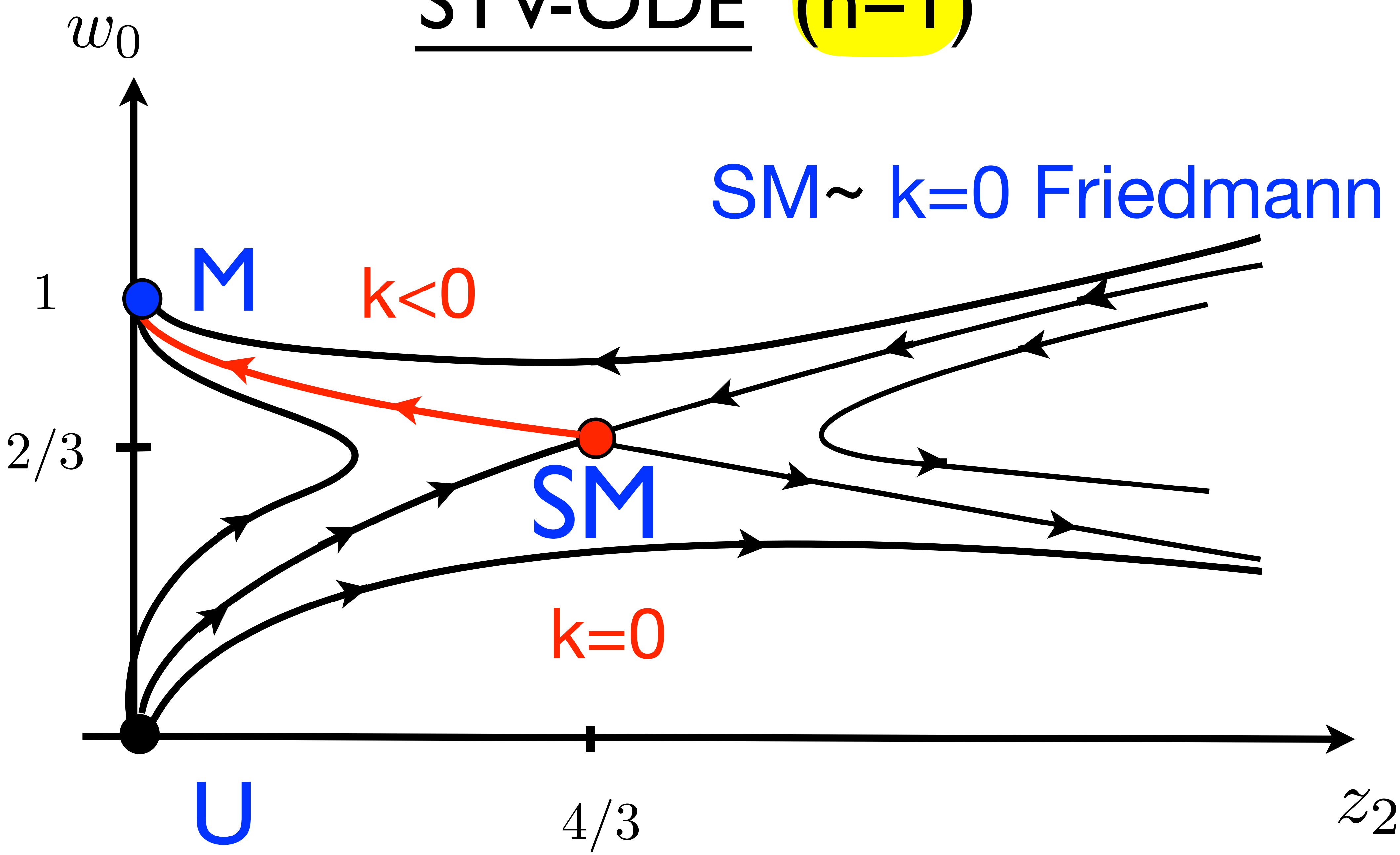

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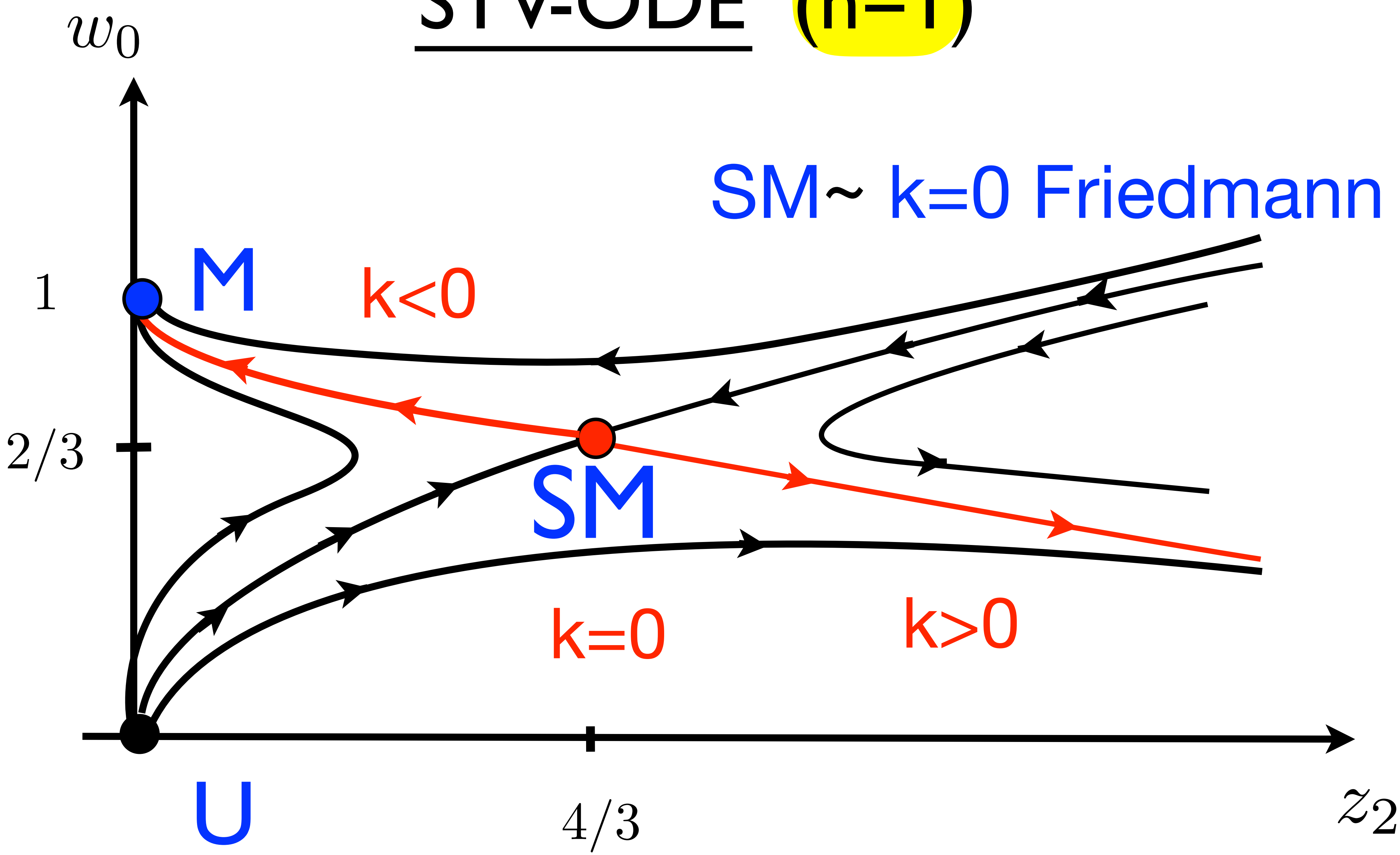

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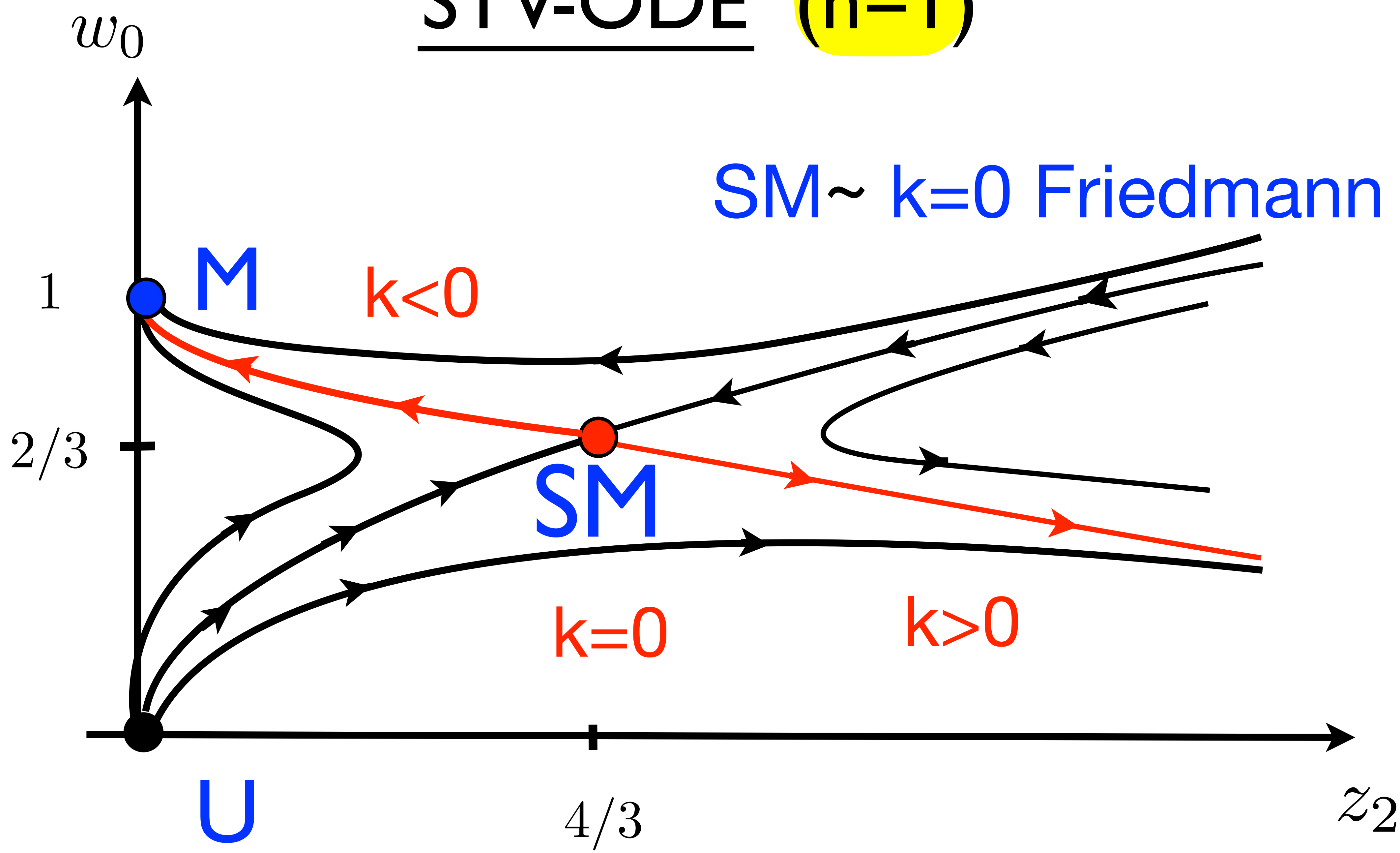

Theorem: The Friedmann spacetimes are SM and unstable manifold of SM.

Proof: Expand exact formulas in powers of ξ

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Proof: to get an exact formula for the trajectory which takes SM to M…

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- Start with exact formula for k<0 Friedmann metric in co-moving coordinates…

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$$R = \Delta_0 \sinh^2 \theta$$

,

STV-ODE (n=1)

Proof: to get an exact formula for the trajectory which takes SM to M…

- Start with exact formula for k<0 Friedmann metric in co-moving coordinates…

$$R = \Delta_0 \sinh^2 \theta\,, \qquad t = \frac{\Delta_0}{2}(\sinh 2\theta - 2\theta)$$

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…and extract the leading order terms…

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…and extract the leading order terms…

$$z_2^F(t) = \tilde{z}_2(\theta(t)), \qquad w_0^F(t) = \tilde{w}_0(\theta(t))$$

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$$z_2^F(t) = \tilde{z}_2(\theta(t)), \qquad w_0^F(t) = \tilde{w}_0(\theta(t))$$

$$\tilde{z}_2(\theta) = \frac{6\big(\sinh 2\theta - 2\theta\big)^2}{\big(\cosh 2\theta - 1\big)^3}, \qquad \tilde{w}_0(\theta) = \frac{\big(\sinh 2\theta - 2\theta\big)\sinh 2\theta}{\big(\cosh 2\theta - 1\big)^2}$$

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-Then check the resulting formula takes SM to M…

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-Then check the resulting formula takes SM to M…

…we know Friedmann is some orbit, so this is it!

STV-ODE (n=1)

Theorem: There exists a solution dependent SSC time translation

$$t \to t - t_*$$

which maps each trajectory to SM or the unstable manifold of SM at order n=1.

Defn: We call this "time since the Big Bang"

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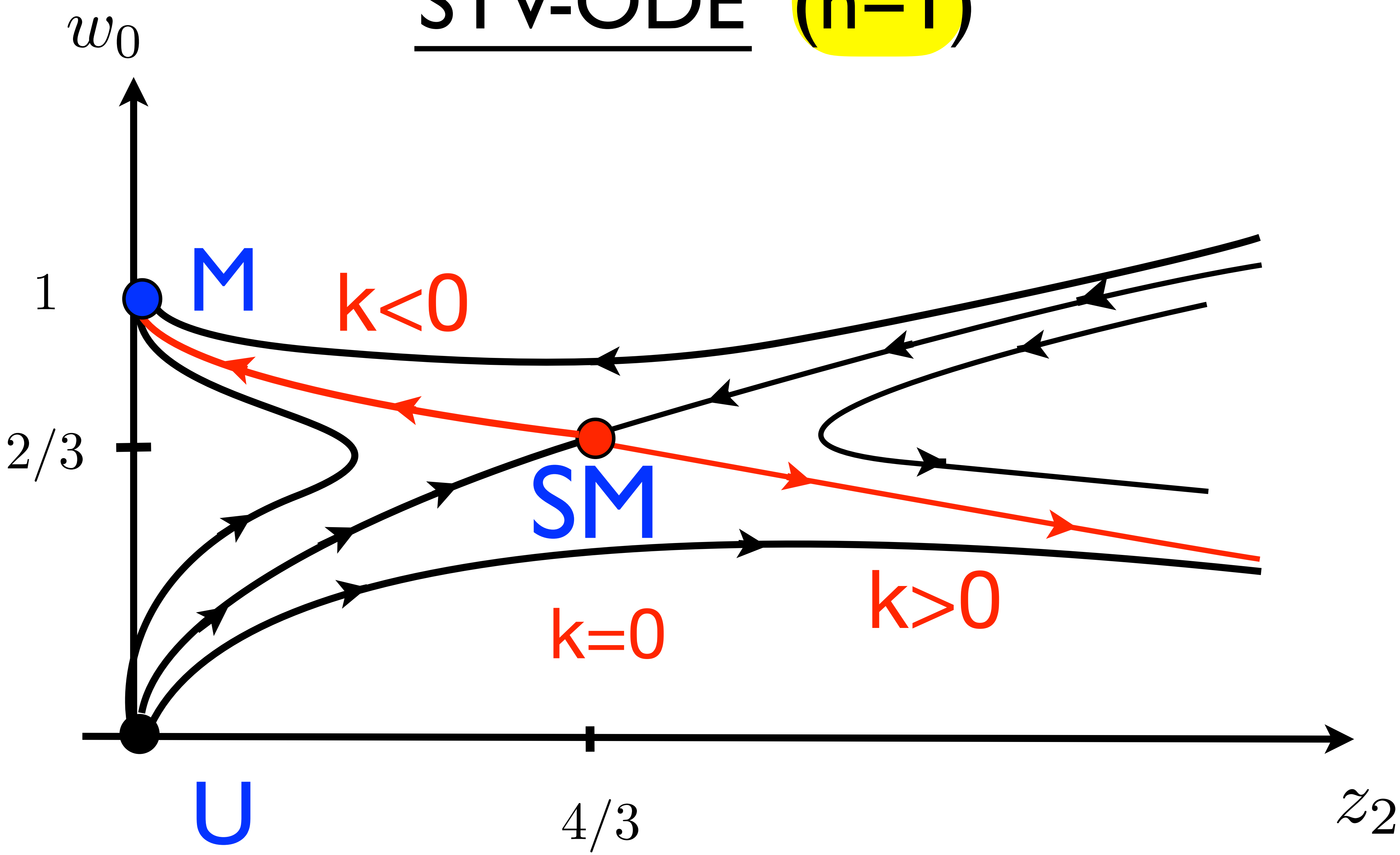
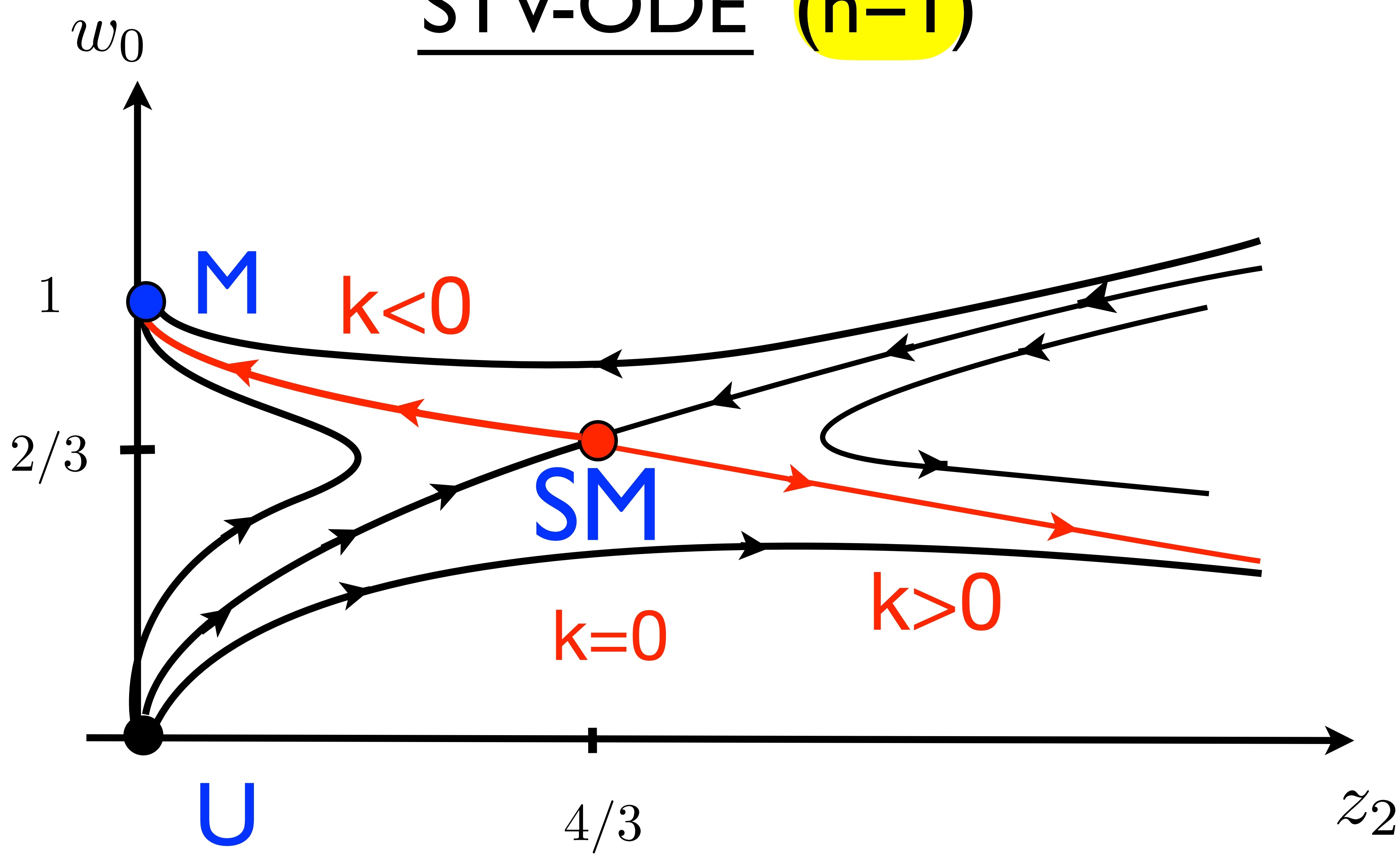
Cor: Every solution of the STV-PDE agrees with Friedmann at leading order n=1.

STV-ODE (n=1)



The final gauge freedom in the STV-ODE

STV-ODE (n=1)



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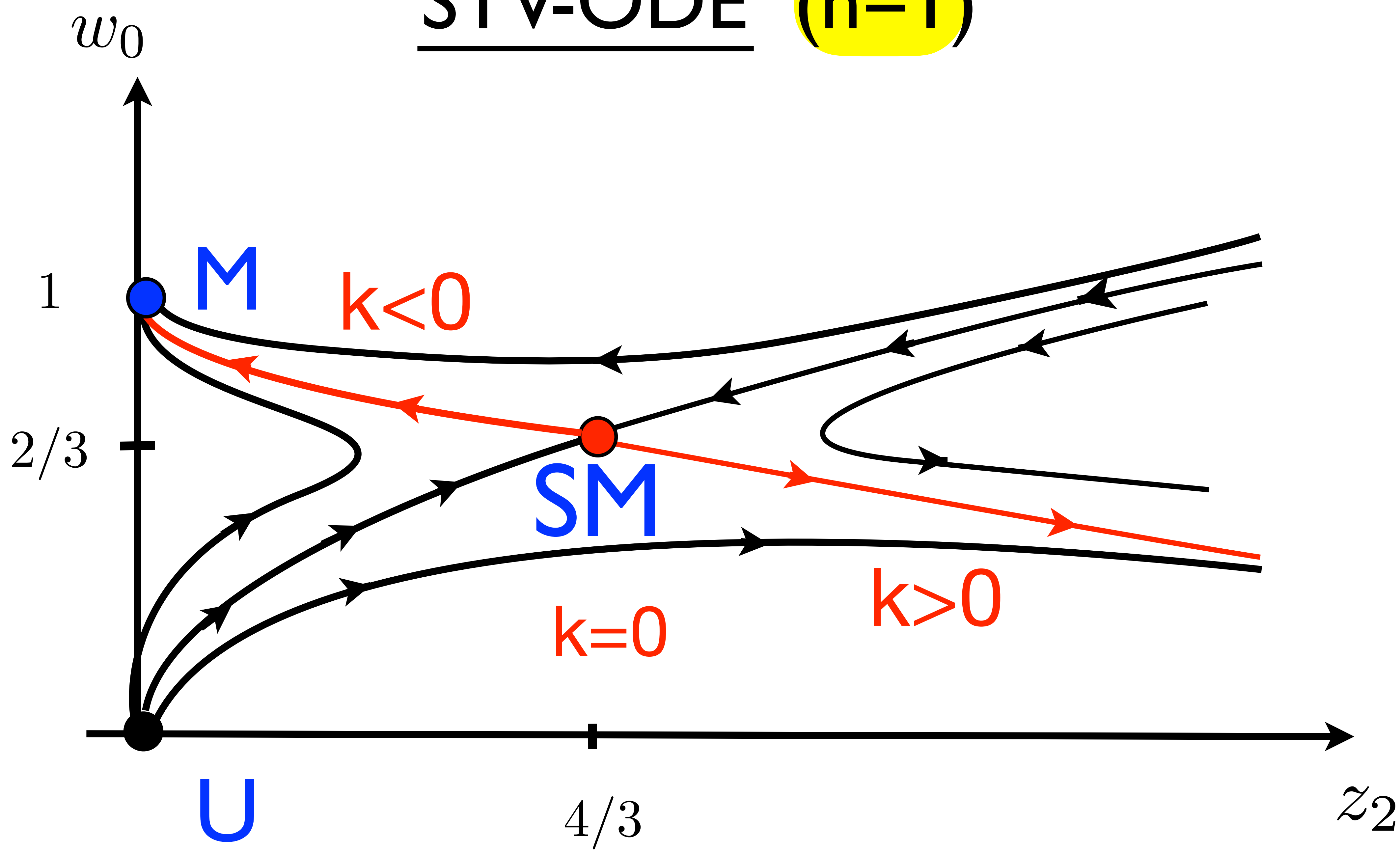
STV-ODE (n=1)

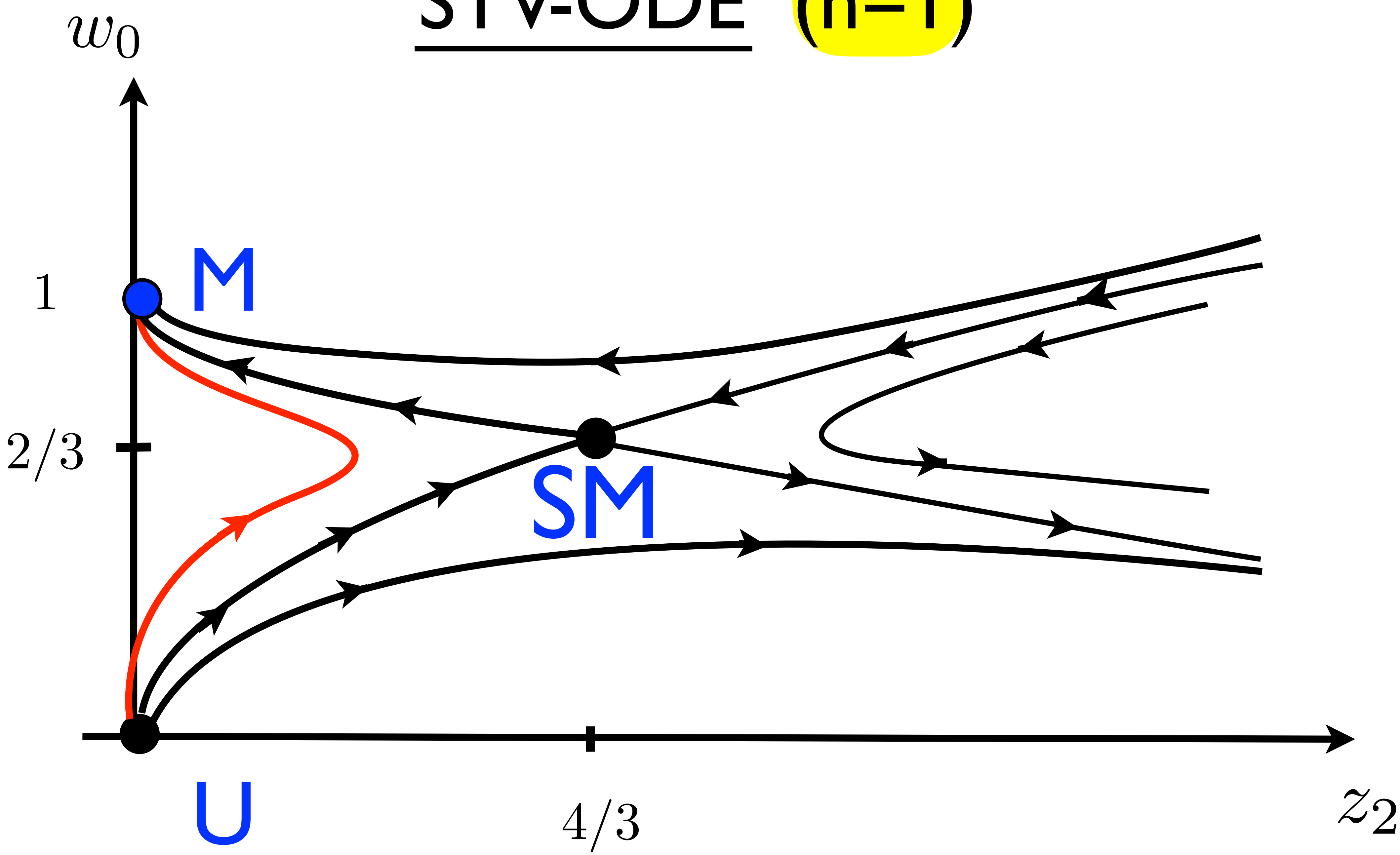


The final gauge freedom in the STV-ODE

$$t \to t - t_*$$

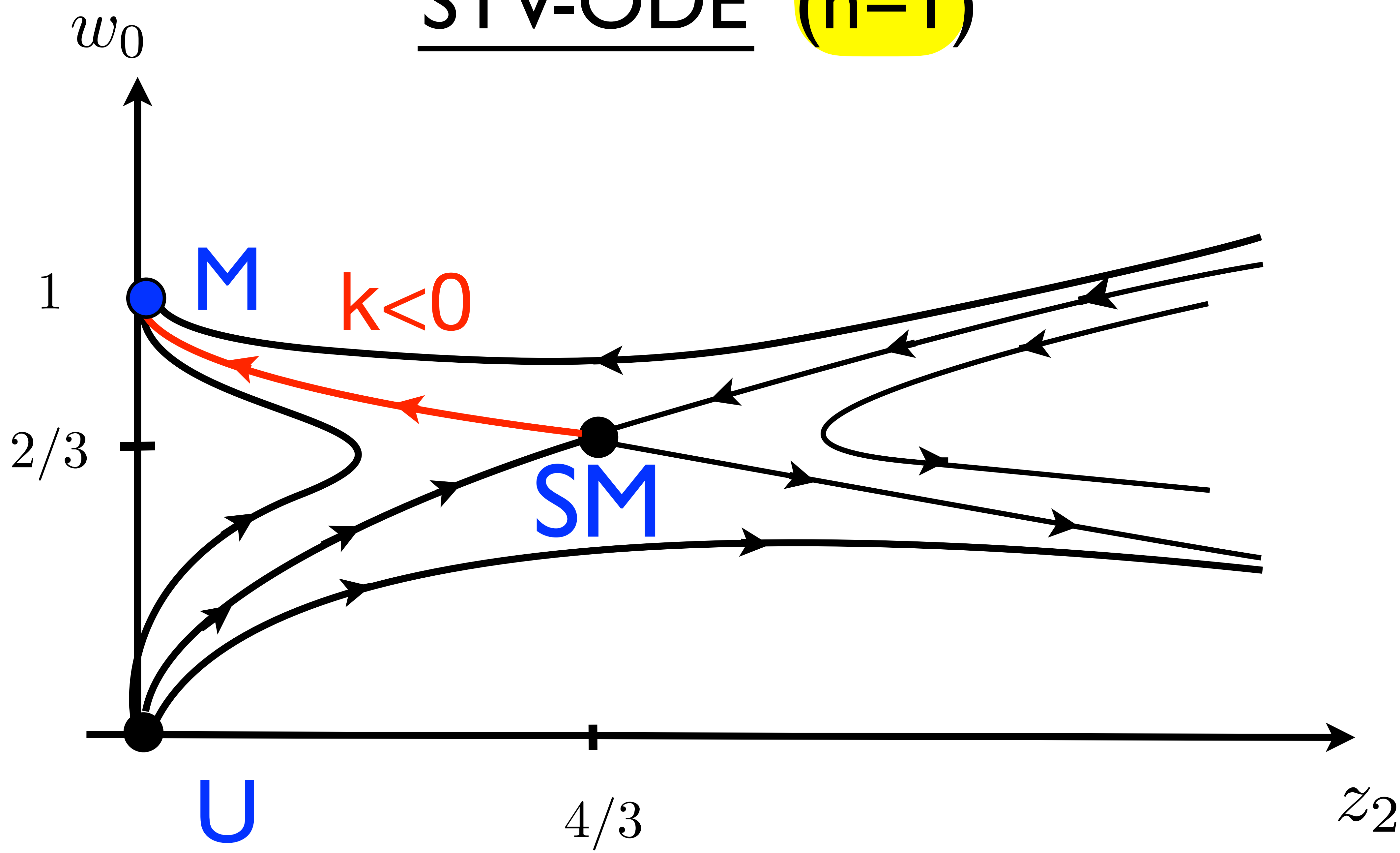
$$\xi = \frac{r}{t} \to \frac{r}{t - t_*}$$

STV-ODE (n=1)



Imposing time since the Big Bang transforms each solution to SM or one of the two branches of the unstable manifold of SM.

STV-ODE (n=1)



Imposing time since the Big Bang transforms each solution to SM or one of the two branches of the unstable manifold of SM.

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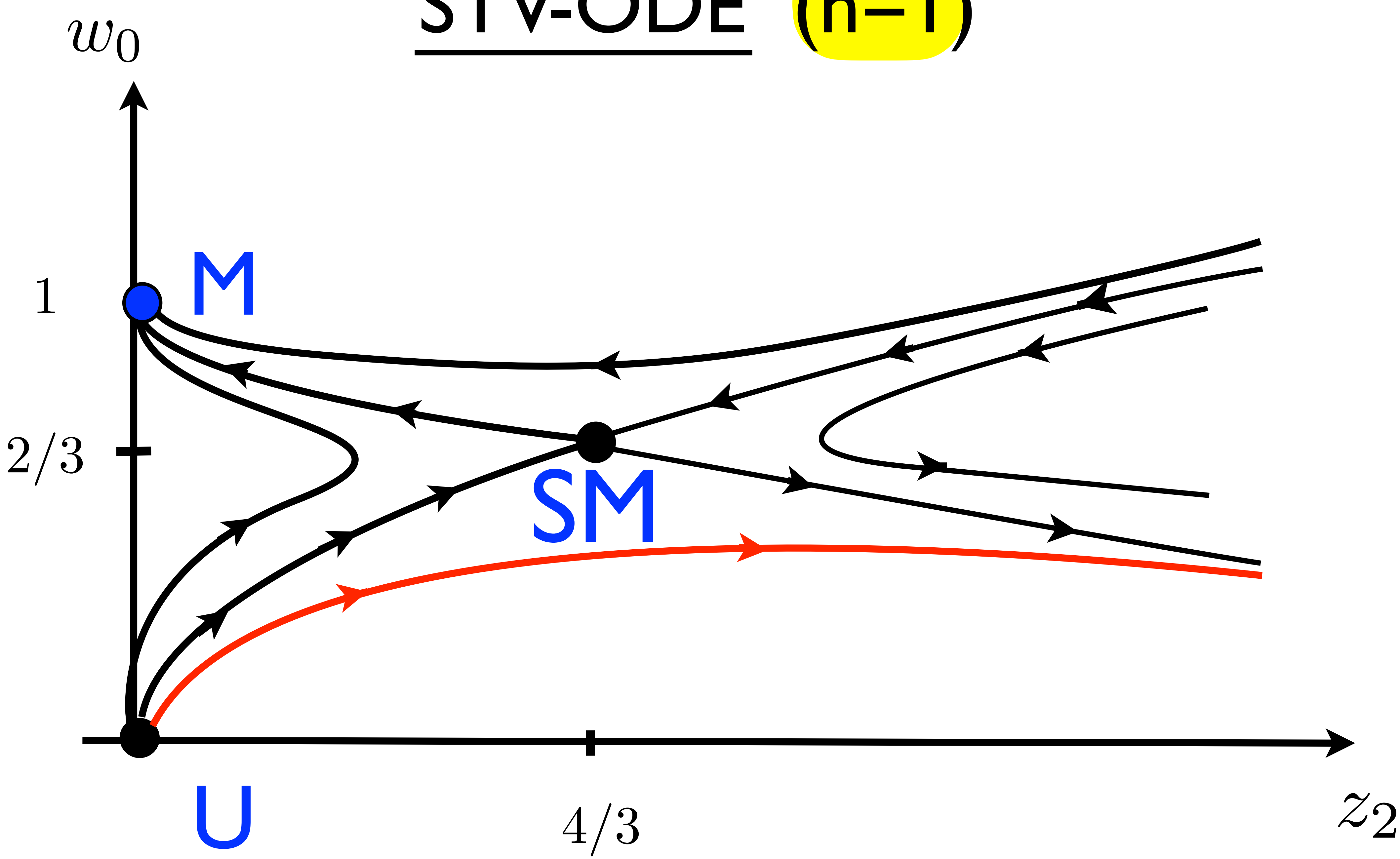

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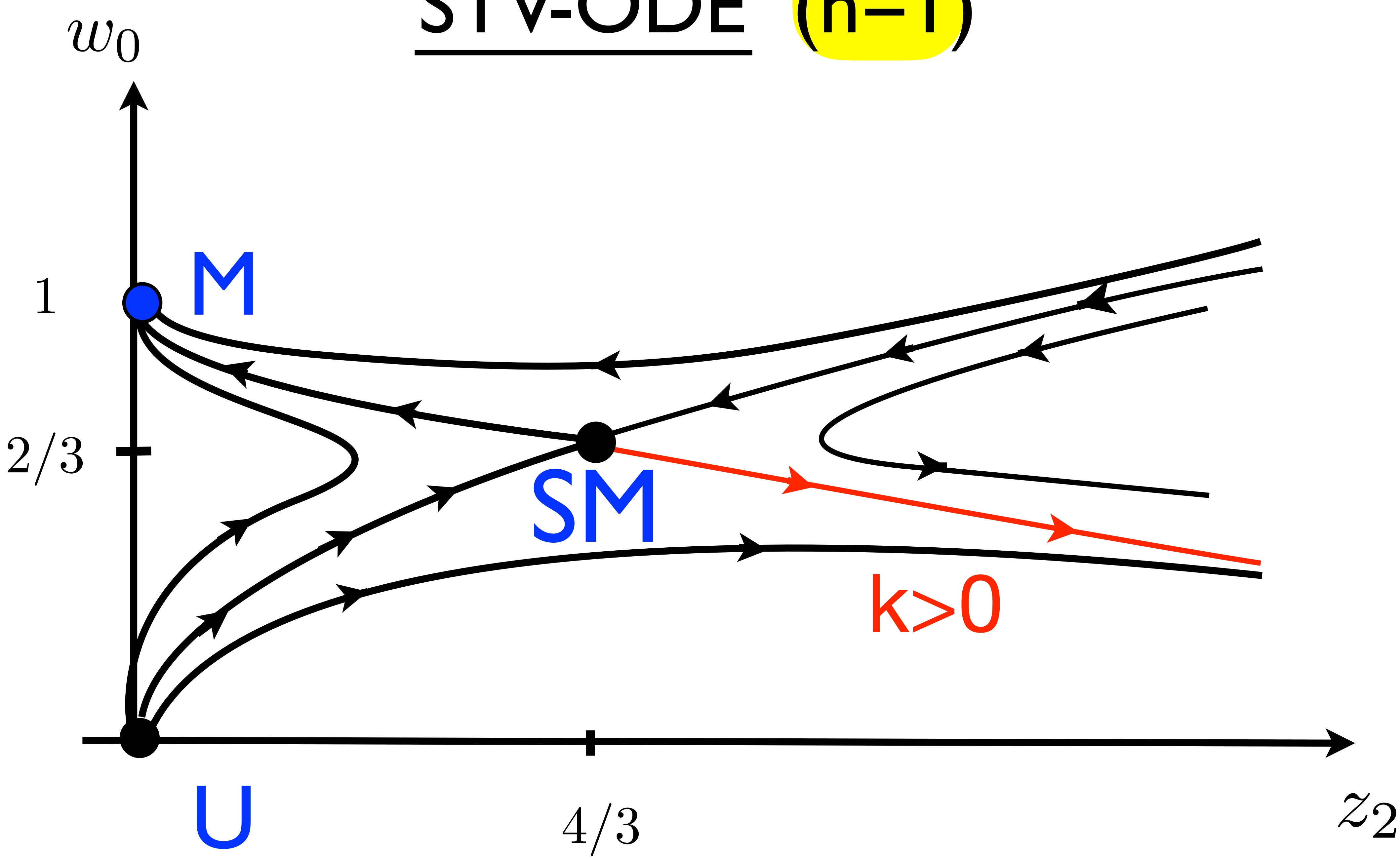

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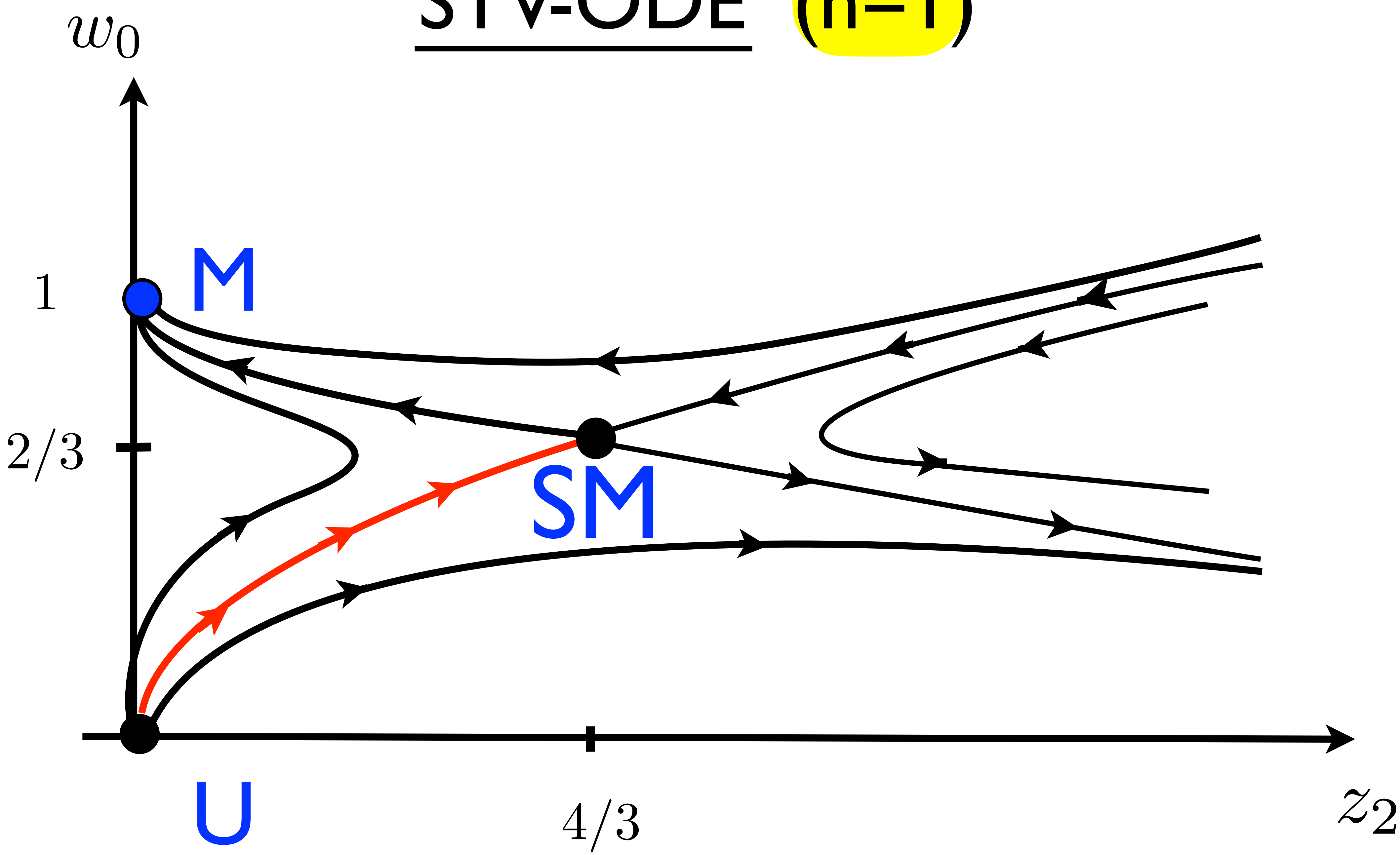

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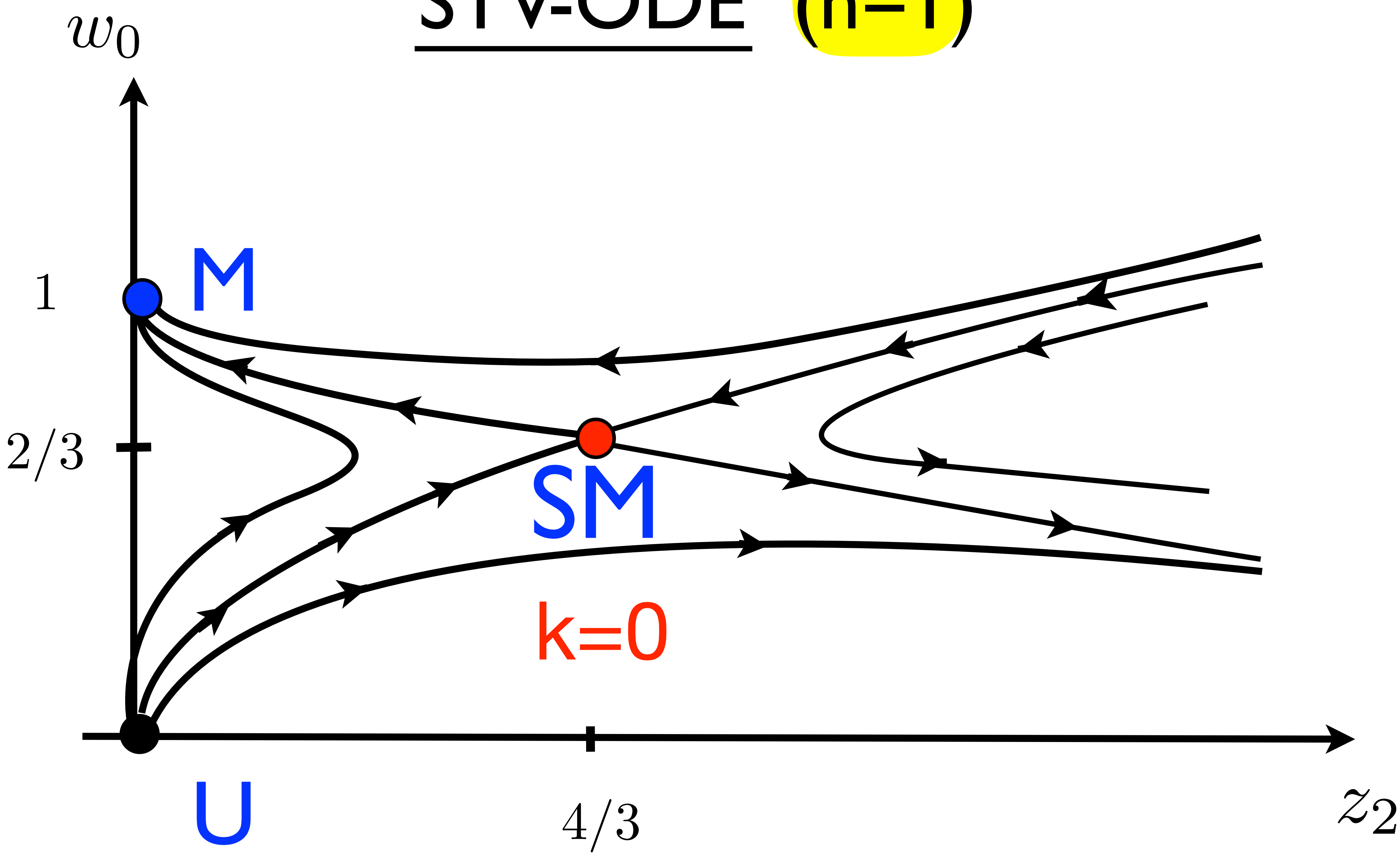

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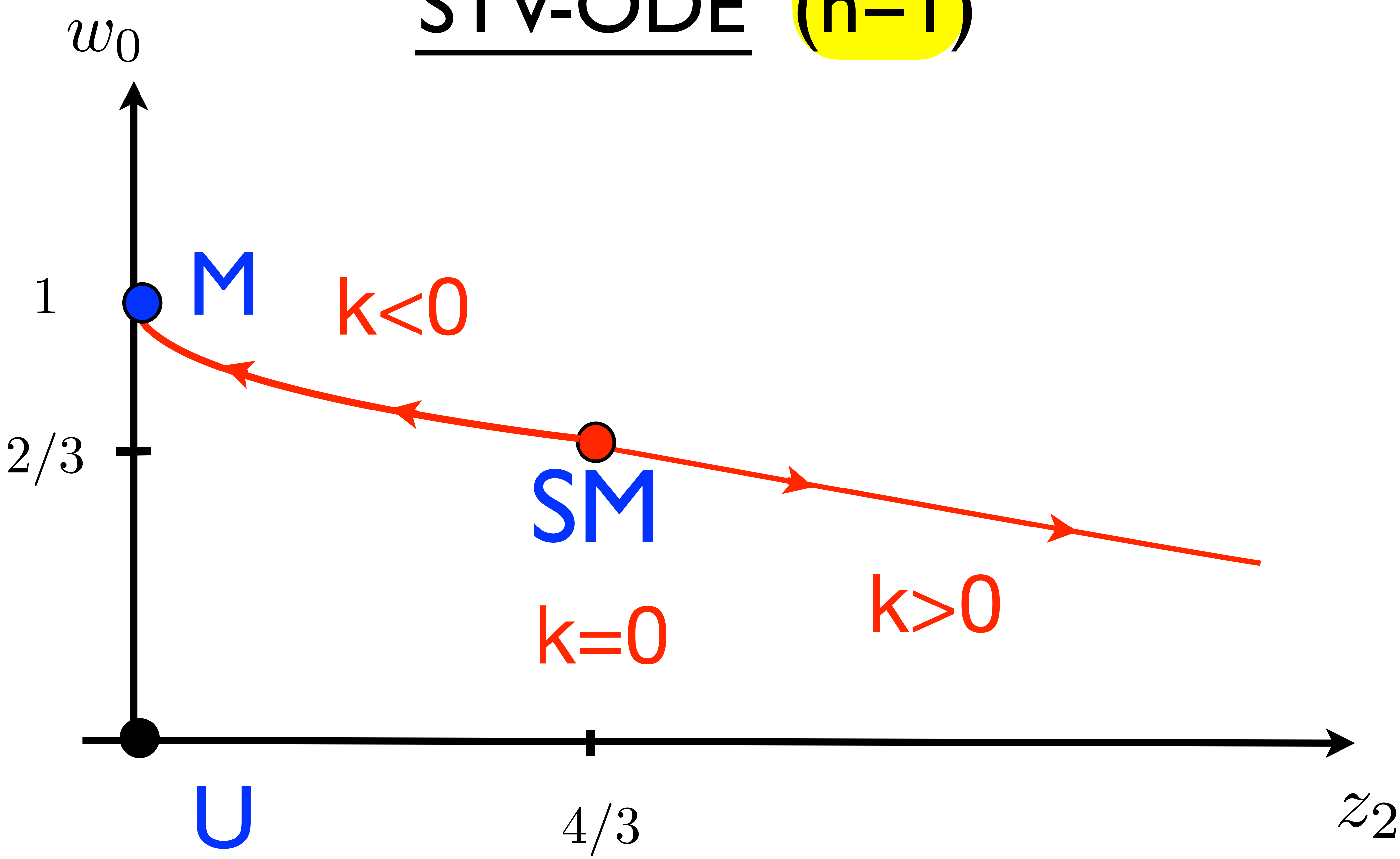
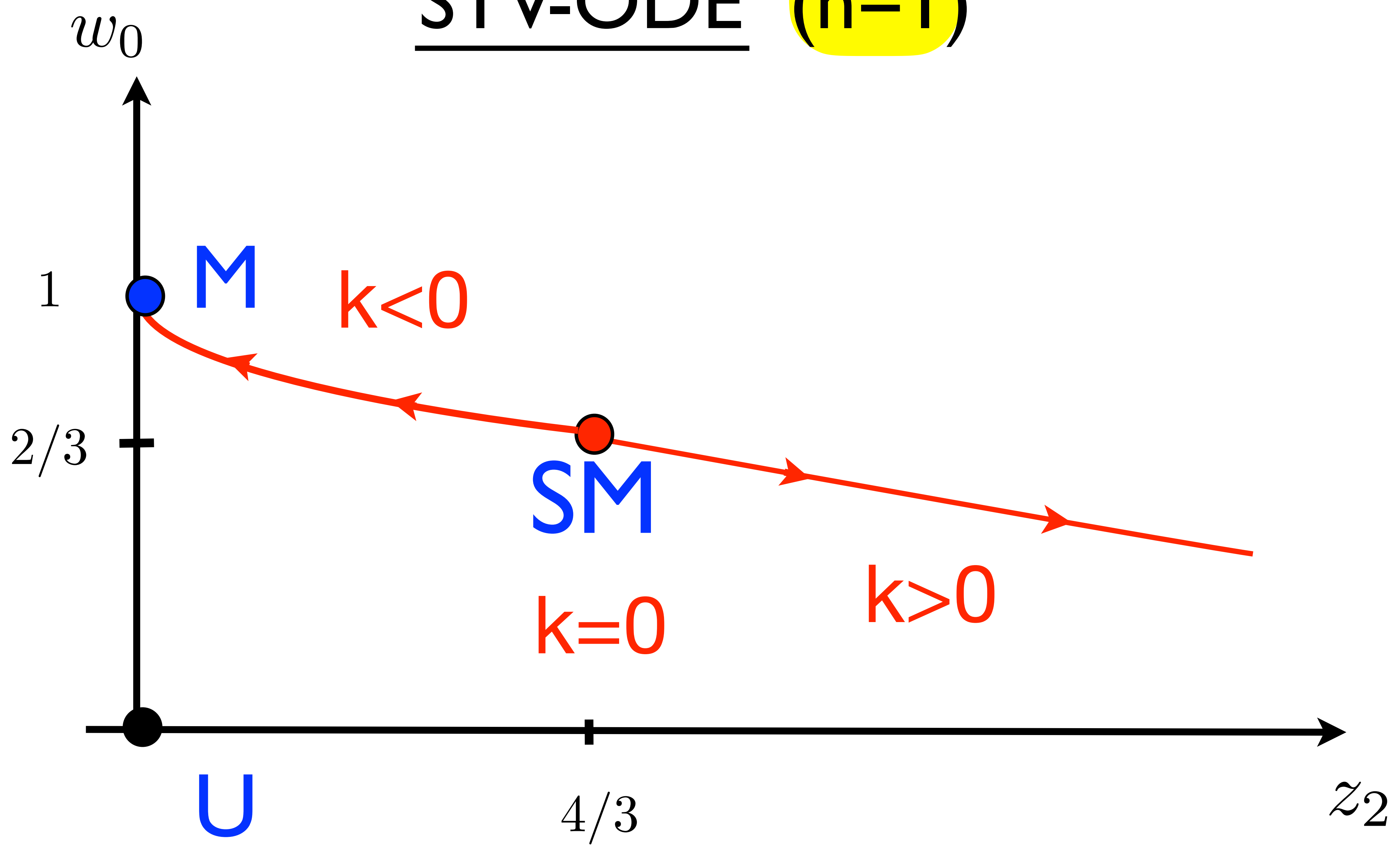


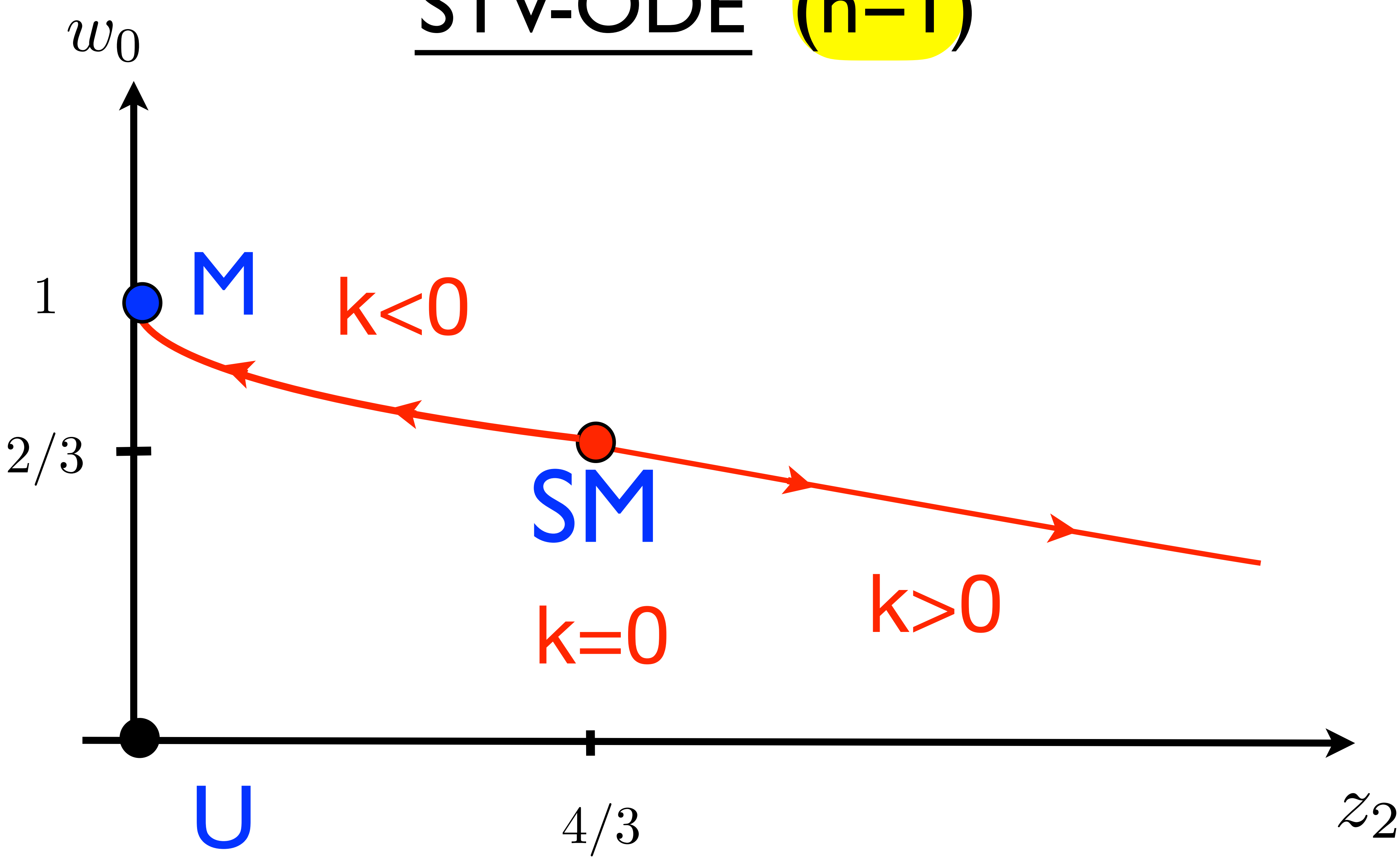
Imposing time since the Big Bang collapses the n=1 phase portrait to just three trajectories…

STV-ODE (n=1)



This eliminates a degree of freedom in STV-ODE

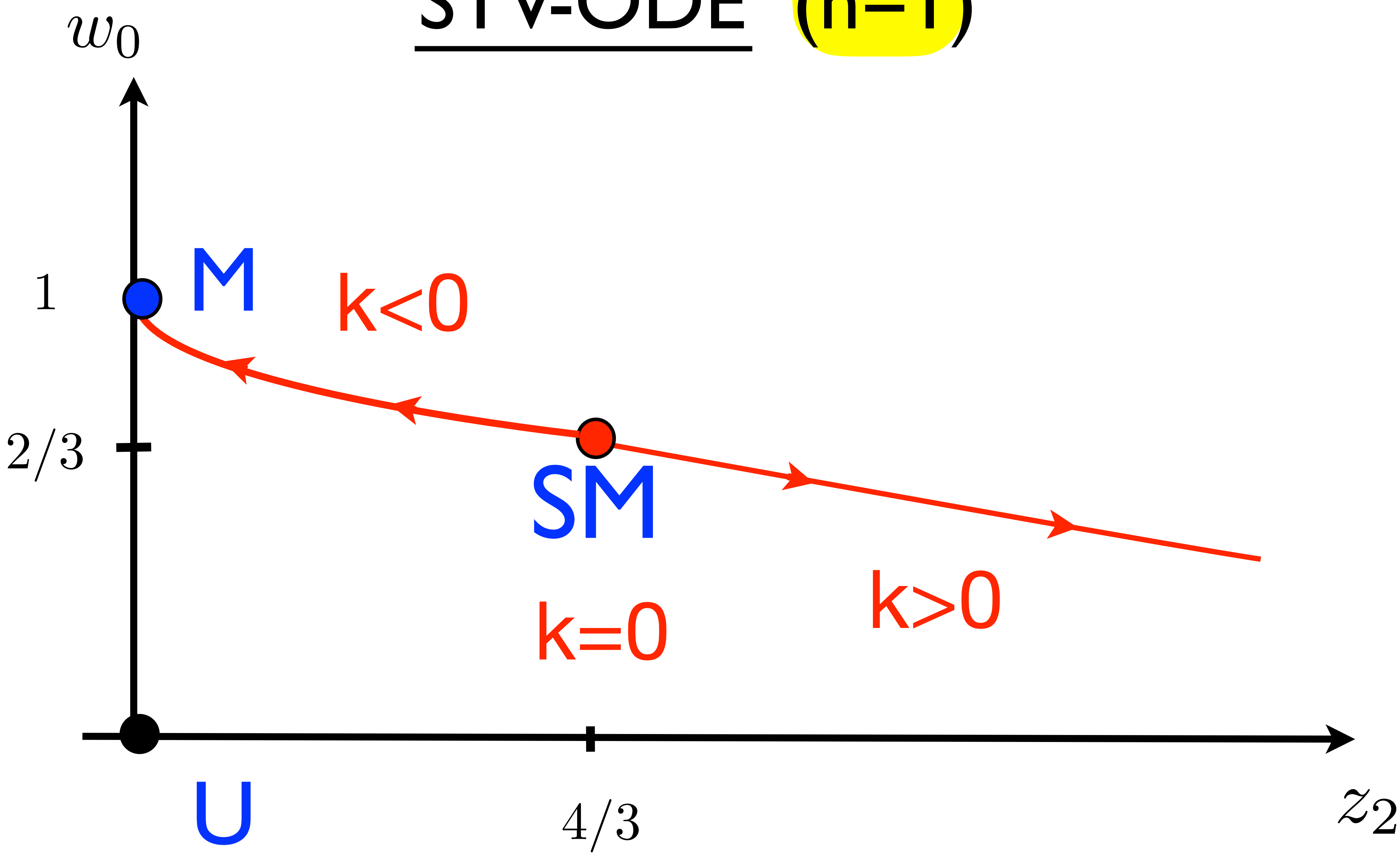
STV-ODE (n=1)



This eliminates a degree of freedom in STV-ODE

…eliminates the rest point U

STV-ODE (n=1)



This eliminates a degree of freedom in STV-ODE

…eliminates the rest point U

…eliminates the negative eigenvalue at SM

STV-ODE (n=1)

- Our formula for the leading order part of $k<0$ Friedmann spacetimes provides an exact formula for a universal spacetime to which every smooth under-dense solution agrees at leading order…

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STV-ODE (n=1)

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"...and extract the leading order terms..."

$$z_2^F(t) = \tilde{z}_2(\theta(t)), \quad w_0^F(t) = \tilde{w}_0(\theta(t))$$

STV-ODE (n=1)

- Our formula for the leading order part of k<0 Friedmann spacetimes provides an exact formula for a universal spacetime to which every smooth under-dense solution agrees at leading order…

"…and extract the leading order terms…"

$$z_2^F(t) = \tilde{z}_2(\theta(t)), \quad w_0^F(t) = \tilde{w}_0(\theta(t))$$

$$\tilde{z}_2(\theta) = \frac{6\left(\sinh 2\theta - 2\theta\right)^2}{\left(\cosh 2\theta - 1\right)^3}, \quad \tilde{w}_0(\theta) = \frac{\left(\sinh 2\theta - 2\theta\right)\sinh 2\theta}{\left(\cosh 2\theta - 1\right)^2}$$

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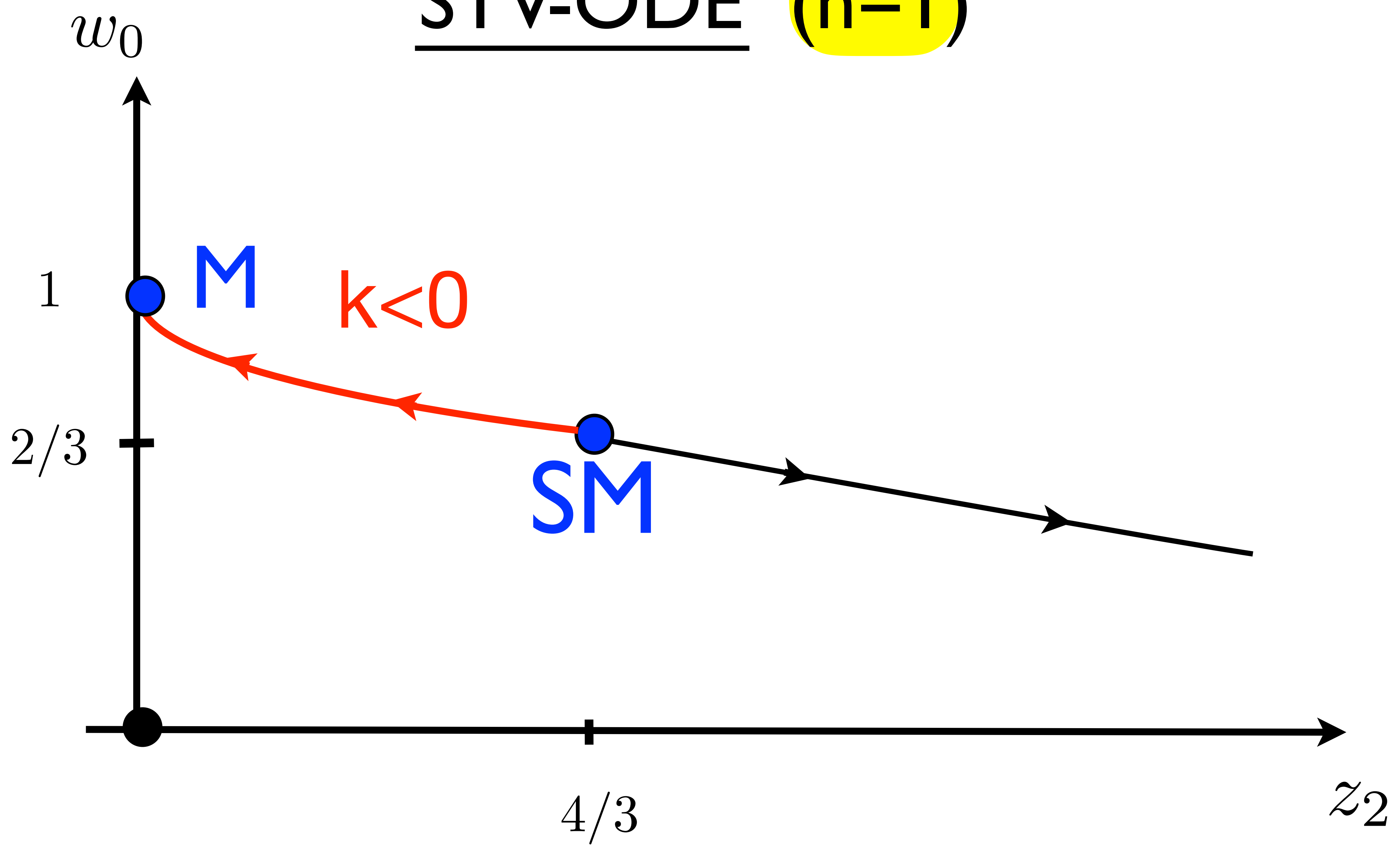
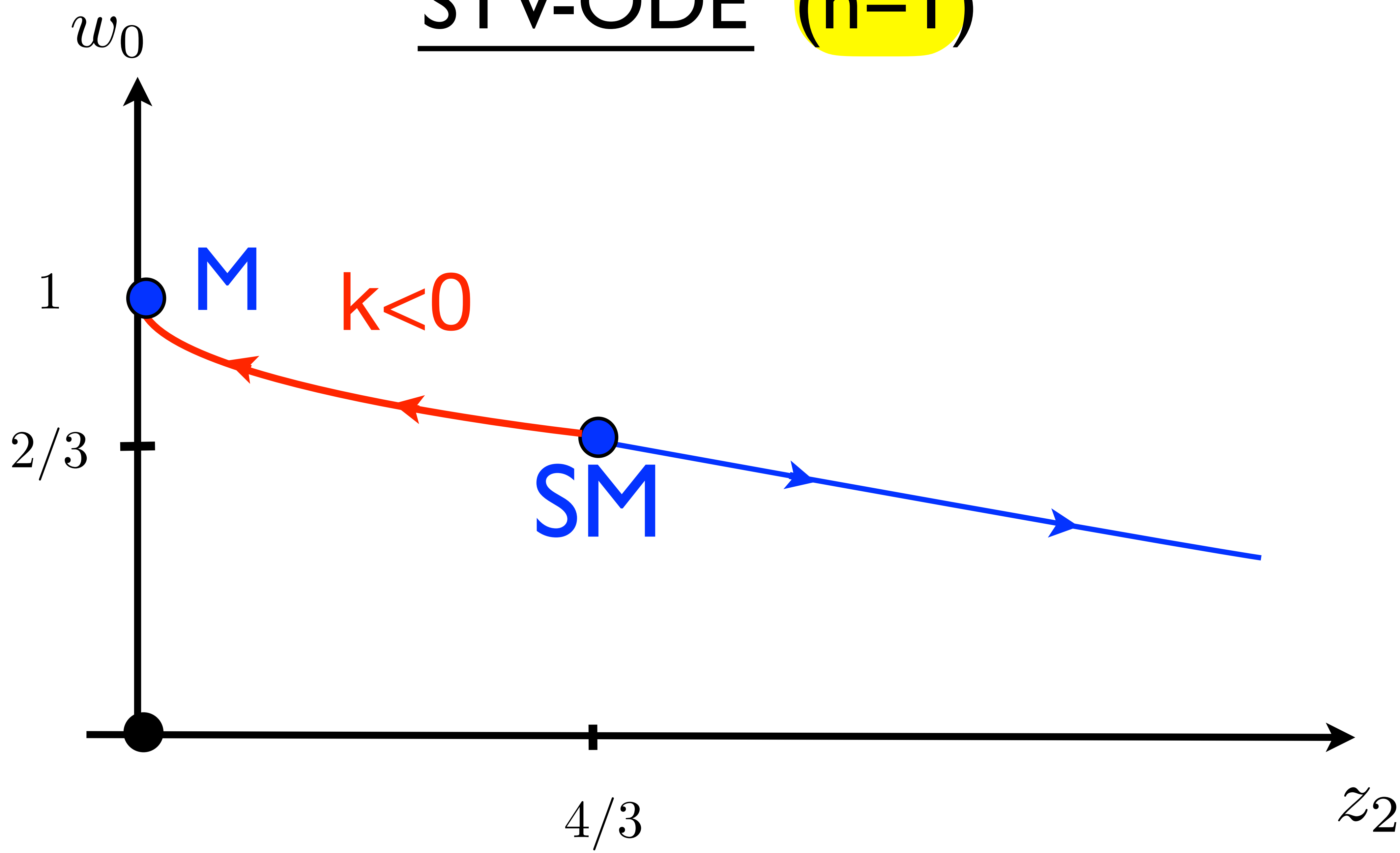
- Similarly for k=0 and k>0...

STV-ODE (n=1)



We say a solution is underdense iff k<0 at n=1…

STV-ODE (n=1)



We say a solution is underdense iff k<0 at n=1…
iff trajectory connects SM to M at order n=1…

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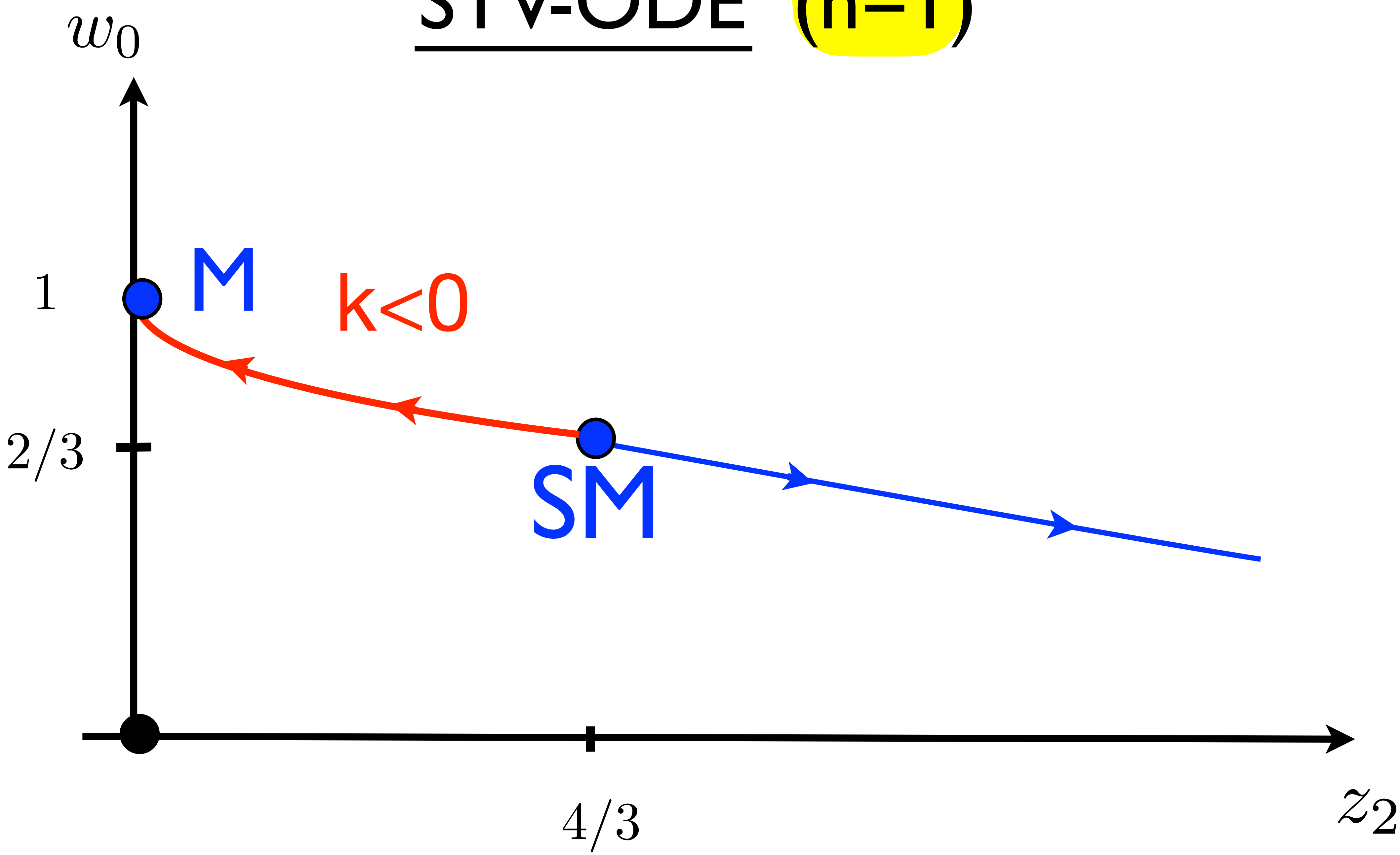
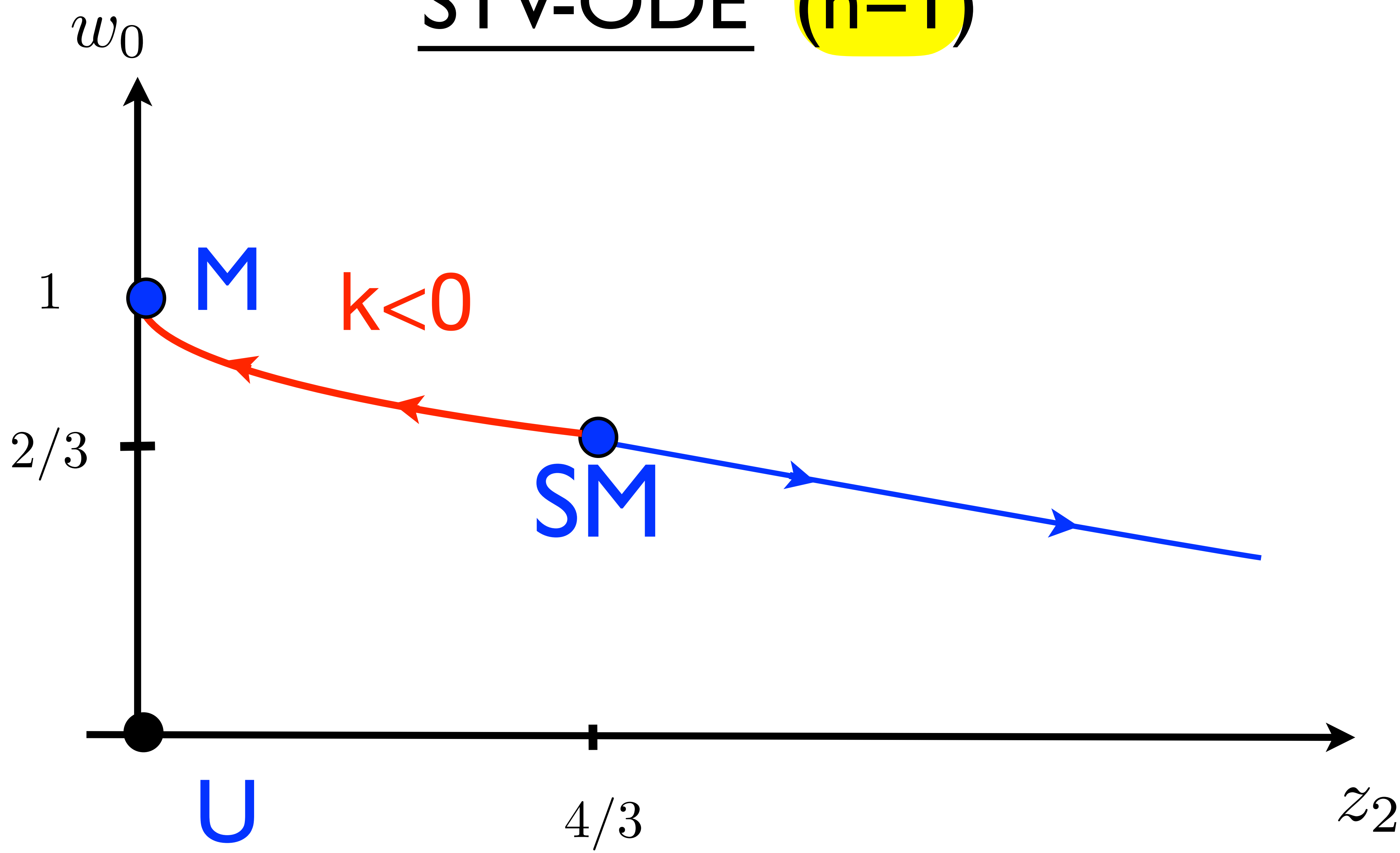


We say a solution is underdense iff k<0 at n=1…

iff trajectory connects SM to M at order n=1…

but we are still free to impose i.c. at each n>1.

STV-ODE (n=1)



(Theorem: If a solution tends to M at order n=1, then it tends to M at every order n>1 as well.)

STV-ODE (n=2)

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Setting n=2:

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$$t\dot{z}_2 = 2z_2 - 3z_2 w_0$$

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$$t\dot{z}_4 = \frac{5}{12} z_2^2 w_0 - 5w_0 z_4 + 4z_4 - 5z_2 w_2$$

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…a 4x4 autonomous system with 3 rest points

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SM or its unstable manifold
only one free parameter

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"time since big bang"

STV-ODE (n=2)

The Rest Points:

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STV-ODE (n=2)

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"time since big bang"

STV-ODE (n=2)

The Rest Points:

M: $\lambda_M = -1, \quad \boldsymbol{R}_M = (0,1,0,1)^T$

SM $= \left(\frac{4}{3}, \frac{2}{3}, \frac{40}{27}, \frac{2}{9}\right)$, M $= (0,1,0,0)$, ~~U $= (0,0,0,0)$~~

"time since big bang"

STV-ODE (n=2)

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Degenerate stable node…
…orbits enter along w-axis

SM $= (\frac{4}{3}, \frac{2}{3}, \frac{40}{27}, \frac{2}{9})$, M $= (0, 1, 0, 0)$, ~~U $= (0, 0, 0, 0)$~~

"time since big bang"

STV-ODE (n=2)

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"time since big bang"

STV-ODE (n=2)

The Rest Points:

SM:

$$\lambda_{A1} = \frac{2}{3}, \qquad \boldsymbol{R}_{A1} = \left(-9, \frac{3}{2}, -\frac{10}{3}, -1\right)^T$$

$$\lambda_{B1} = -1, \qquad \boldsymbol{R}_{B1} = \left(4, 1, \frac{80}{9}, 1\right)^T$$

$$\lambda_{A2} = \frac{4}{3}, \qquad \boldsymbol{R}_{A2} = (0, 0, -10, 1)^T$$

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SM is a regular saddle rest point…

…with 2-dimensional unstable manifold…

STV-ODE (n=2)

The Rest Points:

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$$\lambda_{A1} = \frac{2}{3}, \qquad \boldsymbol{R}_{A1} = \left(-9, \frac{3}{2}, -\frac{10}{3}, -1\right)^T$$

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Theorem: The Friedmann spacetimes are the trajectory of the leading order e-value $\lambda_{A1} = \frac{2}{3}$

STV-ODE (n=2)

The Rest Points:

SM:

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Theorem: The Friedmann spacetimes are the trajectory of the leading order e-value $\lambda_{A1} = \frac{2}{3}$

Proof: Expand exact formulas in powers of ξ.

STV-ODE (n=2)

The Rest Points:

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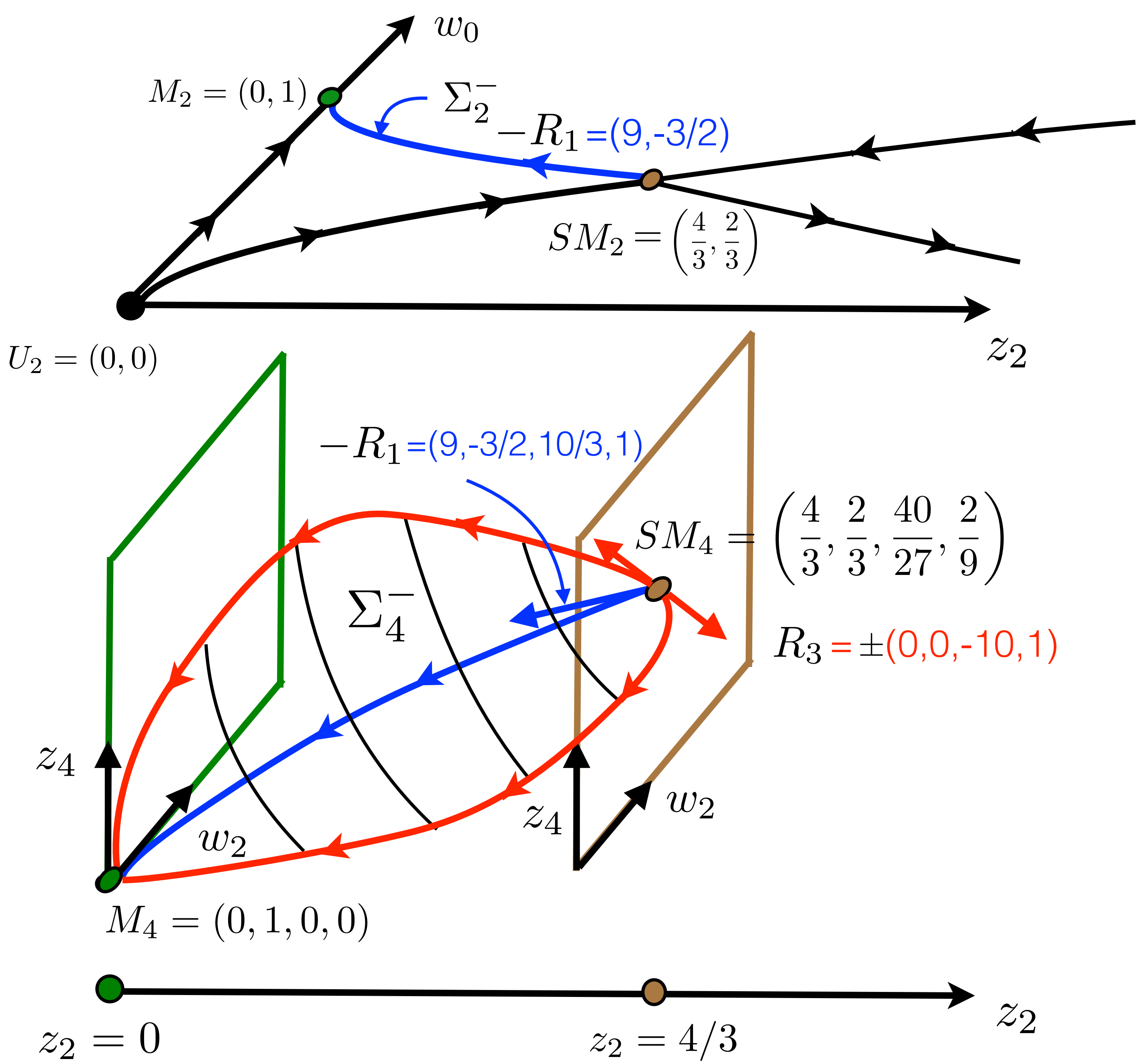
Conclude: The $k \neq 0$ Friedmann spacetimes correspond to one trajectory in the 2-dimensional unstable manifold of SM...

Phase Portrait: STV-ODE (n=2)



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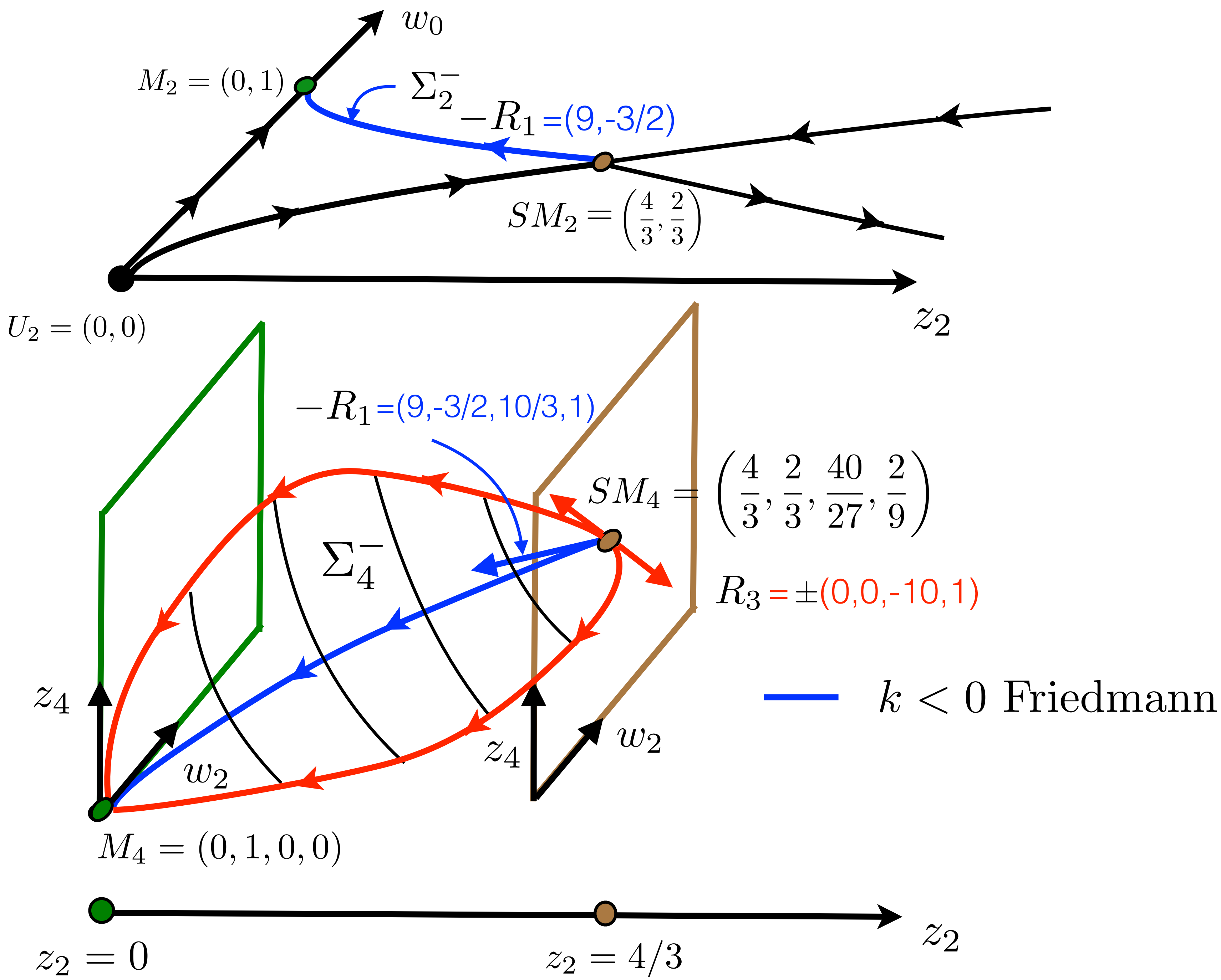
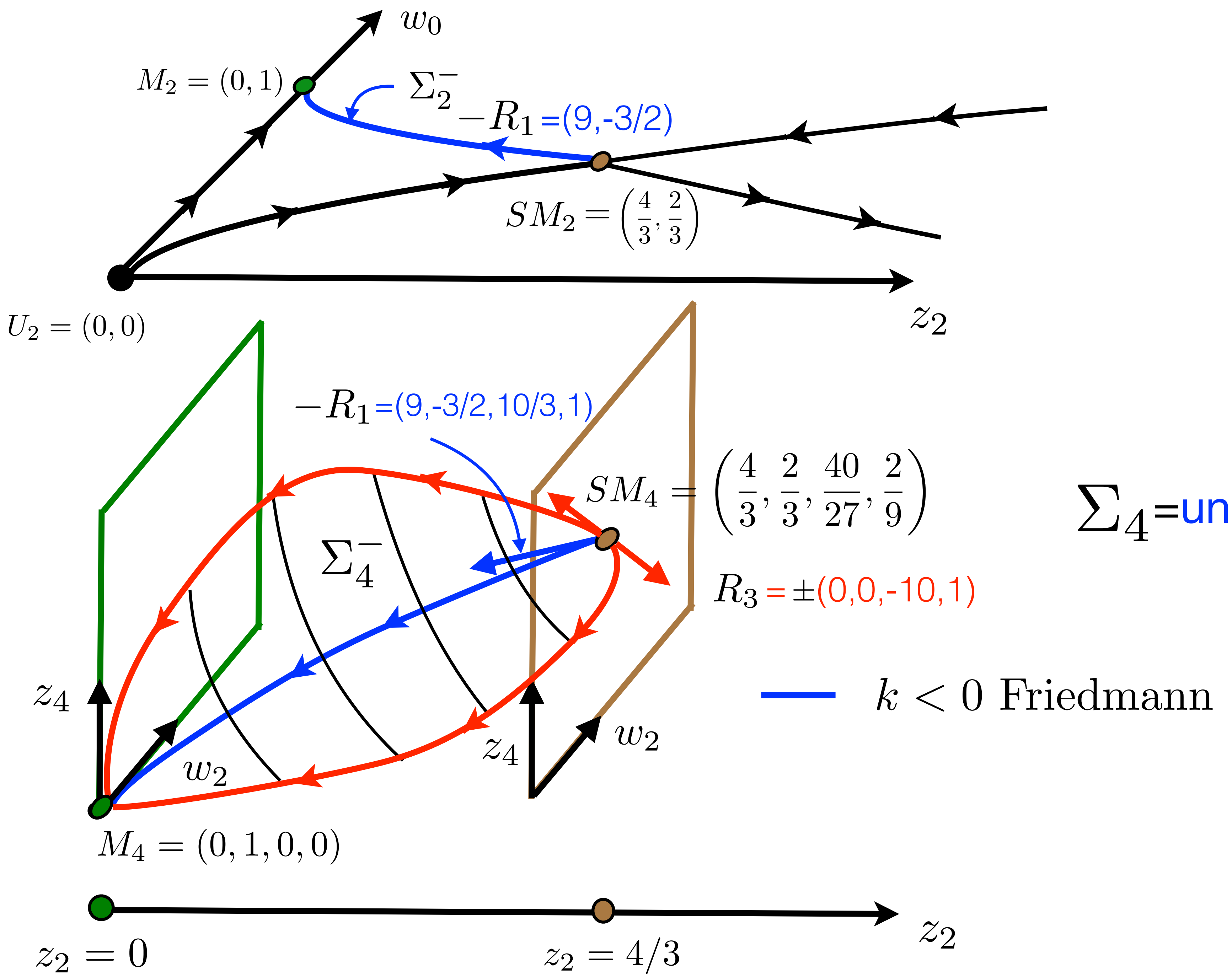
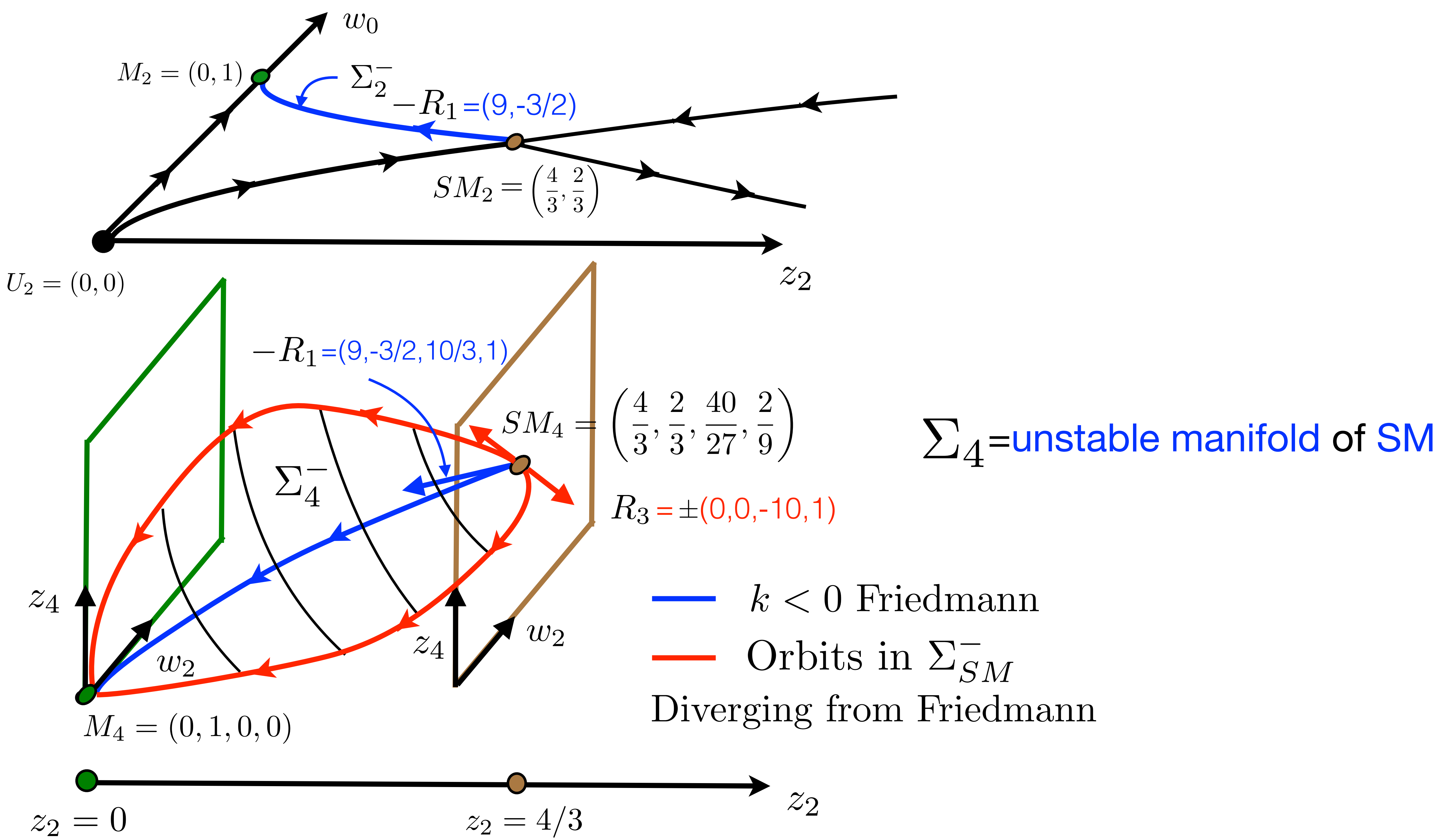


Phase Portrait: STV-ODE (n=2)



Σ_4=unstable manifold of SM

Phase Portrait: STV-ODE (n=2)



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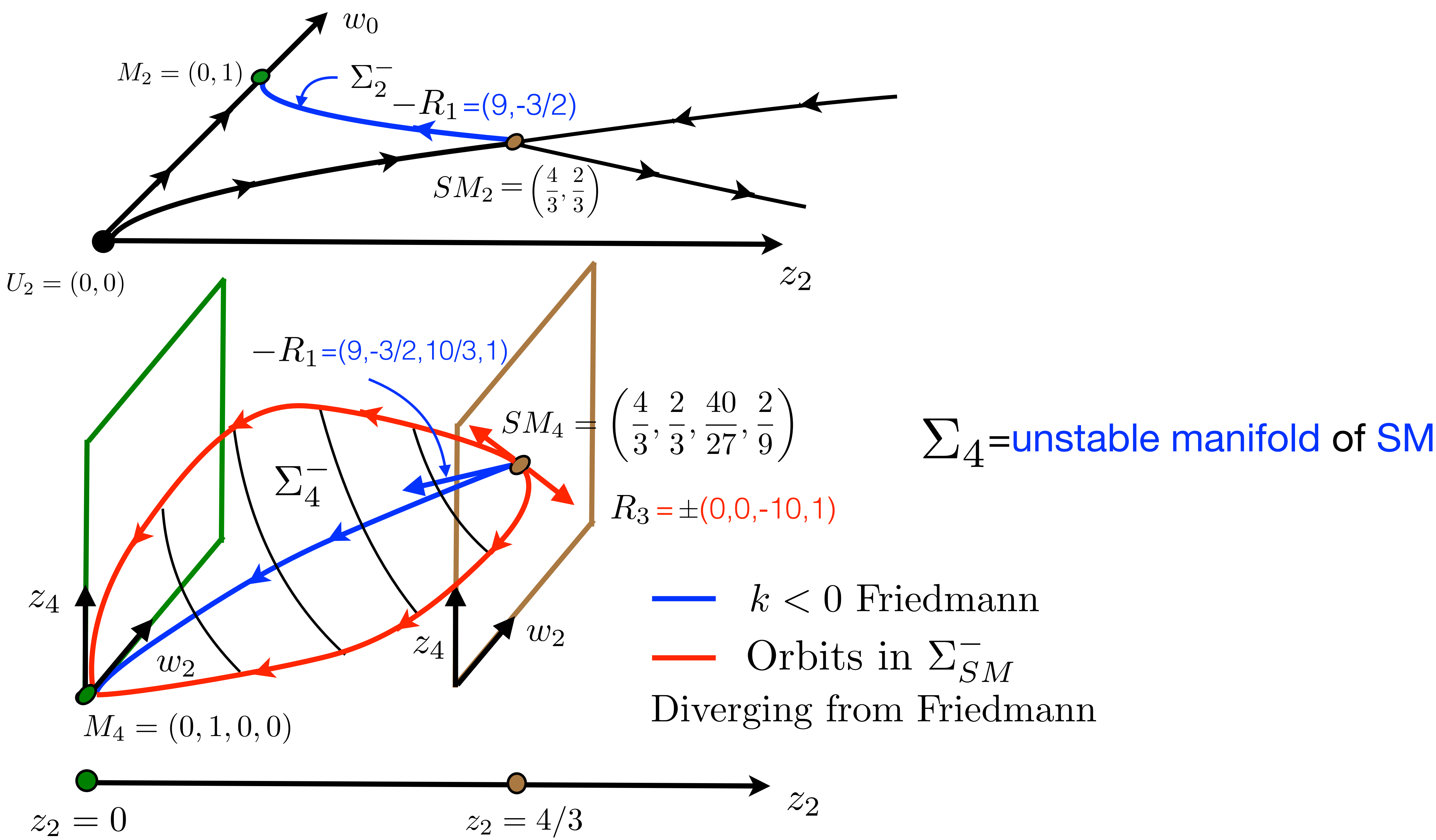
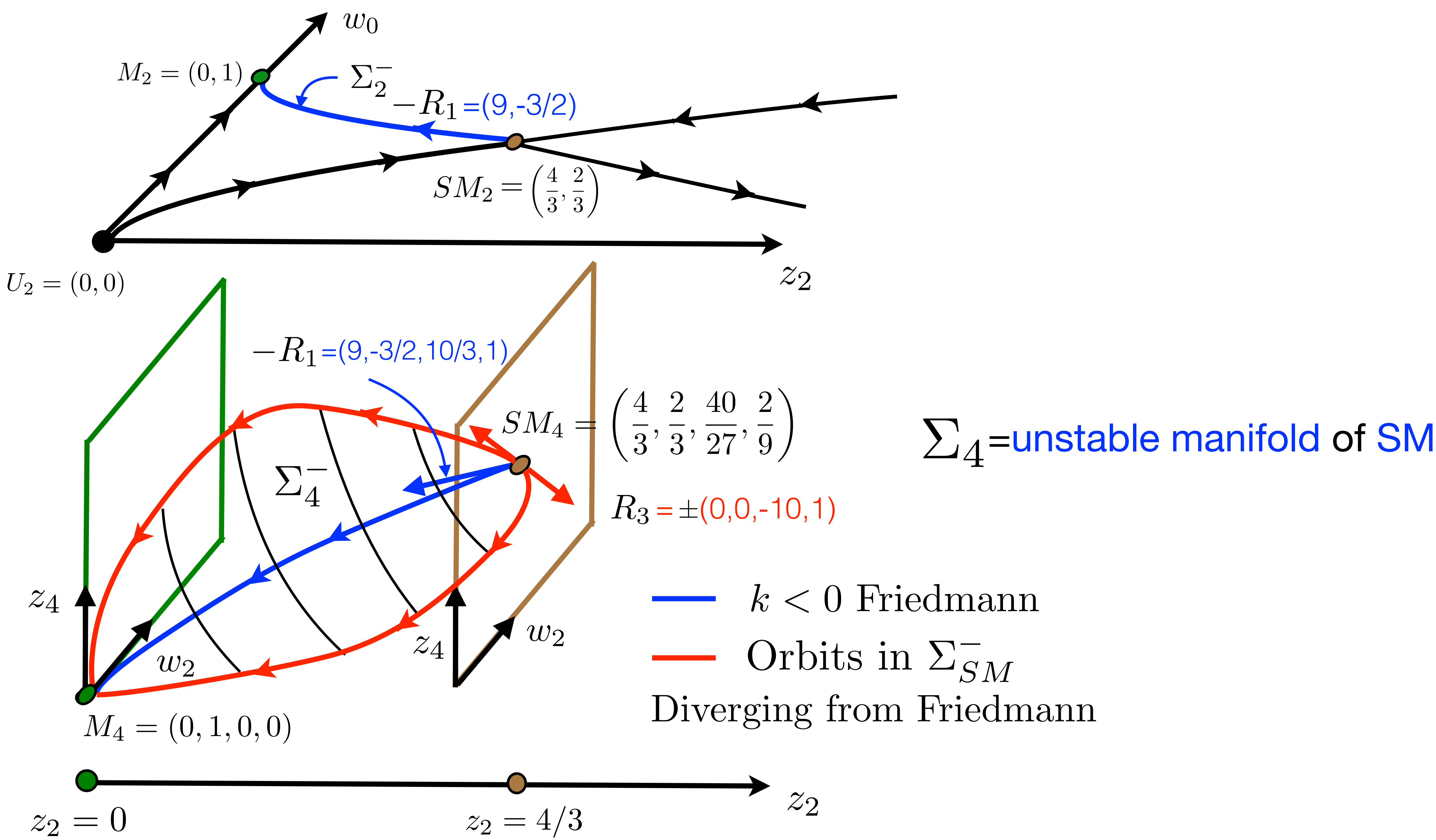


Generically: Solutions agree with Friedman at early times, accelerate away, and then decay back to Friedmann…

Phase Portrait: STV-ODE (n=2)



(Theorem: If a solution tends to M at order n=1, then it tends to M at every order n>1 as well.)

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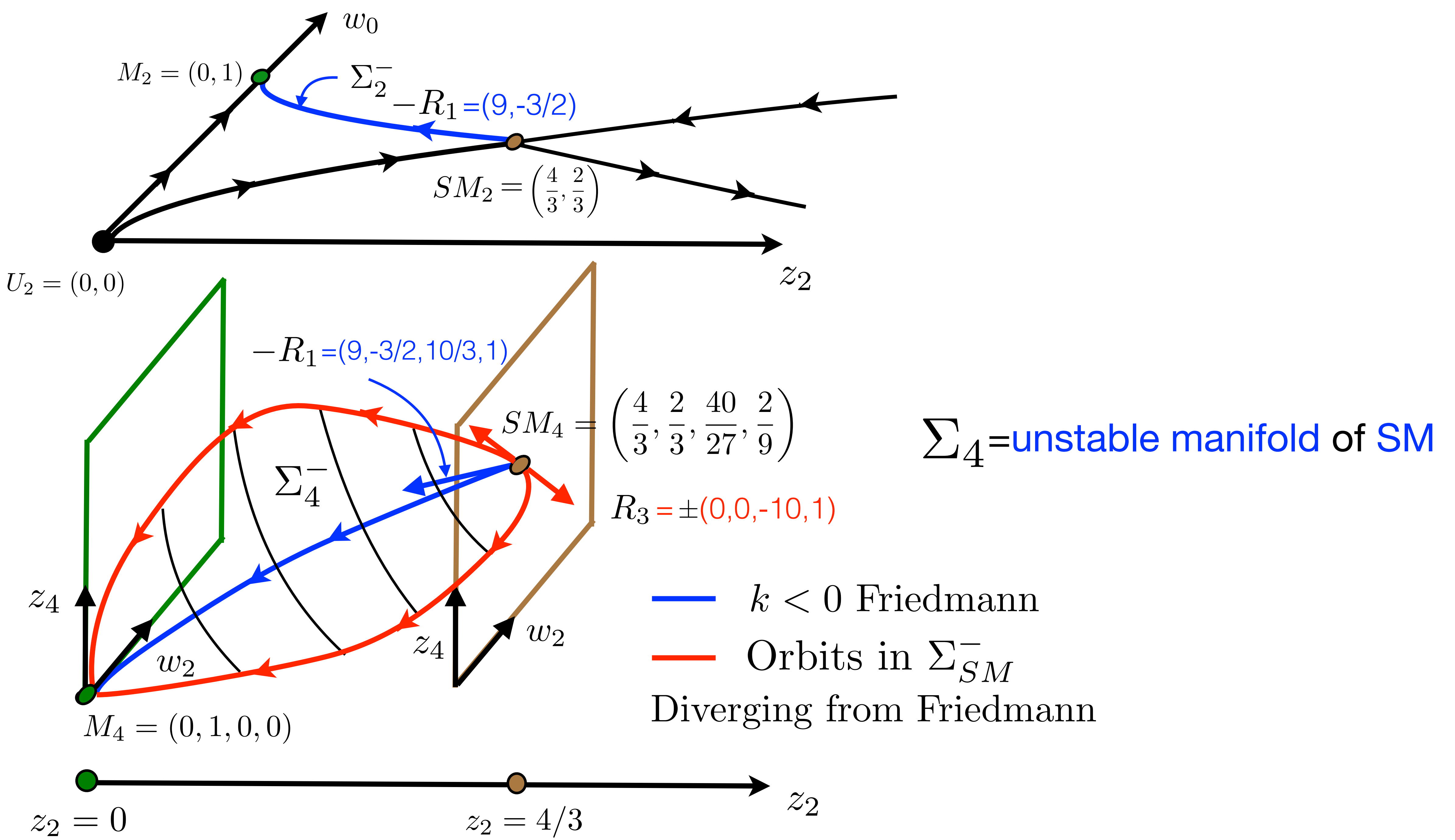
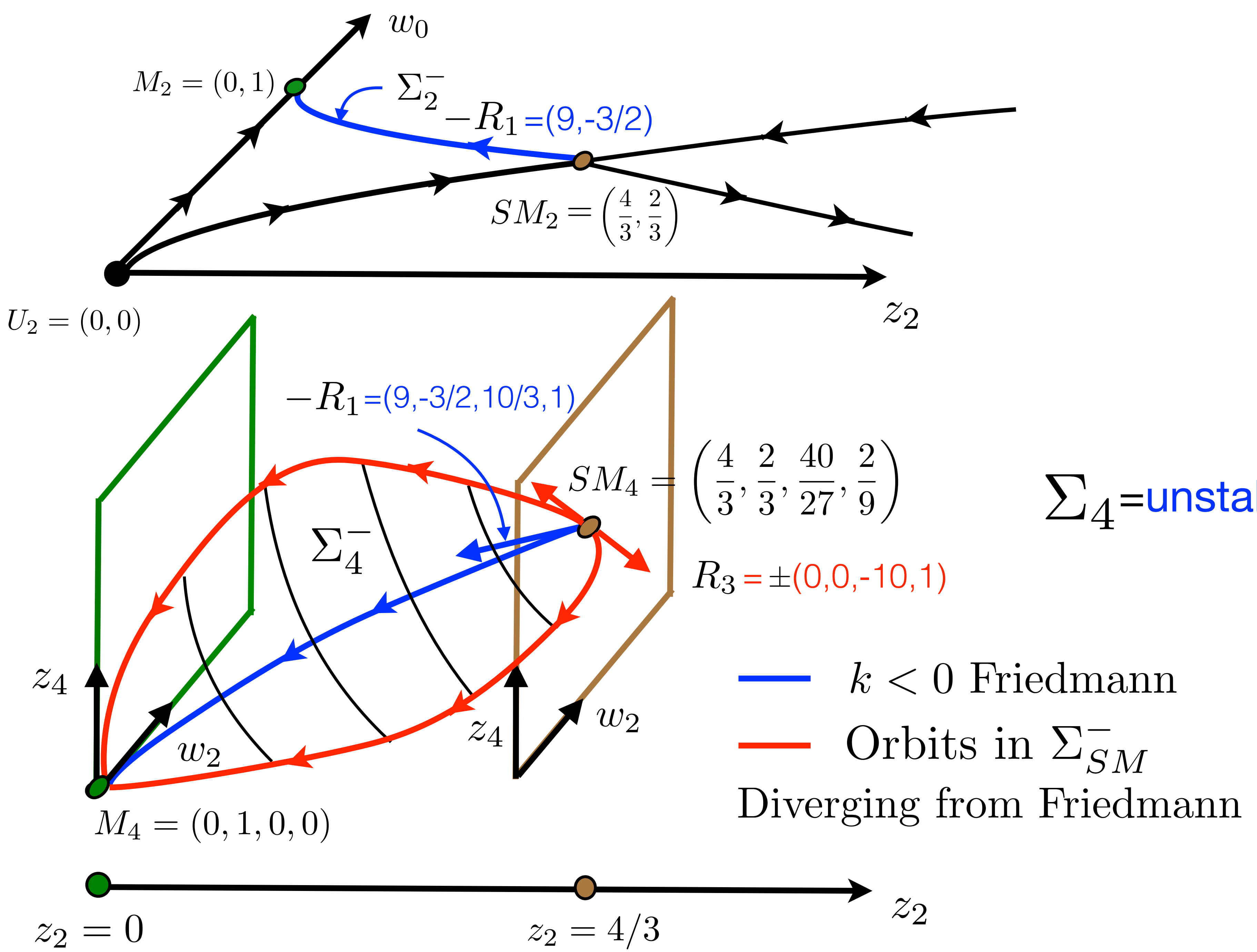


The accelerations fundamentally arise from an instability at Big Bang, not from fluctuations on some length scale…

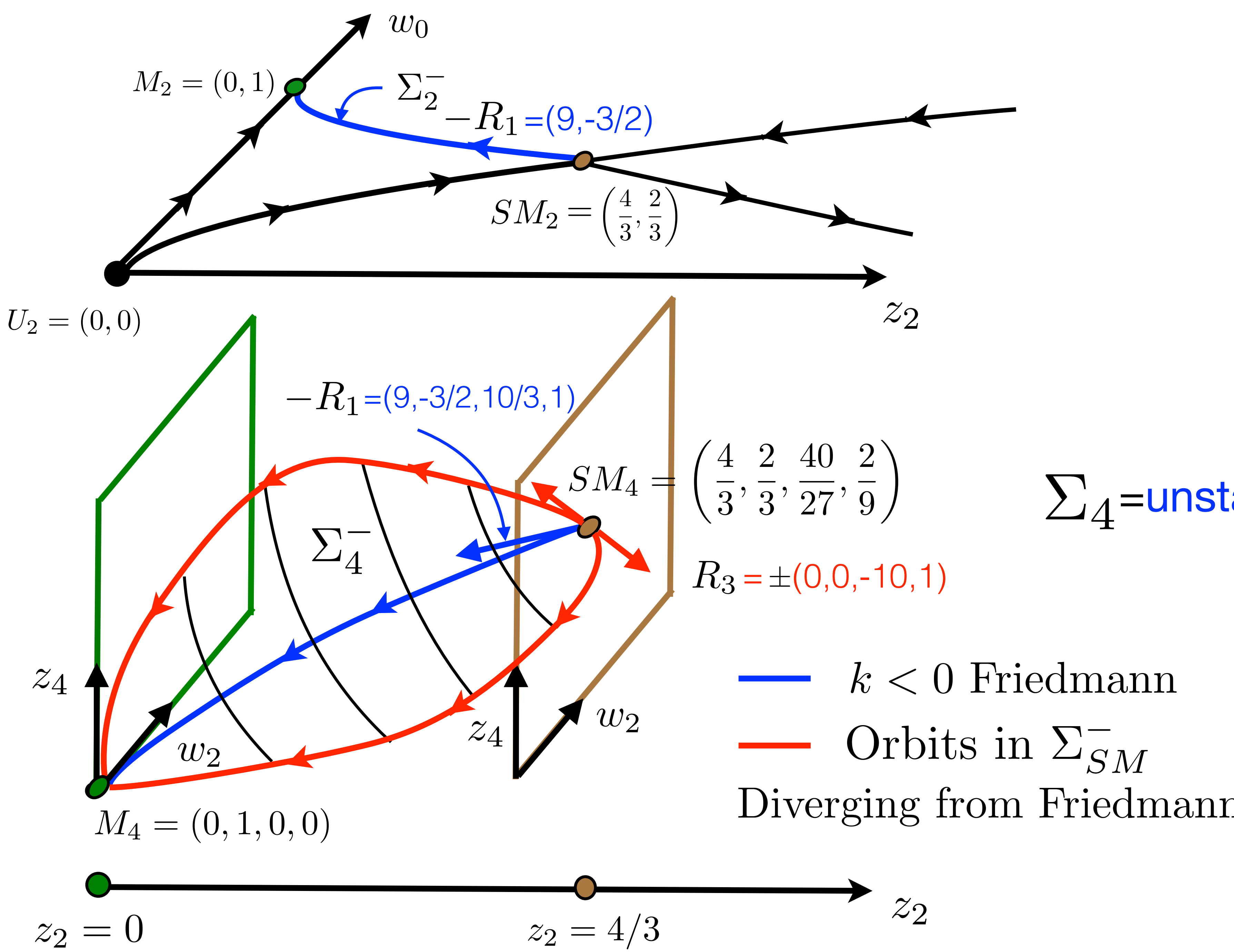
Phase Portrait: STV-ODE (n=2)



Σ_4=unstable manifold of SM

Under-dense solutions all decay back to k<0 Friedmann faster than they decay to Minkowski at M as $t \to \infty$.

<u>Phase Portrait:</u> <u>STV-ODE</u> (n=2)



Σ_4=unstable manifold of SM

The negative eigenvalue at SM implies Big Bang generically self-similar like SM only at leading order n=1…

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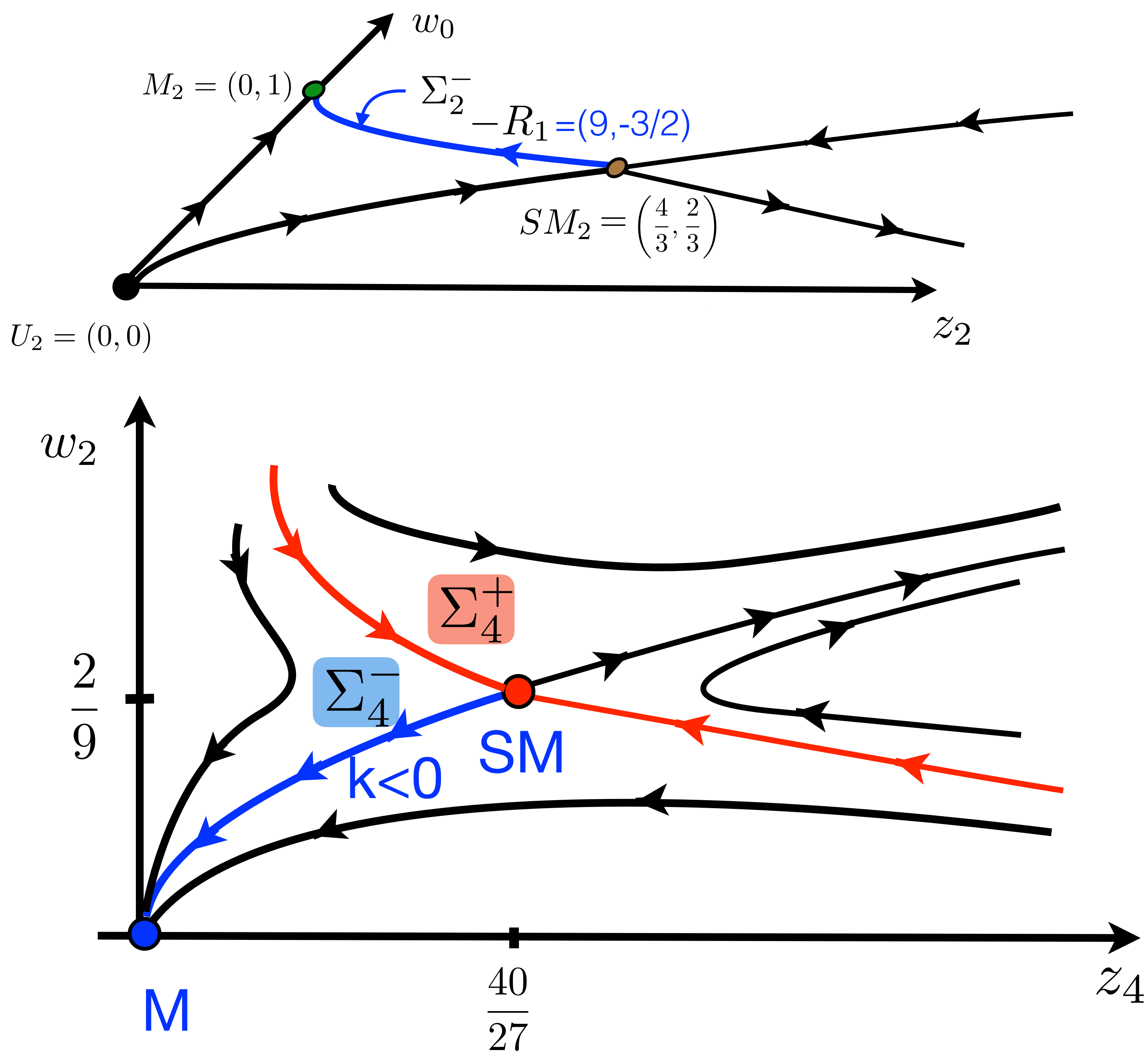
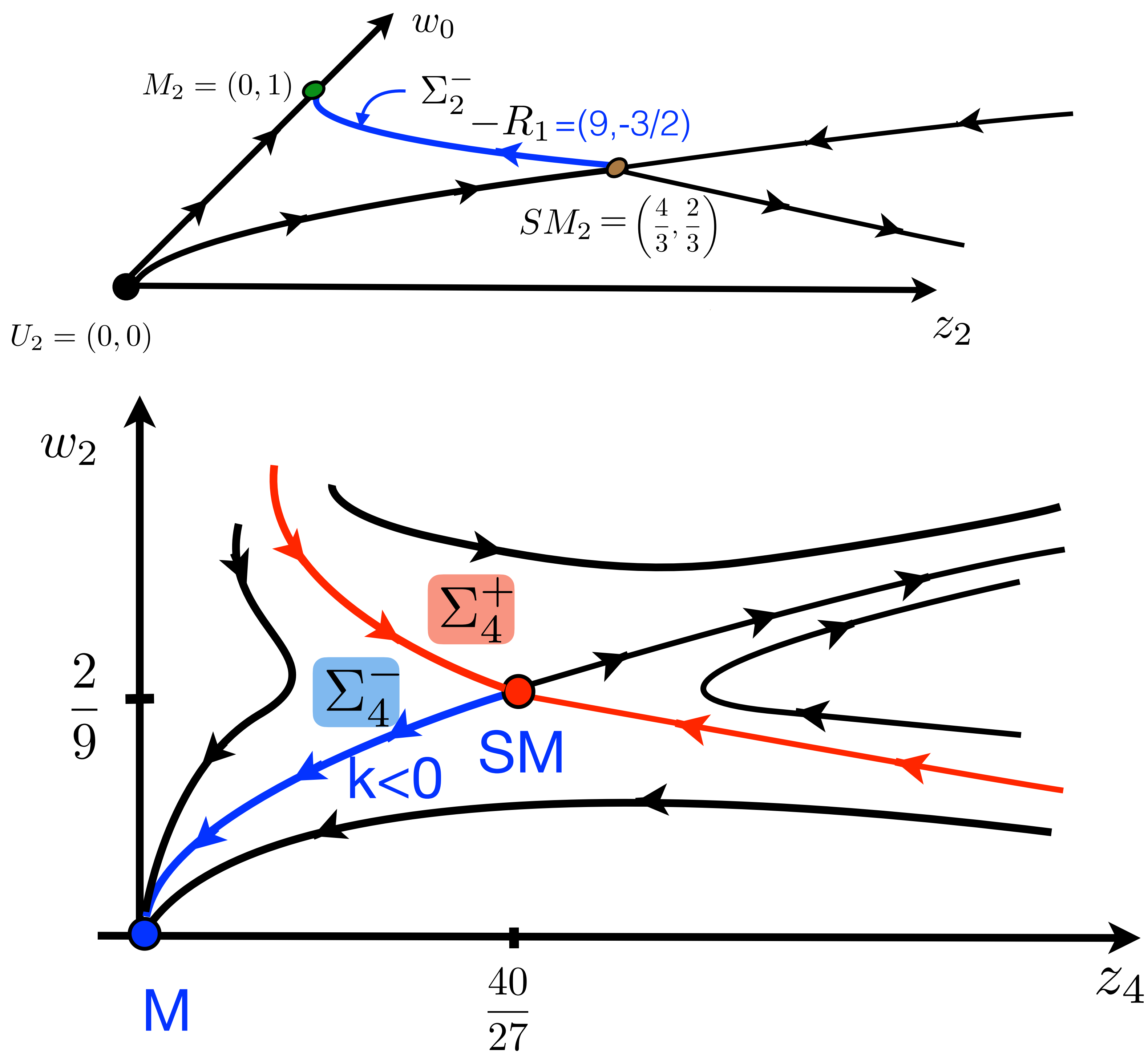


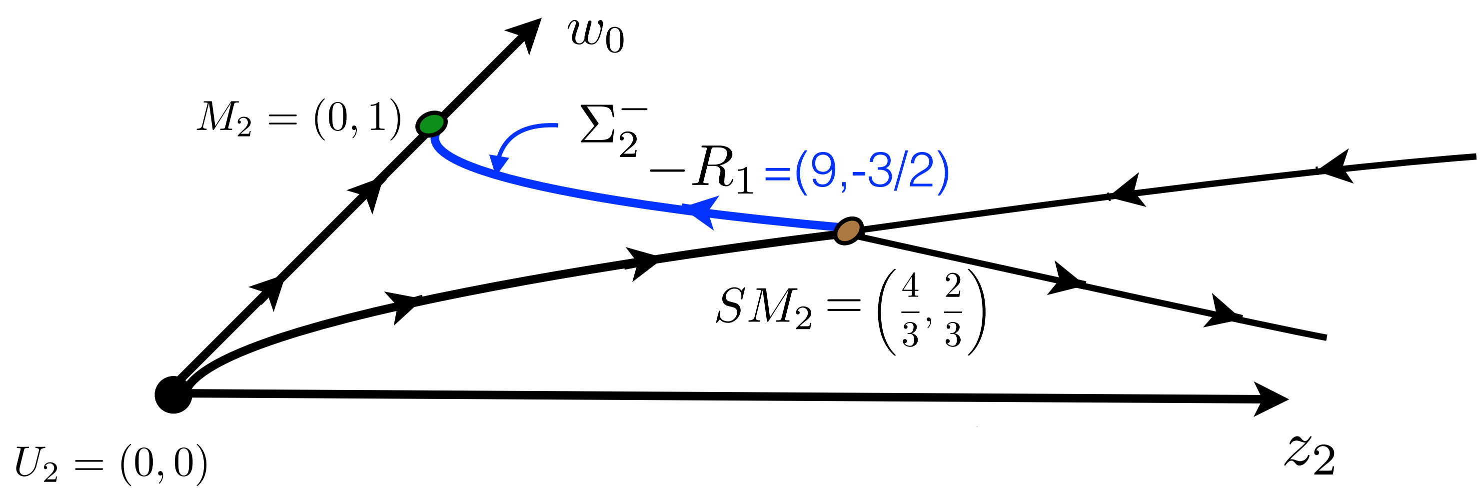
Generically…

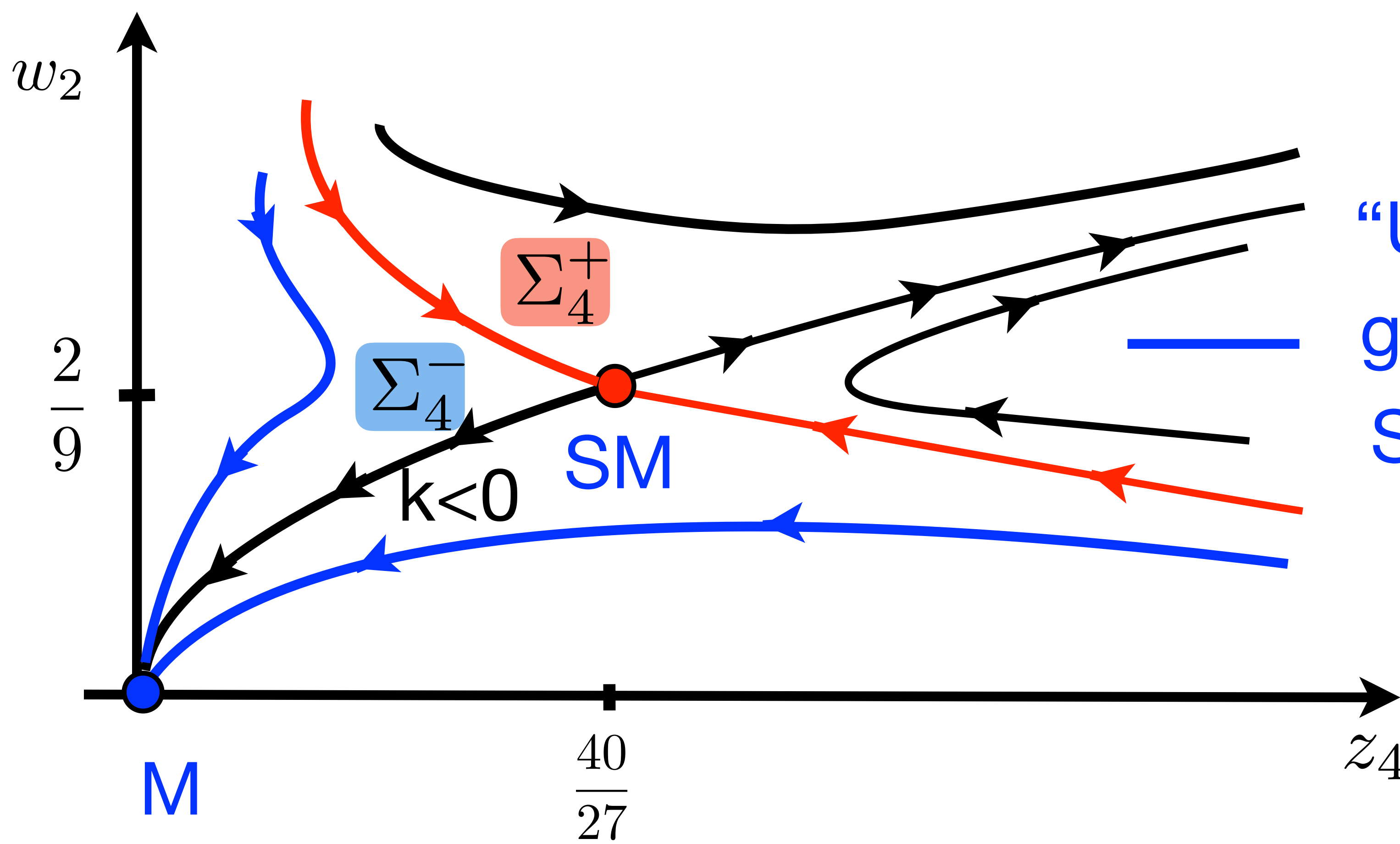
Phase Portrait: STV-ODE (n=2)



Generically…perturbations of k<0 Friedmann do not hit SM in backward time, but go off to infinity instead…

Phase Portrait: STV-ODE (n=2)

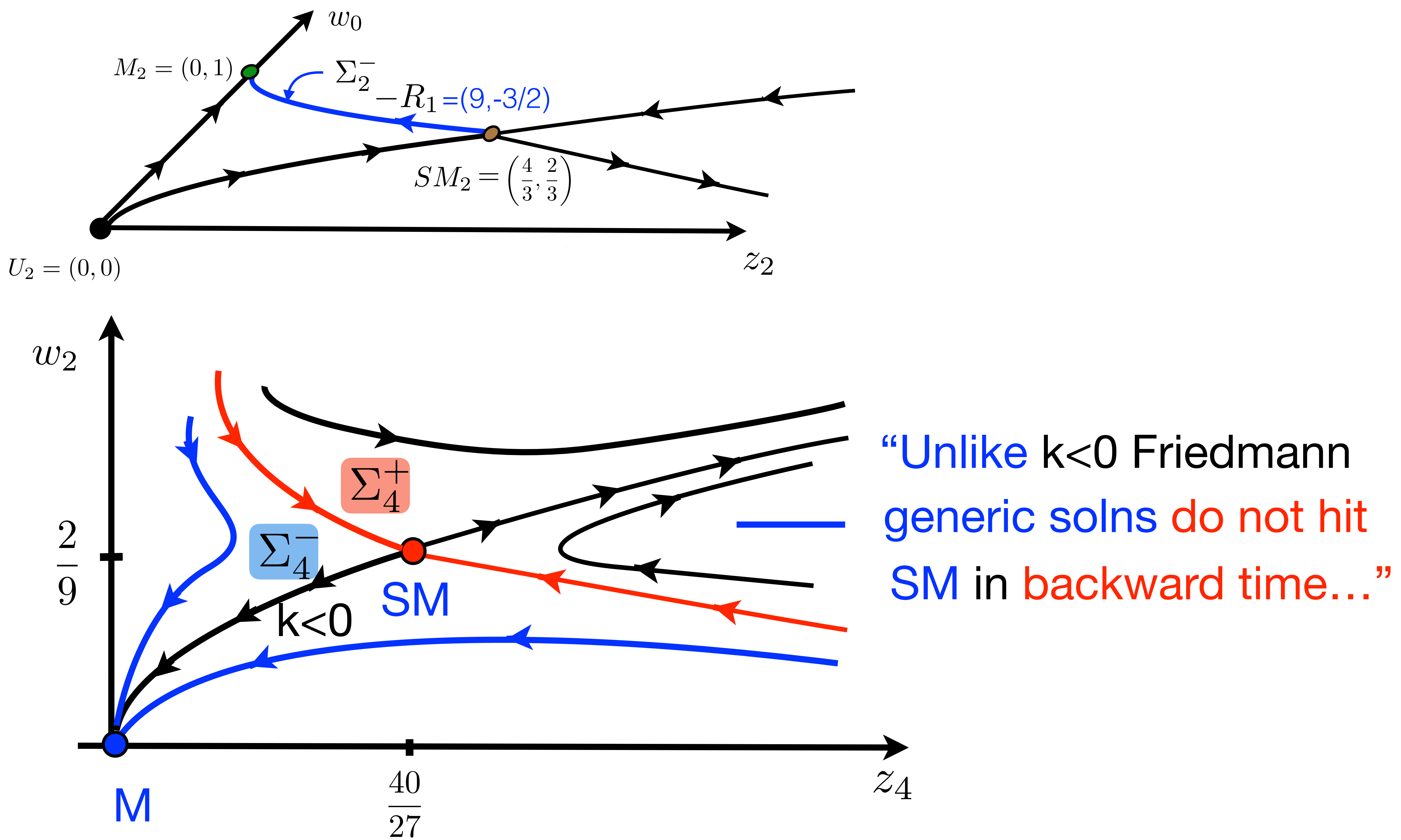




"Unlike k<0 Friedmann generic solns do not hit SM in backward time…"

…so Generically…

Phase Portrait: STV-ODE (n=2)



…so Generically… the Big Bang is self-similar like Friedmann spacetimes only to leading order $n=1$!

STV-ODE (n=3)

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(Complicated…)

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$$t\dot{z}_6 = 6z_6 - 7(z_6 w_0 + z_2 w_4 + z_2 w_0 D_4 + z_2 w_2 D_2 + z_4 w_0 D_2 + z_4 w_2)$$

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$$A_2 = -\frac{1}{3}z_2, \qquad A_4 = -\frac{1}{5}z_4$$

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But phase portrait is determined qualitatively at n=2...

STV-ODE (in general)

$$\frac{d}{d\tau}\boldsymbol{v}_n = \begin{pmatrix} (2n+1)(1-w_0)-1 & -(2n+1)z_2 \\ -\frac{1}{2(2n+1)} & 2n(1-w_0)-1 \end{pmatrix} \boldsymbol{v}_n + \boldsymbol{q}_n$$

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...minimum positive eigenvalue $\Lambda_{B3} = \frac{1}{3}$ at n=3.

Results

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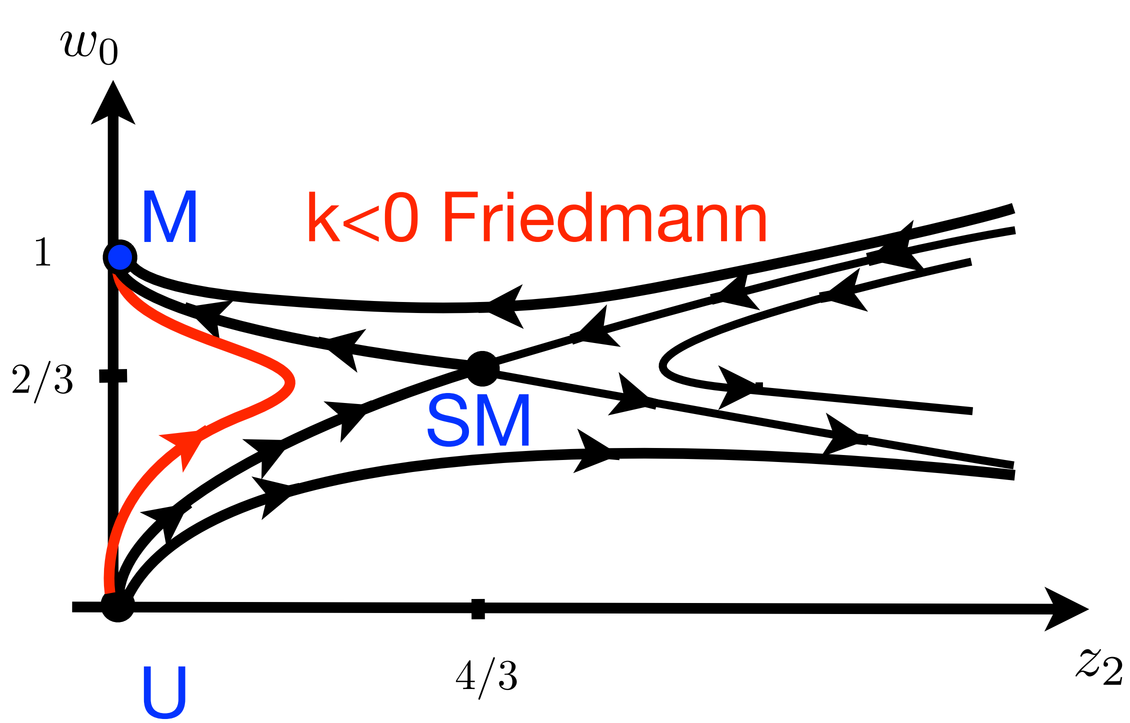

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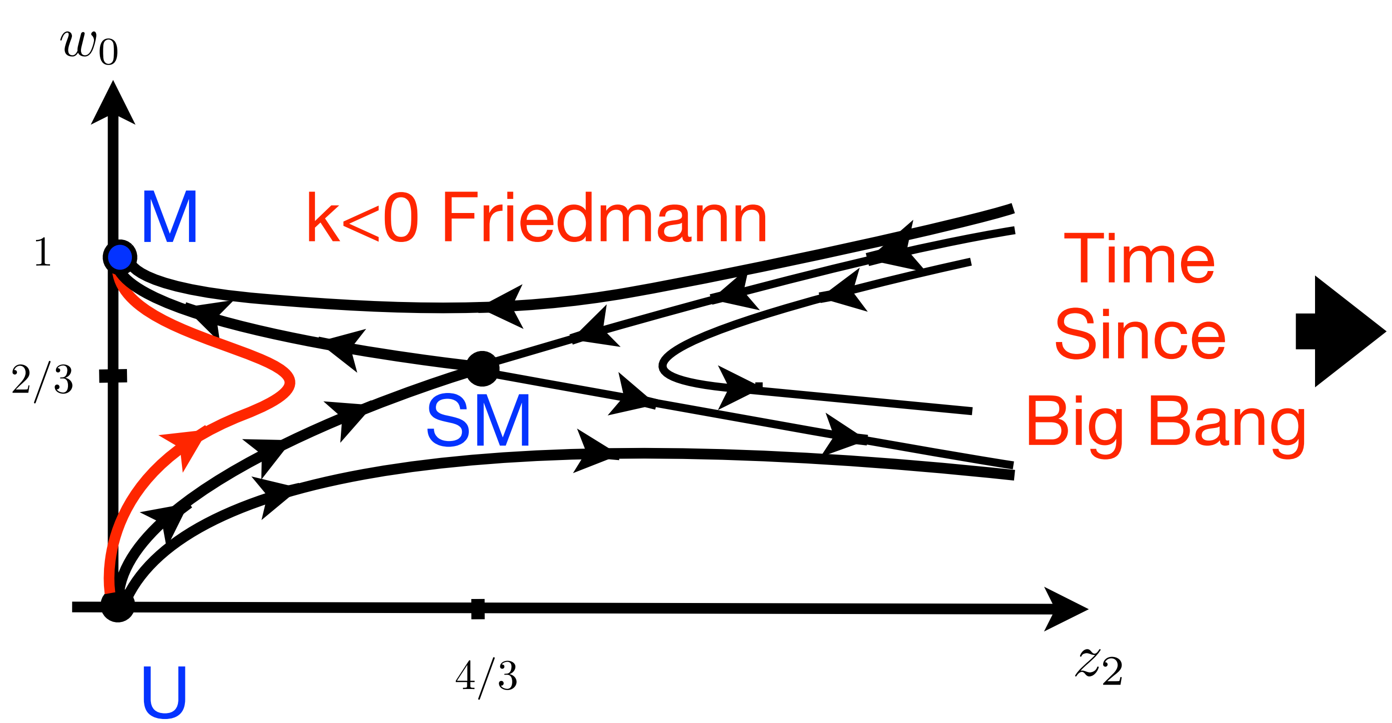

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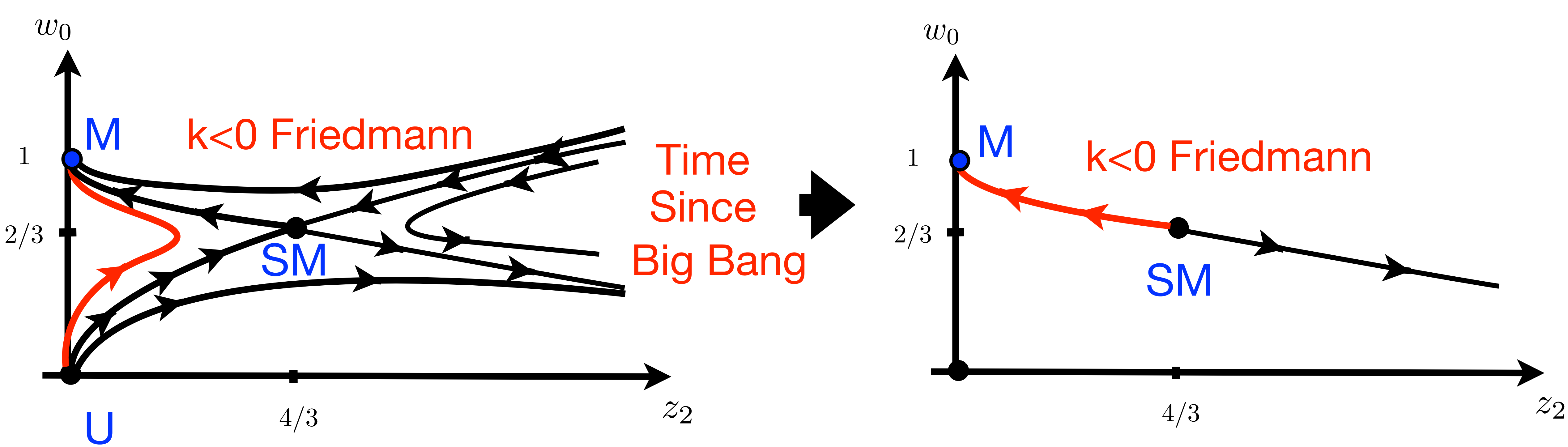

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Solutions in $\mathcal{F}$ which agree at n=1, differ by initial conditions for STV-ODE at orders $n \geq 2$.

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- All eigenvalues of SM are positive for $n \geq 3$!

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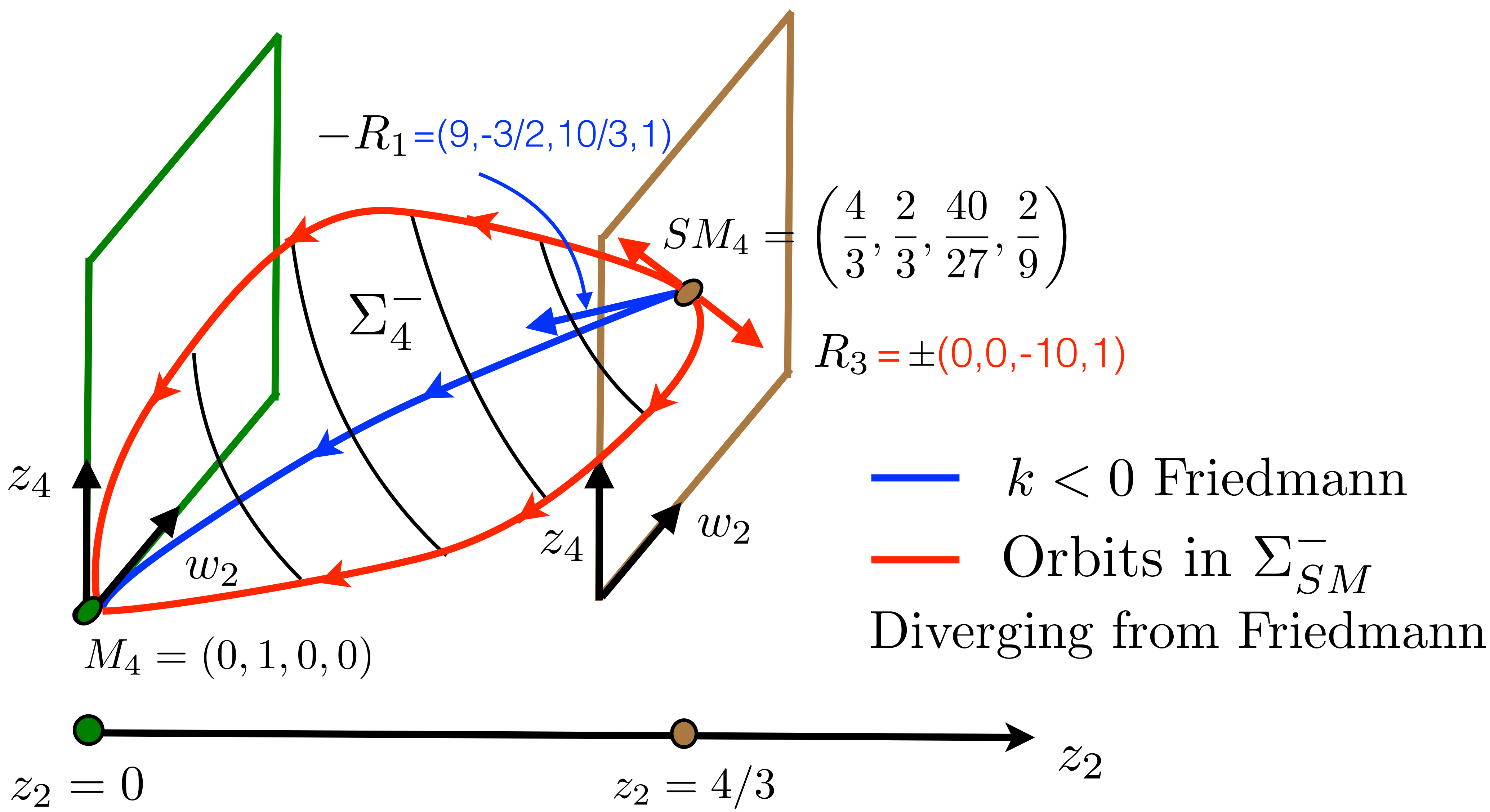

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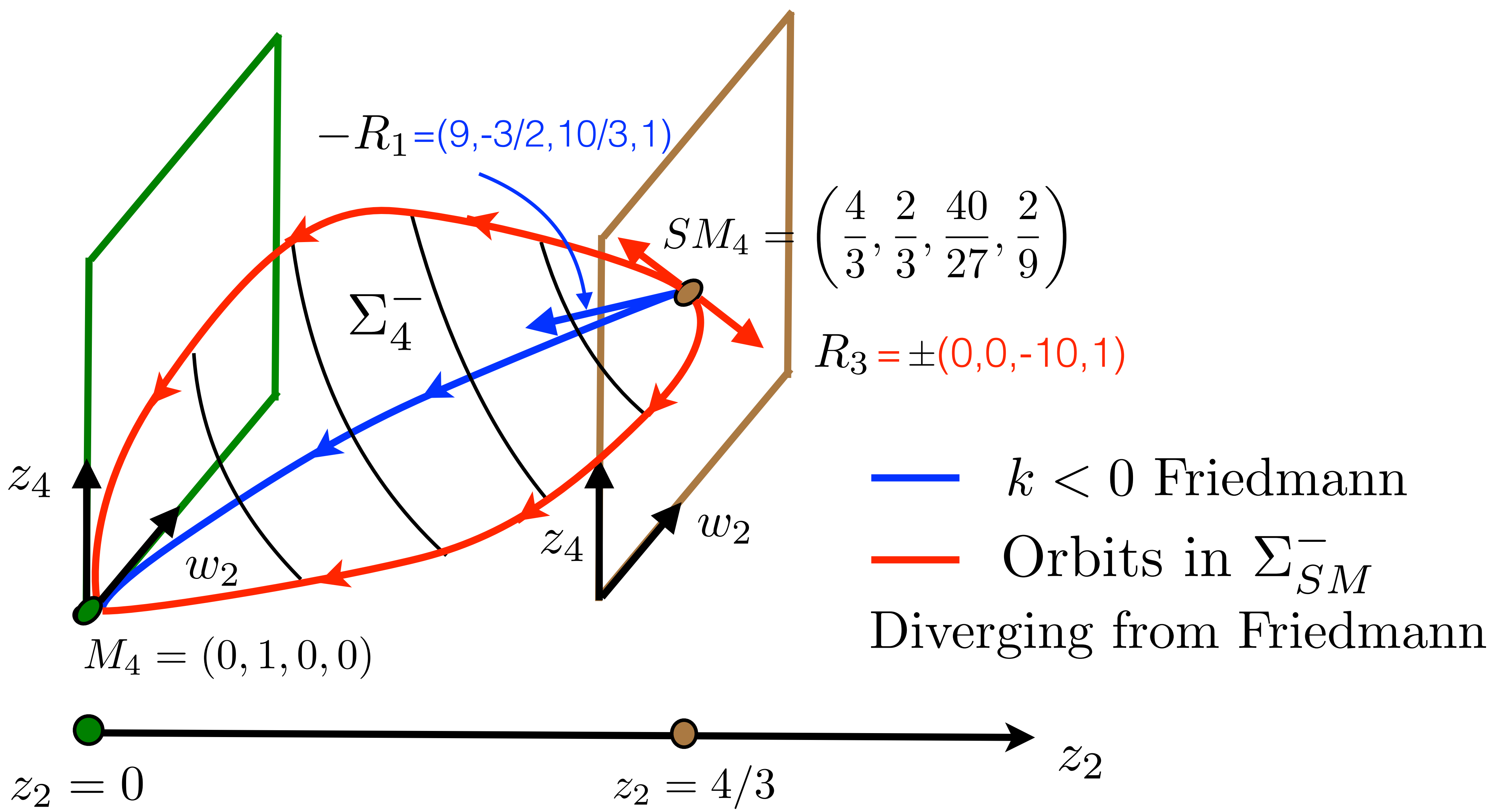

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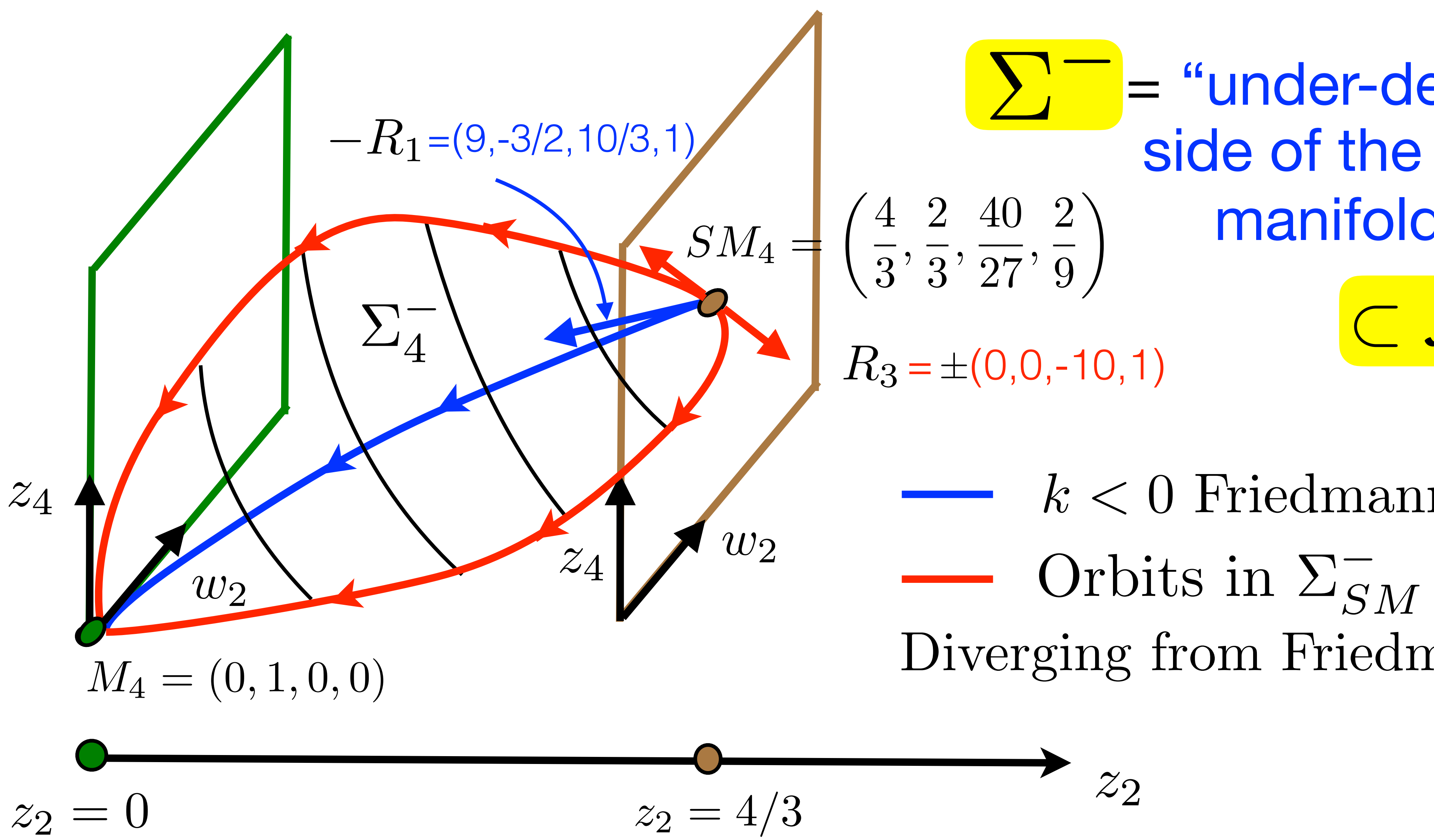

Σ^- = "under-dense" side of the unstable manifold of SM

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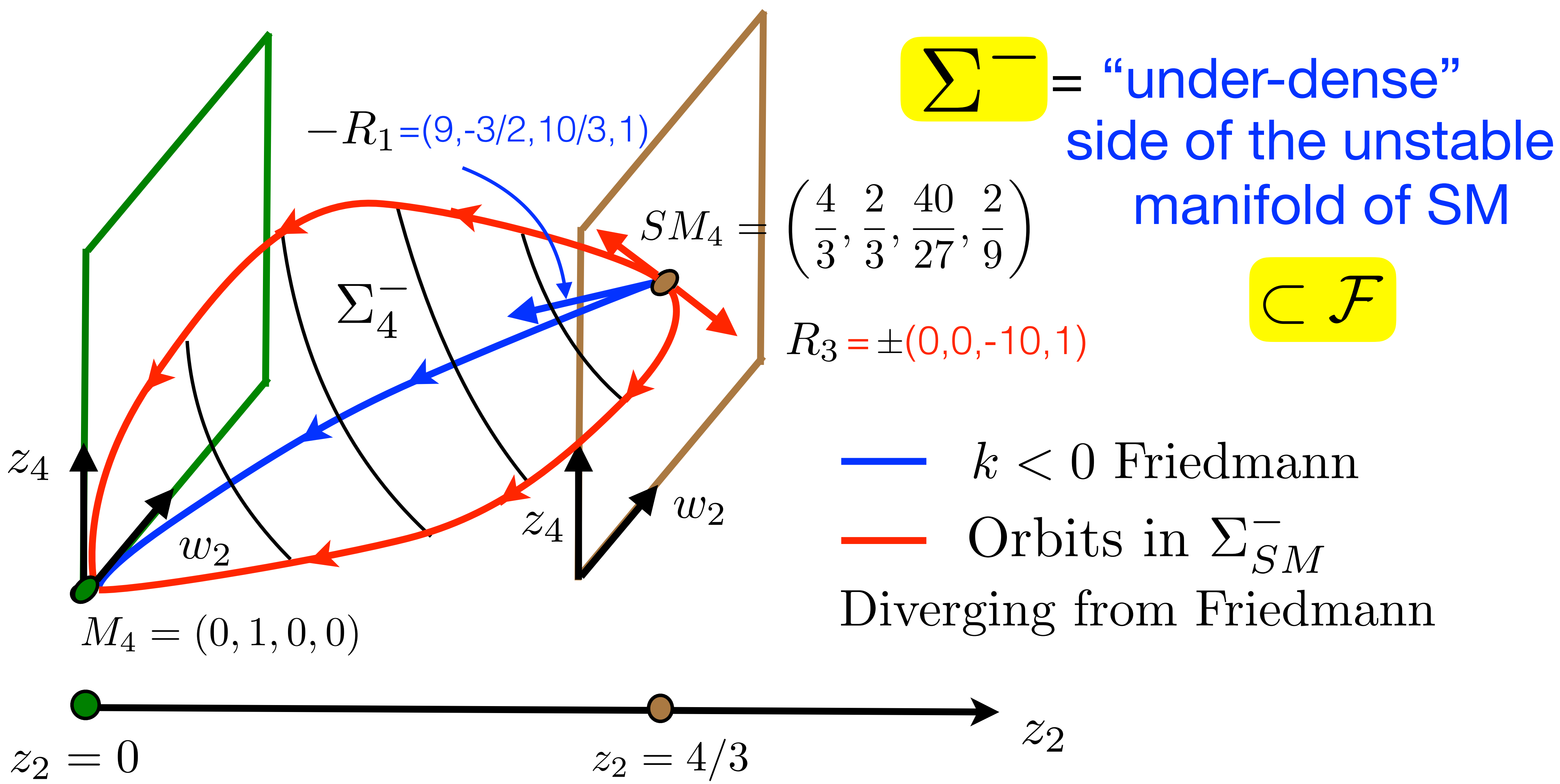
- $\exists$ a second Eigen-direction in Σ^- at order n=2…



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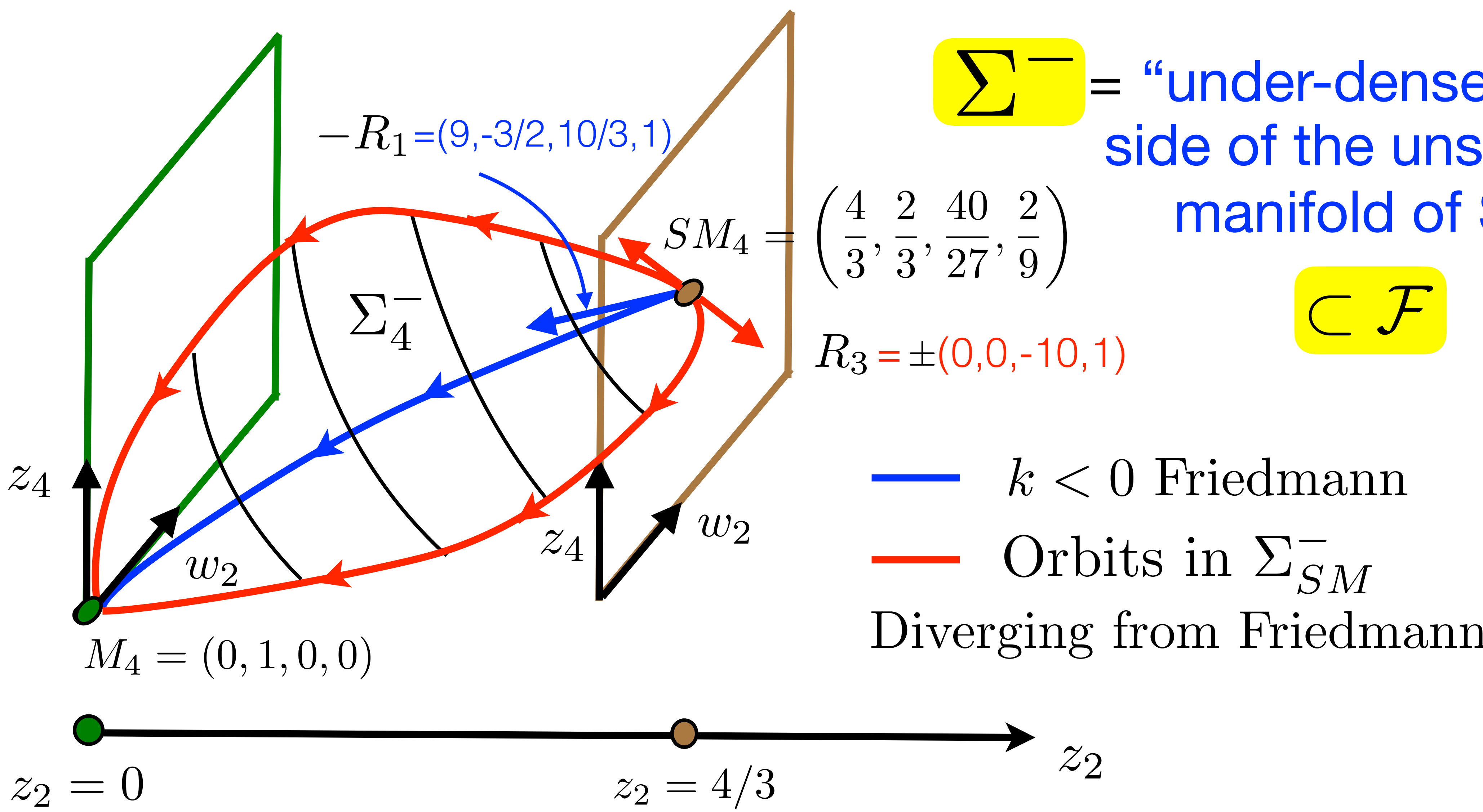
- $\exists$ +2 new Eigen-directions in Σ^- at each $n \geq 2$.



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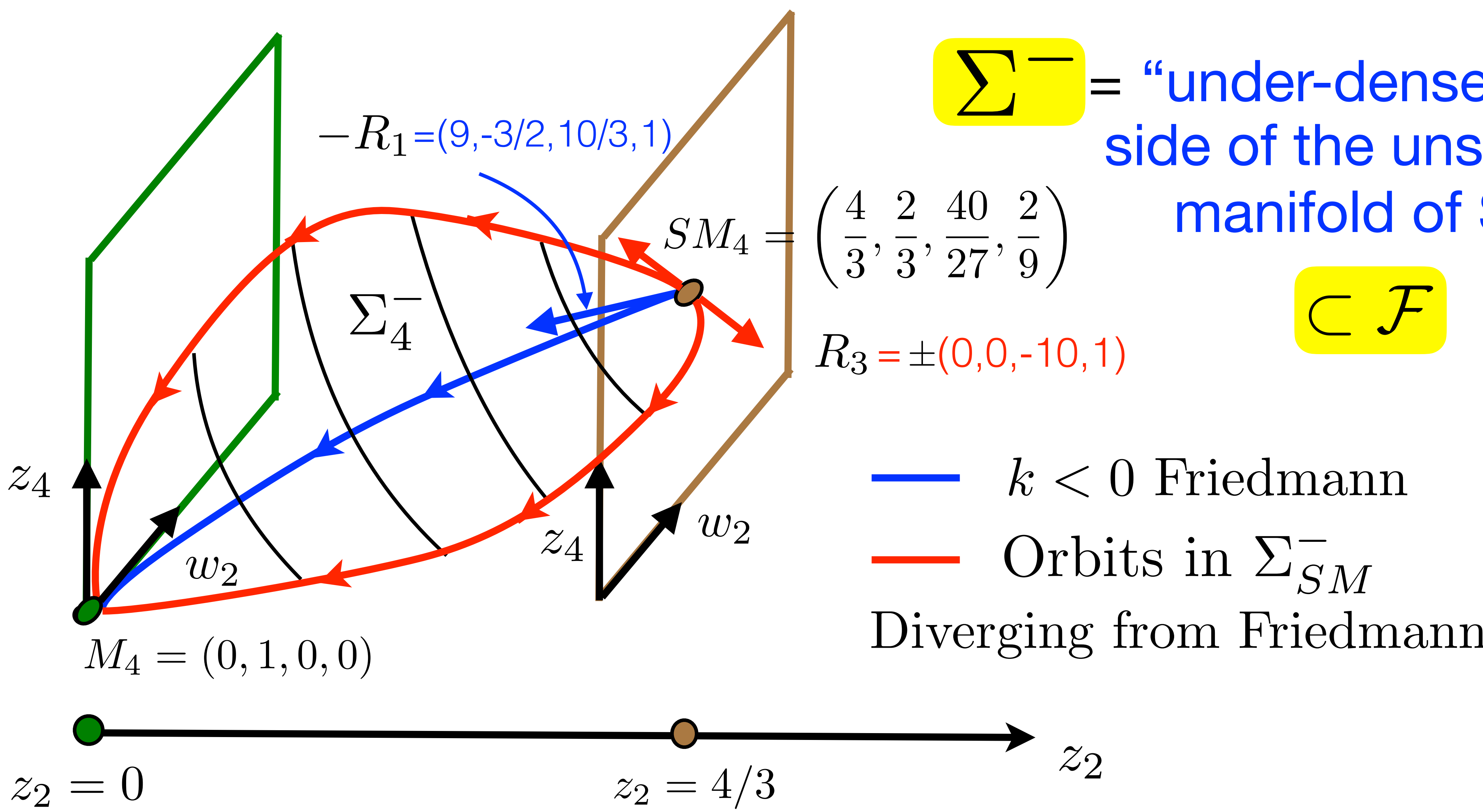
- Solutions in the unstable manifold of SM at order n=2, are in the unstable manifold of SM for all n>2.



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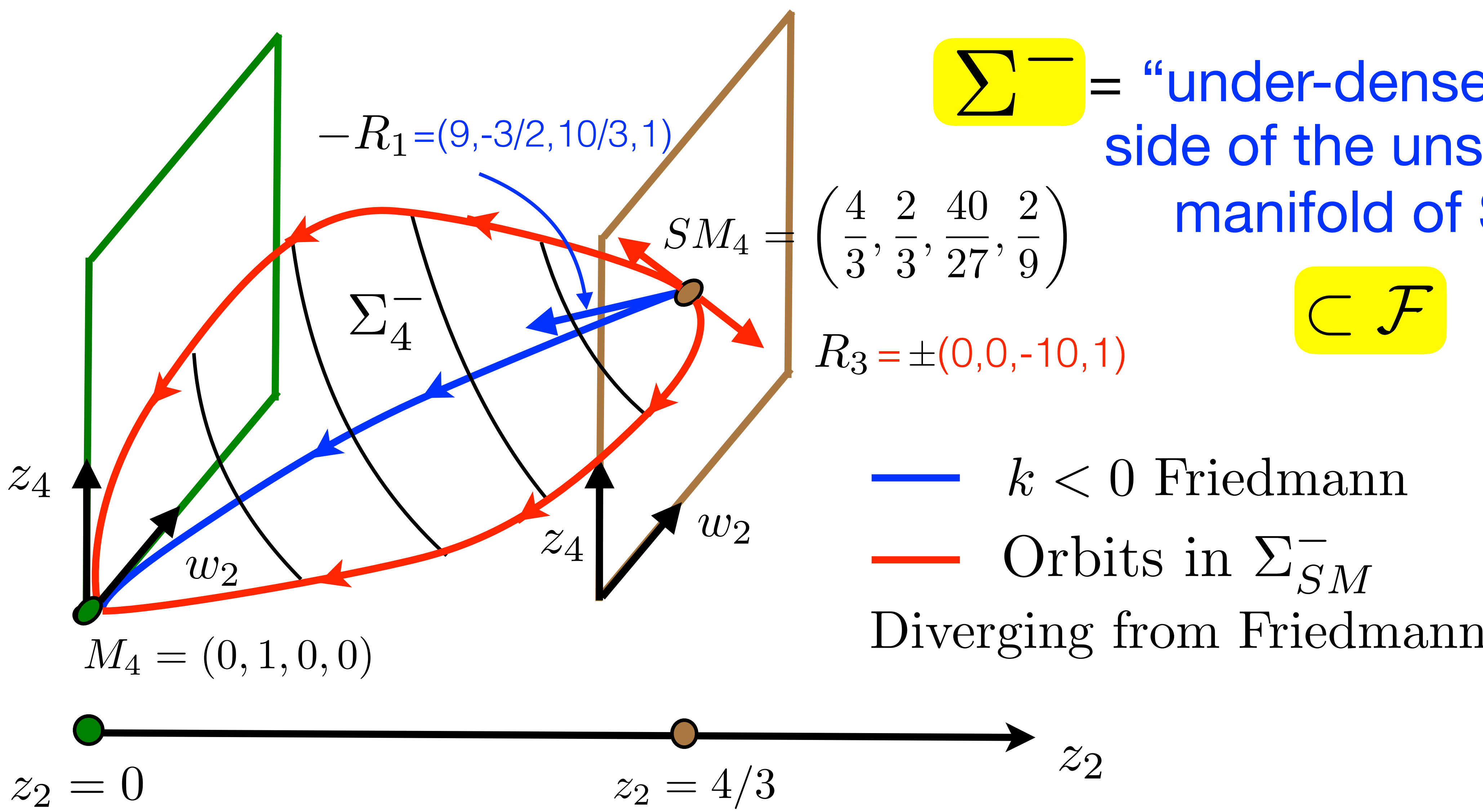
- $n \geq 2$: "Same picture" except more directions to perturb k<0 Friedmann within Σ^-.



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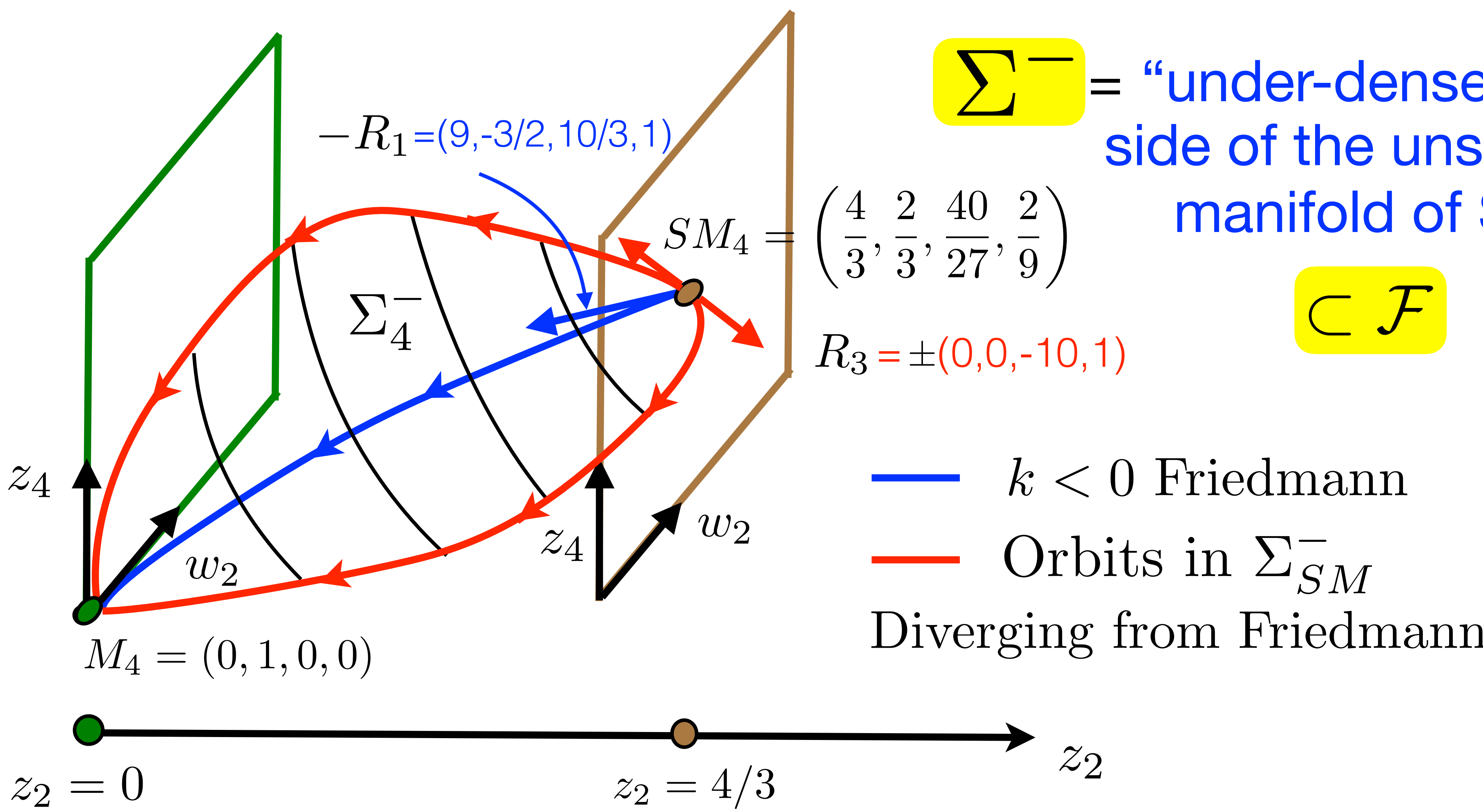
- Conclude: k<0 Friedmann is unstable to perturbation in Σ^- at every order $n \geq 2$!



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- In general…assuming time since the Big Bang…

Σ^{-} is a 2n-3 dimensional space of unstable trajectories…only one of which is k<0 Friedmann!

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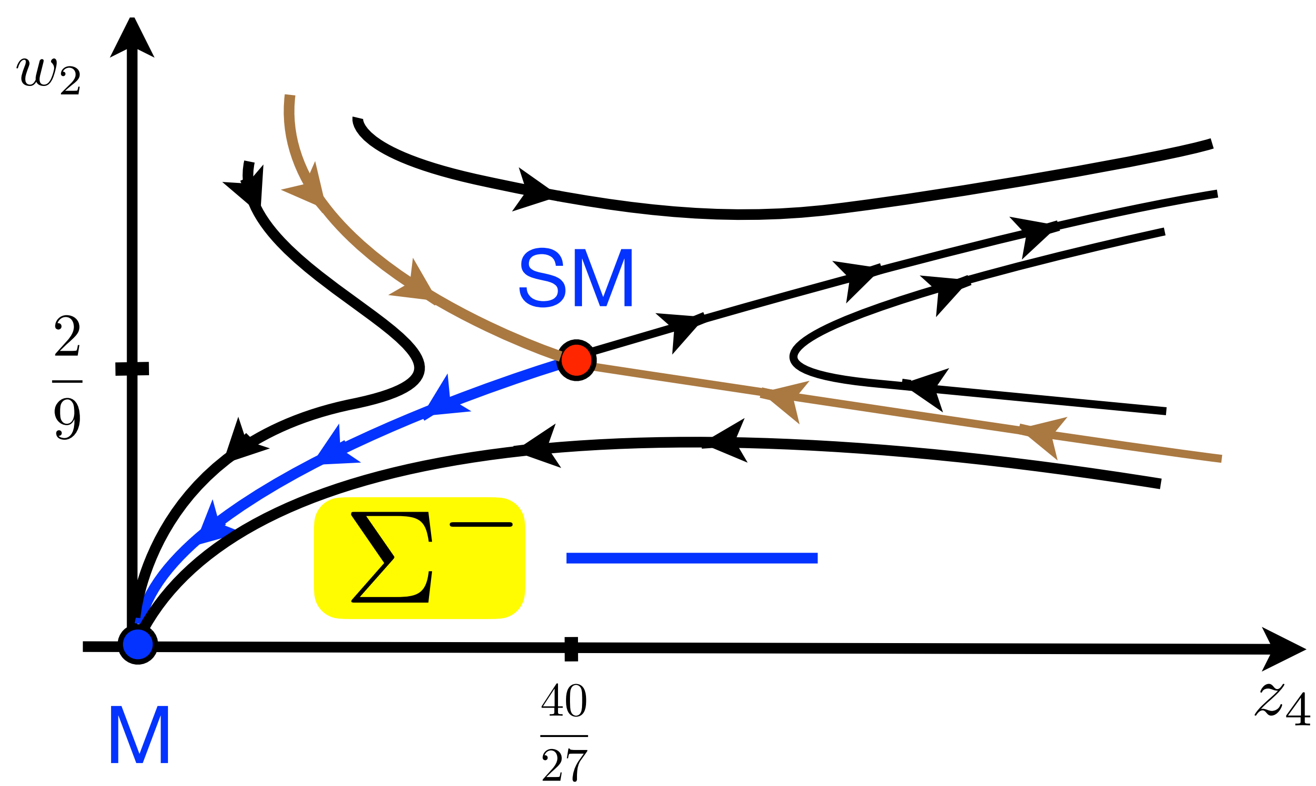

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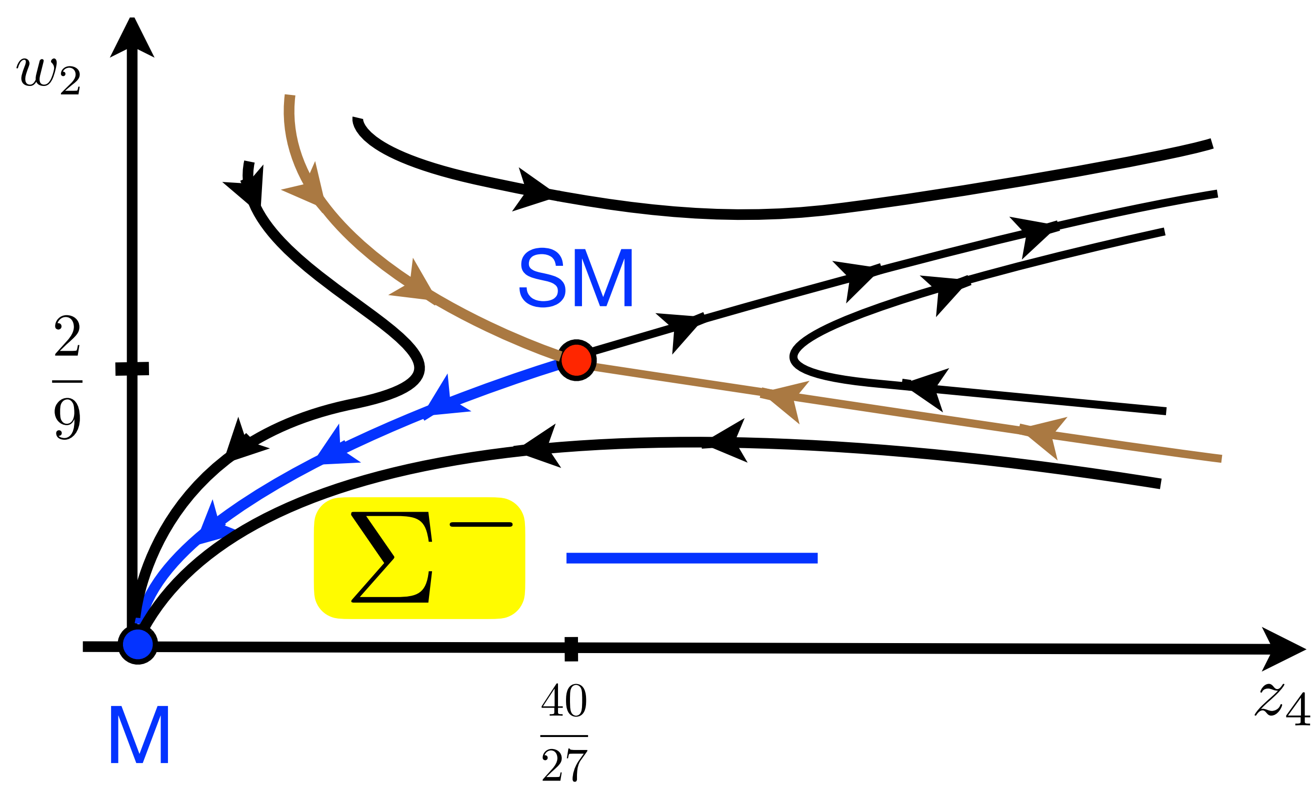
- So it suffices to characterize the instability of k<0 Friedmann within the unstable manifold Σ^- of SM.



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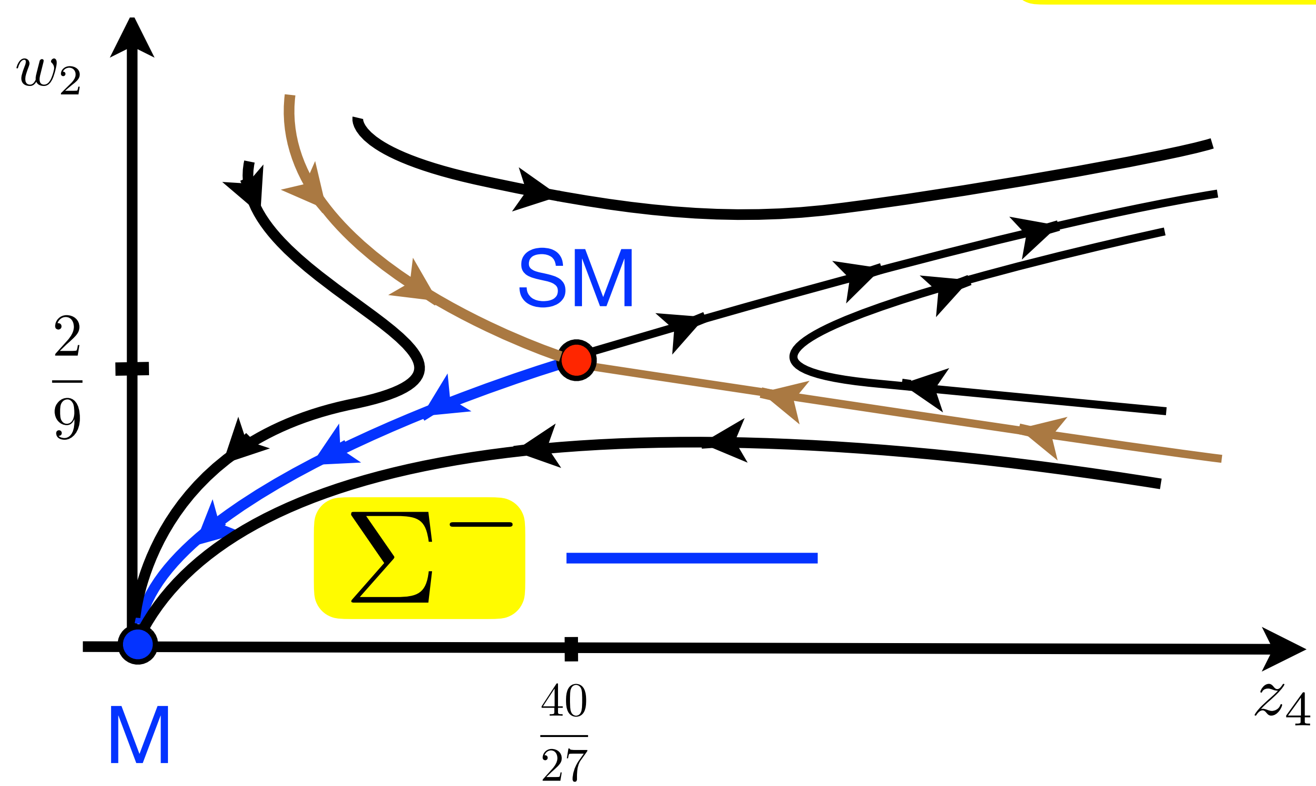
Theorem: Solutions in $\mathcal{F}$ do NOT generically tend to SM in backward time $t \to 0$.



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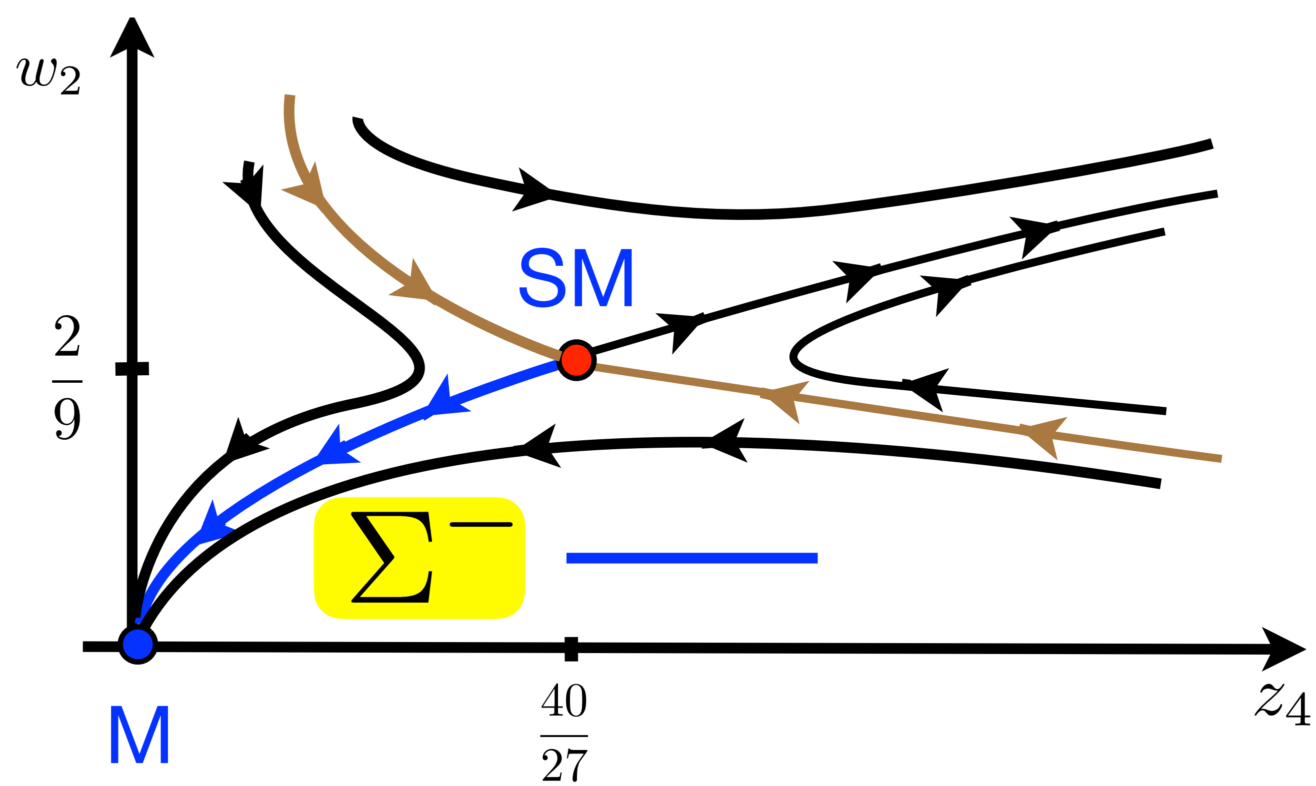
- The Friedmann Big Bang is NOT generic!



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(Same for small perturbations of SM in $\mathcal{F}$.)

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Proof: Taylor's Theorem with decay to M at n=2.

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Theorem: The spatially homogeneous self-similar Big Bang of Friedmann spacetimes is universal at leading order in ξ, but is generically not self-similar beyond leading order.

-In fact, generically: $z_{2n}(t) \to \infty, \quad w_{2n-2}(t) \to \infty$

as $t \to 0$ for all $n \geq 2$

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Problem: Find sharp bounds on i.c.'s which imply convergence of power series to solutions in $\mathcal{F}$:

$$z(t) = \sum_{n=1}^{\infty} z_{2n}(t)\xi^{2n}, \quad w(t) = \sum_{n=1}^{\infty} w_{2n-2}(t)\xi^{2n} \quad \in \mathcal{F}$$

Concluding Remarks:

Define: $\mathcal{F}_n \equiv$ space of approximate solutions

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…we argued this is the hallmark of a "fudge factor"…what you expect to see when you try to add a parameter to the equations in an attempt to fix what is actually an incorrect solution of the original equations…?)

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…just like $k<0$ Friedmann comes from BB?

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- Could this imply a larger region of thermalization?

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…this is the assumption of the current standard model of cosmology!

- With the understanding that solutions generically accelerate away from Friedmann spacetimes before they decay back, couldn't one almost predict the anomalous acceleration before it was observed?

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So $p = \frac{c^2}{3}\rho$ Big Bangs produce $p = 0$ solutions…

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- We conjectured: Enormous pressure and modulus of genuine nonlinearity might imply decay to non-interacting wave pattern by end of radiation…

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Coincidence: The range of possible values of Q is precisely the same as in theory of Dark Energy!

$$.25 \leq Q \leq .5$$

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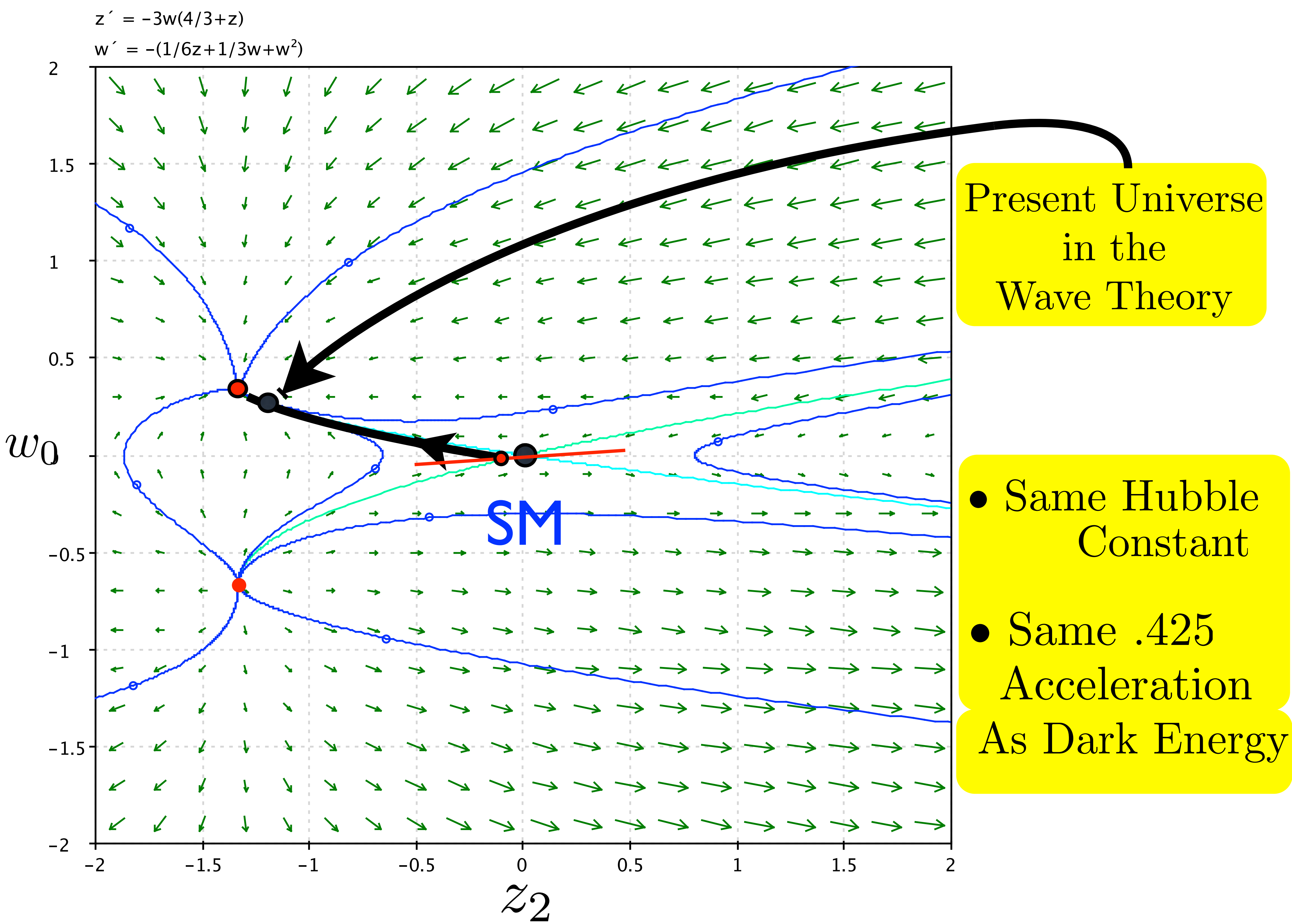

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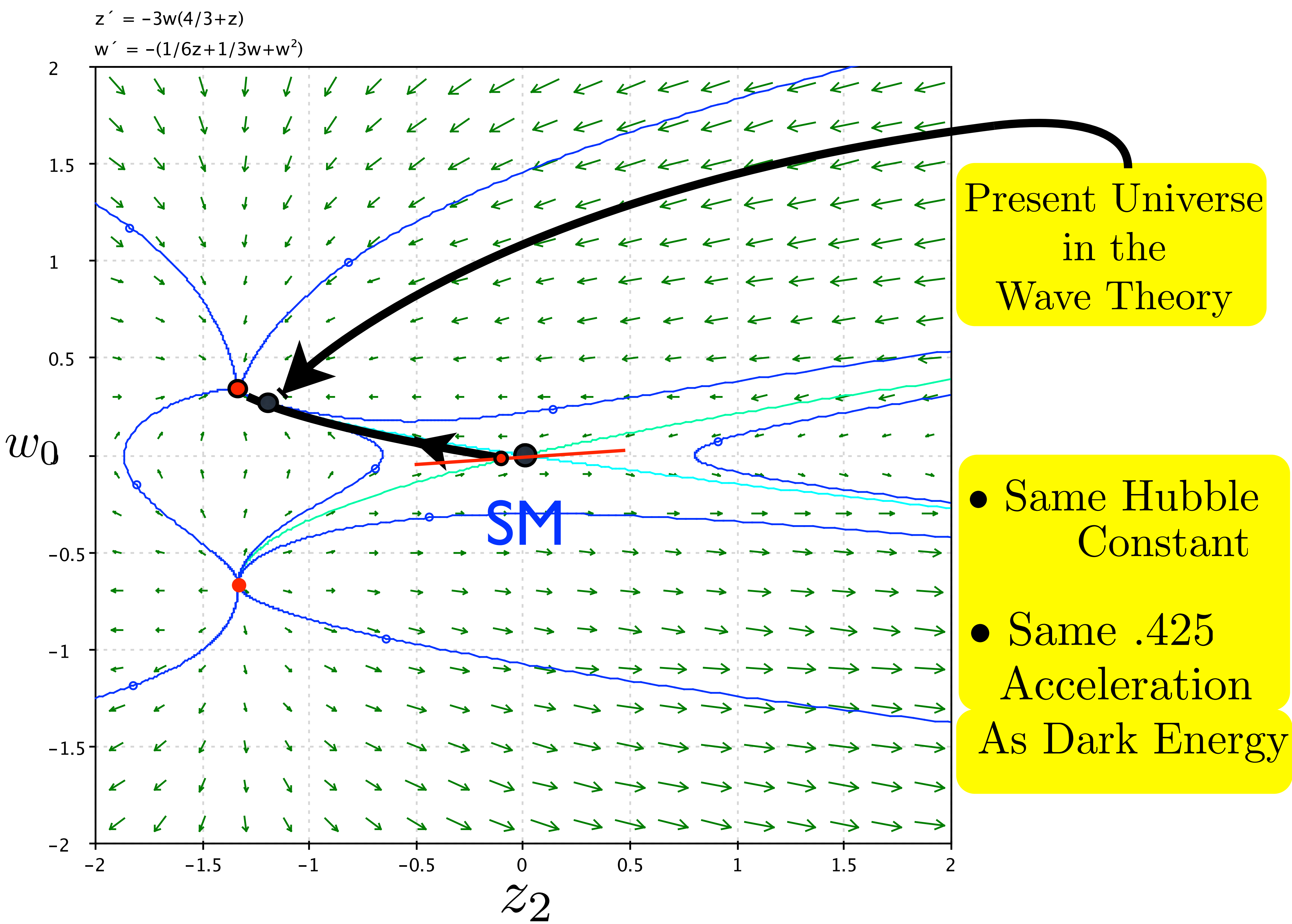
H_0 and Q determine present time at n=1…as well as the acceleration parameter of wave at end of radiation…



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$$H_0 d_\ell = \mathrm{z} + .425\,\mathrm{z}^2 - .1804\,\mathrm{z}^3$$ Dark Energy

$$H_0 d_\ell = \mathrm{z} + .425\,\mathrm{z}^2 + .3591\,\mathrm{z}^3$$ Wave Theory

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Conclude: You need redshift vs luminosity to order n=4 to see where you are in the n=2 phase portrait…

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$z_2(t)$ gives Q

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* Theorem: C_4 determines whether the Big Bang is self-similar like Friedmann spacetimes or not…

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- Does the stability analysis extend to radiation epoch where $p=\frac{c^2}{3}\rho$?

- Final Question: Could the anomalous acceleration of the universe be due to an acceleration away from Friedmann arising directly from the Big Bang, and not from a scale of fluctuation in Friedmann created after the Big Bang?

End

Thank you!!