Name: ______________________

Student ID: ______________________

Final Exam

Instructions: Please read the statements of each individual problem carefully, write your solutions precisely and legibly, and do not cheat. If you have any questions please raise your hand. Good luck!

Problem 1. Let $(X, \mathcal{T})$ be a Hausdorff space, and suppose $x_1, ..., x_n \in X$ for some $n \in \mathbb{Z}_{>0}$. Show that the set $\{x_1, ..., x_n\}$ is closed in $(X, \mathcal{T})$.

Problem 2. Show that $[0, 1) \subset \mathbb{R}_{\text{usual}}$ is homeomorphic to $(0, 1] \subset \mathbb{R}_{\text{usual}}$.

Problem 3. Let $(X, \mathcal{T}_X)$ be a topological space. Let $Y \subset X$ with the subspace topology $\mathcal{T}_Y$. Let $\overline{A}^X$ denote the closure of A in $(X, \mathcal{T}_X)$, and let $\overline{A}^Y$ denote the closure of A in $(Y, \mathcal{T}_Y)$.

a) Prove that $\overline{A}^Y \subset \overline{A}^X$

b) Give an example where $\overline{A}^Y \neq \overline{A}^X$.

Problem 4. True or False? Write TRUE if the statement is true or write FALSE if the statement is false. Give a reason for your answer. You do not need a proof. Just write a brief explanation of your reasoning.

a) $[0,1] \times [0,1) \subset \mathbb{R}^2_{\text{usual}}$ is homeomorphic to $[0,1) \times [0,1) \subset \mathbb{R}^2_{\text{usual}}$.

b) $(0,1) \cup [2,3) \subset \mathbb{R}_{\text{usual}}$ is homeomorphic to $[0,1) \cup [2,3) \subset \mathbb{R}_{\text{usual}}$.

c) $(0,1) \cup (1,2) \cup (2,3) \subset \mathbb{R}_{\text{usual}}$ is homeomorphic to $(0,2) \cup (2,3) \subset \mathbb{R}_{\text{usual}}$

d) $(0,1] \cup [2,3) \subset \mathbb{R}_{\text{usual}}$ is homeomorphic to $[0,1) \cup (2,3] \subset \mathbb{R}_{\text{usual}}$

e) Let $\{a_n\}_{n=1}^{\infty} \subset [0,1] \subset \mathbb{R}_{\text{usual}}$, and let $A = \{a_n | n \in \mathbb{Z}_{>0}\}$. Then $\overline{A} \neq [0,1]$.

f) Given a topological space X and subset $A \subset X$, then the closure of the interior of A contains A. That is, $A \subset \overline{\text{Int}(A)}$.

g) Let A be an open subset of a Hausdorff space. Then A is not compact.

h) If $(X, \mathcal{T})$ is a space and $(A, \mathcal{T}_{\mathcal{A}})$ is connected subspace then $(\overline{A}, \mathcal{T}_{\overline{\mathcal{A}}})$ is also a connected subspace.

Problem 5. Let $(X, \mathcal{T})$ be a compact topological space. Let $C \subset X$ and $\mathcal{T}_C$ be the subspace topology on C induced by $(X, \mathcal{T})$. Show that if C is closed then $(C, \mathcal{T}_C)$ is compact.

Problem 6. Let $(X, \mathcal{T})$ be a topological space. Let $K_1, ..., K_n$ be a finite collection of compact subsets of X. Prove that the union $K = \cup_{i=1}^n K_i$ is a compact subset of X.

Problem 7. Let X be a set and $\mathcal{T}$ and $\mathcal{P}$ be two topologies on X.

a) Prove that if $\mathcal{T} \subset \mathcal{P}$ and $(X, \mathcal{P})$ is compact then $(X, \mathcal{T})$ is compact.

b) Give an example of a set X and two topologies $\mathcal{T}$ and $\mathcal{P}$ on X such that $\mathcal{T} \subset \mathcal{P}$, $(X, \mathcal{T})$ is compact, but where $(X, \mathcal{P})$ is not compact.