

UNIVERSITY OF CALIFORNIA, DAVIS  
FINAL EXAMINATION  
PRE-CALCULUS- MAT 12

Date: 12/10/2013  
Total Marks: 50  
Time: 2 Hours

Name:

ID:

**INSTRUCTIONS**

- (1) **Write your name and ID in the indicated space above.**
- (2) **Answer the FIRST question and ANY FOUR from the remaining questions.**
- (3) **The numbers at the top of each question indicate the distribution of points for all parts of the question. All questions carry 10 points.**
- (4) You must **show your work/calculation** wherever it is necessary to obtain the answer.
- (5) **Write answers only in the space provided.** If necessary use separate sheets for rough work. If extra sheets are needed please staple them with the main answer sheet before submitting.
- (6) State your answers clearly. For example you can draw a box around your answer after you finish computations.
- (7) **Use of *all* electronic gadgets, e.g. mobiles, pagers, smart phones, calculators are prohibited during the exam.**
- (8) Discussion among students during the exam is prohibited.
- (9) **Use of textbooks, or class notes are strictly prohibited.** This is a closed-book exam. Any student trying to use unfair means at any time of the exam may be asked to leave the exam hall and she/he may get a zero score for the exam in such a case.
- (10) **Put down your signature at the bottom of this page.**

*Student's Signature:*



**Question A.** ( $10 \times 1 = 10$ )

State whether the following are TRUE or FALSE. No explanations are required.

- (1) If we know how to draw the curve (i)  $y = f(x + 1)$  then to draw the curve (ii)  $y = f(x)$  we need to translate the curve (i) through a distance 1 to the left hand side.
- (2) The curve  $y = x^{\frac{1}{3}}$  is obtained by reflecting the curve  $y = x^3$  in the  $x$ -axis.
- (3) The numbers 1, 2, 3 are all in the domain of the of the function  $\log_3\left(\frac{2x-1}{2x-3}\right)$ .
- (4) The number 4 is the range of the function  $3e^{2x} + 5$ .
- (5) We have  $\ln 2 < 1$ .
- (6) If  $f(x) = 1 - x$  for all  $x$ , then  $(f \circ f)(x) = 1 - 2x$ .
- (7) There is a function  $g(x)$  such that  $g(5x) = g(x)^5$  for all real number  $x$ .
- (8) Suppose  $a > 1$ ; then for all numbers  $x$  one has  $a^{2\log_a x} = 2x$ .
- (9) For any positive number 'a' we have the conclusion:  $a^x = a^y \Rightarrow x = y$ .
- (10) The number  $1/e$  is rational.

**Question B.** (2 + 2 + 3 + 3)

Suppose  $f(x) = x^3(x - 1)^2$  for all number  $x$ .

- (1) What can you say about the domain and range of  $f(x)$ ?
- (2) Find the  $x$ -intercepts and the  $y$ -intercepts of the curve  $y = f(x)$ .
- (3) Determine the sign of  $f(x)$  on each of the following intervals:  $(-\infty, 0)$ ,  $(1, \infty)$ ,  $(0, 1)$ .
- (4) Draw a rough sketch of the graph of  $f(x)$  using (2), (3). Indicate the nature of the curve near the  $x$ -intercepts as properly as possible.

**Question C.** (2 + 3 + 2 + 3)

Consider the function  $g(x) = \frac{x}{(x+1)(x-2)}$ .

- (1) Find the  $x$ -intercepts and  $y$ -intercepts of the curve  $y = g(x)$ .
- (2) Which lines are asymptotes of the the curve  $y = g(x)$ .
- (3) Determine the sign of  $g(x)$  on the following intervals:  
 $(-\infty, -1)$ ,  $(-1, 0)$ ,  $(0, 2)$ ,  $(2, \infty)$ .
- (4) Draw a rough sketch of the graph of the function  $g(x)$  using (1), (2), (3).

**Question D.** (4 + 6)

- (1) A rectangle is inscribed in a circle of radius 6. Express the area of the rectangle in terms of the width  $x$  (say).
- (2) Find the point of the line  $x - 2y - 5 = 0$  closest to the origin.

**Question E.** (4 + 3 + 3)

**Solve the following equations and inequalities below. (You must show all the intermediate steps through which you arrive at the solutions.)**

(1)  $\log_2(x - 1) + \log_2(x + 2) = 2$

(2)  $2^{3x} = 3^{2x}$

(3)  $\ln x < \ln(2 - x)$

**Question F.** (3 + 2 + 1 + 4)

Consider the function  $G(x) = -\ln(x - 2)$ .

- (1) What can you say about the the domain and the range of the function  $G(x)$ ? Show your work.
- (2) Where does the graph of  $G(x)$  intersect the  $x$ -axis and the  $y$ -axis?
- (3) Which lines are asymptotes of the curve  $y = G(x)$ .
- (4) Draw a rough sketch of the graph of the function  $G(x)$ .



**Question G.** (3 + 2 + 1 + 4)

Consider the function  $F(x) = 1 - 2^x$ .

- (1) What can you say about the the domain and the range of the function  $F(x)$ ? Show your work.
- (2) Where does the graph of  $F(x)$  intersect the  $x$ -axis and the  $y$ -axis?
- (3) Which lines are asymptotes of the curve  $y = F(x)$ .
- (4) Draw a rough sketch of the graph of the function  $F(x)$ ?

