University of California, Davis
Department of Mathematics

Winter Quarter 2020
MAT 180
Special Topics

Course Title: Intro to Physical Mathematics
Professor(s): Tudor Dimofte / Mukund Rangamani

Prerequisites (Tentative):

Either

- MAT 67, 125A, maybe 185A

OR

- PHY 110AB, PHY 115A

Required Text:

The required text for this course is still to be determined.

Course Description:

This course will use several classic problems in quantum physics to motivate and develop mathematical constructions in the realms of algebra, geometry, and topology. We will focus on the dual questions: how can mathematics help organize and conceptualize physical phenomena;and how can physics inspire new ideas in mathematics? This will be a capstone course, and will require students to submit final projects.

We will study 2-3 of the following classical problems:

The quantum-mechanical particle on a manifold

- Introduce basic, relevant aspects of differential geometry, Lie groups, and representation theory
- Use representation theory to organize the spectrum of quantum states
- Abstraction: use quantum systems to produce representations of arbitrary Lie groups (touching on Borel-Weil-Bott theory)

Magnetic monopoles

- Formulate electrodynamics via differential forms, vector bundles, and connections

- Describe magnetic monopoles and "Dirac strings" as nontrivial bundles; introduce basic examples of Chern classes as the electromagnetic field of a monopole
- Topological stability of monopoles/solitons in various dimensions, leading to the concept of winding numbers and homotopy groups

Topological insulators and the quantum Hall effect
- Extends and abstracts the idea of nontrivial bundles and Chern classes
- Understand edge currents as characteristic of a jump in Chern classes, touch on index theorems.

Supersymmetric quantum mechanics
- A natural, direct physical construction of the de Rham complex of a manifold
- Physical proof of the Hodge theorem (relating de Rham cohomology to harmonic forms), by analyzing ground states in supersymmetric quantum mechanics.

Two Practical Notes:
1. Many of the topics above sound (and usually are) quite advanced, either in physics or mathematics. We are well aware of this; and nonetheless believe that by building up the mathematics as a natural way to describe physical systems (and conversely, by using math to describe physics in a clear, concrete, formalism), both will become vastly more approachable.
    - Also, our goal is not to teach (e.g.) the full foundations of differential geometry, Lie theory, or fiber bundles; or the full foundations of quantum mechanics or electromagnetism. We will work with a relatively simple, small set of examples that open a window into more complicated phenomena.
2. The two prerequisite choices stated above were decided so that both physics and math majors may take this course. Satisfying either set of prerequisite courses will be sufficient to enroll.

Course Grade:

The grading criteria for this course is still to be determined. However as stated previously, there will be a final project for this course.