Math 16C
Final Exam

Printed Name ______________________________
(FIRST) (LAST)

Signature ______________________________

ID Number ________________

Please Show All Your Work, and Mark Your Answers Clearly.
No Calculators – No Scratch Paper – No Cell Phones

There are **8 pages** of problems. (#4 and #18 are for extra credit.)

You are expected to do your own work, and to adhere to the UCD Code of Academic Conduct.

Simplify all numerical answers, **except in #12 and #17.**

Be sure to use the limit symbol and summation sign where needed.

When you solve a 1st order linear DE, include all steps of the solution; do not just substitute into the formula given in the text.

Please indicate clearly if you continue work on the back of a page.

Please stop working **immediately** when time is called.

Have a Good Summer!

① FIND THE CENTER AND RADIUS OF THE SPHERE WITH EQUATION
$x^2 + y^2 + z^2 - 10x + 4y - 6z + 22 = 0.$

6 PTS

② A TANK INITIALLY CONTAINS 250 GALLONS OF WATER WITH 15 LB OF SALT. WATER CONTAINING 4 LB OF SALT PER GALLON ENTERS THE TANK AT THE RATE OF 8 GAL/MIN, AND THE STIRRED MIXTURE IS DRAINED AT THE RATE OF 5 GAL/MIN. SET UP A DE TO FIND $A(T)$, THE AMOUNT OF SALT IN THE TANK AT TIME T.

8 PTS

③ SKETCH THE REGION IN THE XY-PLANE WHICH CORRESPONDS TO THE DOMAIN OF $f(x,y) = \sqrt{9 - y - x^2}$.

6 PTS

④ USE NEWTON'S METHOD WITH INITIAL ESTIMATE $x_1 = 1$ TO FIND THE NEXT ESTIMATE x_2 FOR THE SOLUTION OF THE EQUATION $e^{3x-3} + x^3 + 8x = 9 - 2\ln x$.

8 PTS (EXTRA CREDIT)

(5) LET $f(x,y) = 3x^2 - x^2y - y^3 + 6y^2$.
GIVEN THAT $f_x = 6x - 2xy$ AND $f_y = -x^2 - 3y^2 + 12y$,

A) FIND ALL THE CRITICAL POINTS FOR f.

7 PTS

B) CLASSIFY EACH CRITICAL POINT AS CORRESPONDING TO A REL. MAXIMUM, REL. MINIMUM, OR SADDLE POINT. (CALCULATE THE VALUE OF D AT EACH CRITICAL POINT.)

12 PTS

(6) FIND THE SOLUTION OF THE DE $y' = x\sqrt{y}\, e^{x/4}$ WHOSE GRAPH PASSES THROUGH THE POINT $(4,9)$.

12 PTS

(7) FIND THE SOLUTION OF THE DE $xy' - 2y = 6x^4 + 5x^3 + 9x^2 + 8$ WHOSE GRAPH PASSES THROUGH THE POINT $(1, 10)$.

12 PTS

(8) A BALL IS DROPPED FROM A HEIGHT OF 80 FT, AND EACH BOUNCE IS $\frac{11}{15}$ TIMES AS HIGH AS THE PREVIOUS BOUNCE. FIND THE TOTAL VERTICAL DISTANCE TRAVELED BY THE BALL.

8 PTS

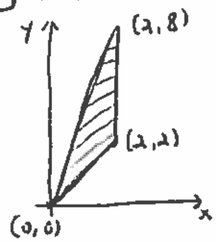
(9) SET UP, BUT DO NOT EVALUATE, A DOUBLE INTEGRAL FOR THE VOLUME OF THE SOLID BOUNDED BY THE TOP HALF OF THE SPHERE $x^2 + y^2 + z^2 = 117$ AND THE PARABOLOID $z = \frac{1}{4}(x^2 + y^2)$.

9 PTS

(10) EVALUATE $\displaystyle\int_0^2 \int_{x^3}^8 \frac{27x^2}{\sqrt{y^2+36}}\, dy\, dx$ BY REVERSING THE ORDER OF INTEGRATION.
(SKETCH, AND SHADE, THE REGION OF INTEGRATION.)

14 PTS

(11) FIND THE AVERAGE VALUE OF $f(x,y) = \dfrac{18y}{(x^3+2)^2}$ ON THE TRIANGULAR REGION SHOWN:



13 PTS

(12) USE THE FIRST 4 NONZERO TERMS OF A MACLAURIN SERIES TO APPROXIMATE
$\int_0^{.4} x^2 e^{-x^5}\,dx$, AND FIND AN UPPER BOUND FOR THE ERROR IN YOUR ESTIMATE.

12 PTS

(13) USE LAGRANGE MULTIPLIERS TO FIND THE MAX. AND MIN. VALUES OF
$f(x,y,z) = 2x - 3y - z$ ON THE ELLIPSOID $(x+2)^2 + 3(y-1)^2 + 2z^2 = 120$.

14 PTS

(14) DETERMINE IF EACH OF THE FOLLOWING SERIES CONVERGES OR DIVERGES, JUSTIFY YOUR ANSWERS COMPLETELY, AND IDENTIFY EACH TEST YOU USE.

A) $$\sum_{n=1}^{\infty}\left(\frac{9(4^{n+1})}{5^{n-1}} - \frac{2}{\sqrt{n}}\right)$$

8 PTS

B) $$\sum_{n=1}^{\infty}\frac{4^n+6^n}{6^{n+2}+5^n}$$

7 PTS

C) $$\sum_{n=1}^{\infty}\frac{2^n(n+4)!}{n^n}$$

9 PTS

(14) D) $\sum_{n=1}^{\infty} \frac{n+10}{n^3+n^2+2}$

9 PTS

(15) FIND THE OPEN INTERVAL OF CONVERGENCE FOR THE POWER SERIES

$\sum_{n=1}^{\infty} (-1)^n \frac{n+6}{5^n n^9} (x-3)^n$. (WRITE YOUR ANSWER IN INTERVAL NOTATION.)

10 PTS

(16) APPROXIMATE $\sqrt{6}$ USING THE FIRST 4 TERMS OF THE TAYLOR SERIES FOR $f(x) = \sqrt{x}$ CENTERED AT $c = 4$, AND SIMPLIFY EACH TERM IN YOUR ANSWER.

12 PTS

(17) APPROXIMATE $\int_0^{.3} \frac{x^2}{x^4+2}\,dx$ USING THE FIRST 3 NONZERO TERMS OF A MACLAURIN SERIES, AND FIND AN UPPER BOUND FOR THE ERROR.

12 PTS

(18) ASSUME THAT THE POPULATION $Y(T)$ OF BACTERIA ON CALLISTO GROWS AT A RATE PROPORTIONAL TO THE SUM OF THE POPULATION AND ITS SQUARE ROOT. IF THERE WERE 100 BACTERIA INITIALLY AND 400 BACTERIA AFTER 6 YEARS, WHEN WILL THERE BE 2500 BACTERIA ON CALLISTO?

15 PTS

(EXTRA CREDIT)