Math 21C
Final Exam

Printed Name ______________________________
(FIRST) (LAST)

Signature ______________________________

ID Number ________________

Please Show All Your Work, and Mark Your Answers Clearly.
No Calculators -- No Scratch Paper -- No Cell Phones

There are **9 pages** of problems. (The last 2 problems are for extra credit.)

You are expected to do your own work, and to adhere to the UCD Code of Academic Conduct.

Simplify all numerical answers, except in #8.

Be sure to use the limit symbol and summation sign where needed.

Please indicate clearly if you continue work on the back of a page.

Be sure to stop working **immediately** when time is called.

Have a Good Summer!

① FIND THE SUM OF THE CONVERGENT SERIES $\sum_{n=1}^{\infty}\left(\frac{28}{5^n} + (-1)^{n+1}\frac{20}{3^n}\right)$.

8 PTS

② FIND THE ANGLE BETWEEN THE PLANES $2X + Y + 2Z = 7$ AND $4X + 8Y + Z = 11$.

7 PTS

③ FIND THE INTERVAL OF CONVERGENCE FOR THE POWER SERIES $\sum_{n=0}^{\infty}(-1)^n \frac{(x-1)^n}{3^n\sqrt{n+1}}$.
(WRITE YOUR ANSWER IN INTERVAL NOTATION.)

11 PTS

④ FIND AN EQUATION OF THE PLANE WHICH PASSES THROUGH THE POINTS $P(1,2,2)$, $Q(2,9,4)$, AND $R(3,5,8)$. (SIMPLIFY YOUR ANSWER.)

8 PTS

⑤ FIND AN EQUATION OF THE TANGENT PLANE TO THE SURFACE $5x^2 - 3xy + 2yz^2 - 4z^3 = -2$ AT $P(2,5,2)$. (SIMPLIFY YOUR ANSWER.)

8 PTS

⑥ IF $f(x,y,z) = 4xz - 5xy + 2yz$ AND $P = (1,1,3)$, FIND

a) THE DIRECTIONAL DERIVATIVE OF f AT P IN THE DIRECTION OF $\vec{v} = \langle 2,-2,1 \rangle$.

b) THE MAXIMAL DIRECTIONAL DERIVATIVE OF f AT P.

10 PTS

(7) USE LAGRANGE MULTIPLIERS TO FIND THE MAX. AND MIN. VALUES OF
$f(x,y,z) = 3x - 2y - 3z$ ON THE ELLIPSOID $(x-6)^2 + 2(y+4)^2 + 3z^2 = 350$.

13 PTS

(8) APPROXIMATE $\int_0^{.8} x^4 e^{-x^3}\,dx$ USING THE FIRST 3 NONZERO TERMS OF A MACLAURIN SERIES, AND FIND AN UPPER BOUND FOR THE ABSOLUTE VALUE OF THE ERROR. (YOU DO NOT HAVE TO SIMPLIFY NUMERICALLY.)

12 PTS

⑨ LET $f(x,y) = 2x^3 + xy^2 - 2y^2 - 15x^2$, SO
$f_x = 6x^2 + y^2 - 30x$ AND $f_y = 2xy - 4y$.

A) FIND ALL THE CRITICAL POINTS FOR f.

7 PTS

B) CLASSIFY EACH CRITICAL POINT AS CORRESPONDING TO A LOCAL MAX., LOCAL MIN., OR SADDLE POINT. (CALCULATE THE VALUE OF D AT EACH POINT.)

11 PTS

⑩ FIND THE DISTANCE FROM $P(8,-1,3)$ TO THE LINE WHICH PASSES THROUGH THE POINTS $Q(3,5,2)$ AND $R(2,9,5)$.

9 PTS

(11) IF $g(t) = f(t^2+5t,\ t^3+2t)$ WHERE f IS DIFFERENTIABLE, EXPRESS $g'(2)$ IN TERMS OF THE PARTIAL DERIVATIVES OF f.

8 PTS

(12) LET $\vec{a} = \langle 10, 9, 5 \rangle$ AND $\vec{b} = \langle 2, 1, -1 \rangle$. IF $\vec{a} = \vec{u} + \vec{v}$ WHERE $\vec{u}$ IS PARALLEL TO $\vec{b}$ AND $\vec{v}$ IS ORTHOGONAL TO $\vec{b}$, FIND $\vec{u}$ AND $\vec{v}$.

8 PTS

(13) FIND PARAMETRIC EQUATIONS FOR THE TANGENT LINE TO THE CURVE
$\vec{r}(t) = (\sin 4t + 5)\vec{i} + (3t + 8\cos t)\vec{j} + (2e^{-3t})\vec{k}$ AT THE POINT WITH $t=0$.

9 PTS

(14) SHOW WHETHER EACH OF THE FOLLOWING SERIES CONVERGES OR DIVERGES. (JUSTIFY YOUR ANSWERS COMPLETELY.)

A) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n+2}{n+6}\right)^n$

7 PTS

B) $\displaystyle\sum_{n=1}^{\infty} \frac{(\ln n)^3}{n^2}$

7 PTS

(15) APPROXIMATE $\sqrt{23}$ USING THE FIRST 4 TERMS OF THE TAYLOR SERIES FOR $f(x)=\sqrt{x}$ AT $a=25$, AND SIMPLIFY EACH TERM IN YOUR ANSWER. (YOU DO NOT HAVE TO COMBINE THE TERMS, OR ESTIMATE THE ERROR.)

12 PTS

(16) SHOW WHETHER EACH OF THE FOLLOWING SERIES CONVERGES ABSOLUTELY, CONVERGES CONDITIONALLY, OR DIVERGES. (JUSTIFY YOUR ANSWERS COMPLETELY.)

A) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{3^n (n+5)!}{n^n}$

4 PTS

B) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n-1}{n^2+35}$

12 PTS

(17) FIND THE POINT ON THE PLANE $2X + Y - 3Z = 20$ WHICH IS CLOSEST TO THE POINT $P(8, 3, 9)$.

10 PTS

(18) FIND THE FIRST 4 NONZERO TERMS OF THE MACLAURIN SERIES FOR $f(x) = \sin^{-1} x$, GIVEN THAT $f'(x) = \dfrac{1}{\sqrt{1-x^2}}$ AND $f(0) = 0$. (SIMPLIFY THE COEFFICIENTS.)

12 PTS

(19) FIND THE SUM OF THE CONVERGENT SERIES $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n(n+1)}{4^n}$ USING A POWER SERIES.

10 PTS (EXTRA CREDIT)

(20) LET $f(x,y) = xy + \frac{12}{x} + \frac{18}{y}$ FOR $x > 0$ AND $y > 0$.
GIVEN THAT $(2,3)$ IS THE ONLY CRITICAL POINT FOR f, SHOW THAT f HAS AN ABSOLUTE MINIMUM AT THIS POINT.

14 PTS (EXTRA CREDIT)