

Math 145  
Final Exam

Printed Name \_\_\_\_\_  
(FIRST) (LAST)

Student ID Number \_\_\_\_\_

**Please Show All Your Work.**

**No Calculators -- No Scratch Paper -- No Cell Phones**

There are **8 pages** of problems (and 250 points).

**You are expected to do your own work, and to  
adhere to the UCD Code of Academic Conduct.**

**Be sure to give explanations of your answers,**  
except in #3, #13A, and #16.

**You may leave combinations, factorials, and exponents in your answers,**  
unless indicated otherwise.

Please indicate clearly if you continue work on the back of a page.

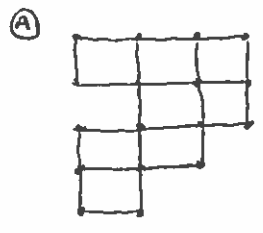
**Be sure to stop working immediately when time is called;**  
if you do not, you are subject to a deduction from your test score.

**Have a good summer!**

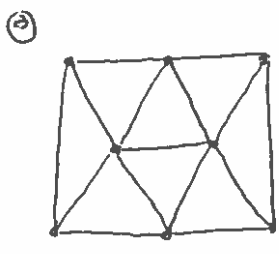
① FIND THE NUMBER OF WAYS TO CHOOSE A JURY OF 12 PEOPLE FROM A GROUP OF 20 MEN AND 30 WOMEN SO THAT THE JURY HAS AT LEAST 1 MAN AND AT LEAST 2 WOMEN.

8  
PTS

④ FOR EACH OF THE FOLLOWING GRAPHS,  
i) STATE WHETHER THE GRAPH IS BIPARTITE AND BRIEFLY EXPLAIN YOUR REASONING, AND  
ii) FIND ITS CHROMATIC NUMBER.



8  
PTS



10  
PTS

⑤ FIND THE NUMBER OF WAYS TO GIVE 5 BOOKS TO 9 STUDENTS IF

A) THE BOOKS ARE DISTINCT, AND

1) NO STUDENT CAN GET MORE THAN 1 BOOK, \_\_\_\_\_

2) ANY STUDENT CAN GET MORE THAN 1 BOOK. \_\_\_\_\_

16  
PTS

B) THE BOOKS ARE IDENTICAL, AND

1) NO STUDENT CAN GET MORE THAN 1 BOOK. \_\_\_\_\_

2) ANY STUDENT CAN GET MORE THAN 1 BOOK. \_\_\_\_\_

④ IN HOW MANY WAYS CAN TESTS BE RETURNED TO A CLASS OF 10 STUDENTS SO THAT EXACTLY 6 STUDENTS GET THEIR OWN TEST? (WRITE YOUR ANSWER AS AN INTEGER.)

P.2

10  
PTS

⑤ FIND THE NUMBER OF WAYS TO GIVE 14 PRESENTS TO 6 CHILDREN SO THAT EVERY CHILD GETS A PRESENT

A) IF THE PRESENTS ARE IDENTICAL.

8  
PTS

B) IF THE PRESENTS ARE DISTINCT.

12  
PTS

⑥ FIND THE NUMBER OF  $n$ -DIGIT SEQUENCES WHICH CAN BE MADE USING THE DIGITS  $1, \dots, 6$  IF  $1$  AND  $2$  EACH OCCURS AN EVEN NUMBER OF TIMES,  $4$  OCCURS AN ODD NUMBER OF TIMES, AND  $5$  OCCURS AT LEAST ONCE.




14  
PTS

⑦ FIND THE NUMBER OF PERMUTATIONS OF  $S = \{4 \cdot a, 5 \cdot b, 8 \cdot c, 7 \cdot d, 6 \cdot e\}$  WHICH HAVE EXACTLY 2 ADJACENT  $e$ 'S.

13  
PTS

9) Let  $a_n$  be the number of nonnegative integer solutions of  
 $x + y + z + t + u + v + 4r + 5s = n$  with  $t \leq 3, u \leq 4, v \leq 8$ .  
 Use generating functions to find a formula for  $a_n$ .

14  
 pts

10) Find all nonisomorphic trees of order 6, given that the nonisomorphic  
 trees of order 5 are  and  and .

11  
 pts

11) Let  $c_n$  be the number of  $n$ -letter codewords using the letters  $a, b, c$  which do not contain the sequences aa, ab, or bb. Find a recurrence relation satisfied by  $c_n$ .

10  
pts

12) Find the number of collections of 50 coins that can be made from a selection of 60 nickels, 30 dimes, 15 quarters, and 10 half-dollars (if coins of the same type are identical).

13  
pts

13) a) FIND THE NUMBER OF LABELED TREES OF ORDER 10. \_\_\_\_\_

4  
PTS

b) DRAW A TREE WITH PRÜFER CODE 35154.

7  
PTS

c) FIND THE NUMBER OF LABELED TREES OF ORDER 10 WITH EXACTLY 4 LEAVES  
(USING THAT THE LEAVES CORRESPOND TO THE MISSING DIGITS IN THE PRÜFER CODE)

12  
PTS

14) GIVE AN EXAMPLE OF EACH OF THE FOLLOWING, AND BRIEFLY JUSTIFY YOUR ANSWERS!

a) A GRAPH  $G$  WITH A HAMILTON CYCLE BUT NO CLOSED EULERIAN TRAIL.

9  
PTS

b) A GRAPH  $G$  WITH A CLOSED EULERIAN TRAIL BUT NO HAMILTON CYCLE.

9  
PTS

15) a) PROVE THAT IF  $G$  IS A TREE OF ORDER  $n$ , THEN  $G$  HAS  $n-1$  EDGES.  
 (USE THE DEFINITION THAT A TREE IS A CONNECTED GRAPH WITH NO CYCLES.)

9  
 P13

b) SHOW THAT  $K_{3,3}$  IS NOT A PLANAR GRAPH. (NOTICE THAT  $K_{3,3}$  HAS NO 3-CYCLES.)

9  
 P13

16) SIMPLIFY EACH OF THE FOLLOWING EXPRESSIONS:

a) 
$$\sum_{k=0}^{24} (-1)^k \binom{24}{k} 5^k 8^{24-k} = \underline{\hspace{2cm}}$$

5  
 P13

b) 
$$\binom{11}{0} + \binom{11}{1} + \binom{11}{2} + \dots + \binom{11}{11} = \underline{\hspace{2cm}}$$

3  
 P13

c) 
$$\binom{16}{1} - \binom{16}{2} + \binom{16}{3} - \binom{16}{4} + \dots - \binom{16}{16} = \underline{\hspace{2cm}}$$

4  
 P13

d) 
$$\binom{7}{7} + \binom{8}{7} + \binom{9}{7} + \dots + \binom{44}{7} = \underline{\hspace{2cm}}$$

4  
 P16



17) LET  $G$  BE A BIPARTITE GRAPH WITH BIPARTITION  $A, B$  SUCH THAT  $\deg(v) = d$  FOR EVERY VERTEX  $v$  IN  $G$ .

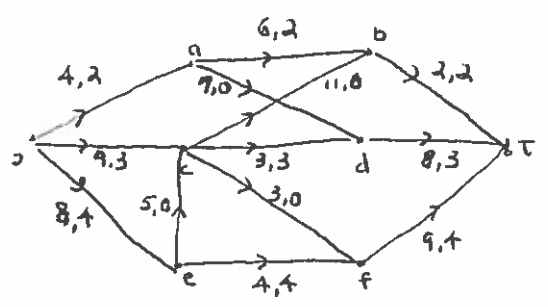
a) SHOW THAT  $|A| = |B|$ .

4  
pts

b) SHOW THAT  $G$  HAS A PERFECT MATCHING (A MATCHING THAT SATURATES EVERY VERTEX)

9  
pts

18) STARTING WITH THE FLOW GIVEN BELOW,  
a) FIND A MAXIMUM FLOW  $F$  AND ITS VALUE  $\text{val}(F)$ .  
b) SHOW THAT  $F$  IS MAXIMUM BY FINDING A CUT  $C$  WITH  $\text{val}(F) = \text{cap}(C)$ .  
(IN PART A), INDICATE THE PATHS YOU USE.)



15  
pts