Cluster algebra was introduced by Fomin and Zelevinsky in early 2000’s, and it has appeared in many branches of mathematics, including combinatorics, representation theory, and low dimensional topology. This course will be an introductory course to cluster algebra. We will discuss some basic results including Laurent phenomenon, finite type classification, and positivity. We will also learn about some basic examples of cluster algebras such as Grassmannian, double Bruhat cells, and augmentation variety of Legendrian links. Familiarity with some basic abstract algebra is recommended.

The course will cover a number of fundamental topics in advanced linear algebra and matrix analysis including the following: norms on matrix spaces, positivity and order structures, convexity, matrix inequalities, and variational principles. The selection of topics will be guided by important applications in quantum information theory, quantum probability, and quantum physics, such as entropies and related inequalities, completely positive maps, quantum channel capacities, Perron-Frobenius results for quantum channels, spectral gap and log-Sobolev inequalities and more.

Prerequisites: Basic linear algebra and finite-dimensional analysis.

Text: R. Bhatia, Matrix Analysis (Springer GTM 169), supplemented by additional literature and instructor’s notes.

The course provides a panorama of rapid developments in pure mathematics research in the topics of the title that took place in the last dozen years, in which the instructor has played an active role. We will employ a ground-up mechanism starting with the simplest model to describe the excitement of the research frontier from the current perspective. The purpose is to give an invitation to active research projects beginning concurrently with the course. The instructor will organize several international conferences in 2024-25, including ones at RIMS-Kyoto in Summer 2024 and MPI MiS, Leipzig, in Spring 2025. Possible summer funding and travel grants to students may be provided by the instructor’s NSF-FRG 2152257 that carries the same title.