

Math 12
Final Exam

Printed Name _____
(FIRST) (LAST)

Signature _____

ID Number _____

**Please Show All Your Work, and Mark Your Answers Clearly.
No Calculators – No Scratch Paper – No Cell Phones**

There are **8 pages** of problems. (Problems #22 and #24 are for extra credit.)

**You are expected to do your own work, and to
adhere to the UCD Code of Academic Conduct.**

Simplify all numerical answers.

Please indicate clearly if you continue work on the back of a page.

In problems **#3** and **#4**, write your answers using interval notation.

In problems **#15, #18, and #22**, give units for your answer.

Please stop working immediately when time is called.

Have a Good Winter Break!

① FIND THE CENTER AND RADIUS OF THE CIRCLE $x^2 + y^2 + 8x - 10y + 5 = 0$.

P. 1

7
pts

② FIND THE SLOPE-INTERCEPT FORM FOR THE LINE WHICH IS PERPENDICULAR TO THE LINE $2x - 6y = 1$ AND WHICH PASSES THROUGH THE VERTEX OF THE PARABOLA $y = x^2 - 4x + 9$.

8
pts

③ SOLVE THE INEQUALITY $\frac{x^2 - 3x - 4}{x^3(x-3)^2} < 0$.

8
pts

④ FIND THE DOMAIN OF $f(x) = \sqrt{\frac{16 - x^2}{x^2 + 3x}}$.

8
pts

5) SIMPLIFY THE FOLLOWING EXPRESSION, SO THAT LOGARITHMS OF PRODUCTS, QUOTIENTS, AND POWERS DO NOT APPEAR. (SHOW ALL WORK.)

$$\ln \left(\frac{x^6 \sqrt{x^2+9}}{e^{x^3} (x-5)^7} \right)$$

7
pts

6) USE AN ADDITION FORMULA TO FIND THE VALUE OF $\cos 105^\circ$.

7
pts

7) IF $f(x) = \sqrt{9x^2+4}$, FIND AND SIMPLIFY $\frac{f(x+h) - f(x)}{h}$.

8
pts

8) GIVE THE EQUATION OF THE GRAPH WHICH RESULTS FROM PERFORMING EACH OF THE FOLLOWING OPERATIONS ON THE GRAPH OF $y = \sqrt{x}$!

A) SHIFT 3 UNITS TO THE LEFT, AND THEN REFLECT IN THE Y-AXIS.

B) REFLECT IN THE X-AXIS, SHIFT 4 UNITS UP, AND THEN STRETCH HORIZONTALLY BY A FACTOR OF 5.

C) SHIFT 8 UNITS TO THE RIGHT, REFLECT IN THE LINE $y=x$, AND THEN SHIFT 6 UNITS DOWN.

9
pts

④ EVALUATE EACH OF THE FOLLOWING:

1) $e^{3\ln 2} + (\ln e)^5$

2) $22 \log_6 \sqrt{6} + 7^{2 \log_7 5}$

9
PTS

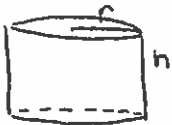
3) $\log_{27} 99 - \log_{27} 11$

⑩ 1) FIND $\sin \theta$ IF $\cos \theta = \frac{15}{17}$ AND $\frac{3\pi}{2} < \theta < 2\pi$,

8
PTS

2) FIND $\cos \theta$ IF $\tan \theta = \frac{12}{5}$ AND $\pi < \theta < \frac{3\pi}{2}$,

⑪



THE TOTAL VOLUME OF A RIGHT CIRCULAR CYLINDER IS $30\pi \text{ cm}^3$.
EXPRESS THE TOTAL SURFACE AREA OF THE CYLINDER AS A FUNCTION
OF r , THE RADIUS OF THE BASE, AND SIMPLIFY YOUR ANSWER.

9
PTS

12) FIND EACH OF THE FOLLOWING NUMBERS:

6
pts

a) $\tan^{-1} 1 = \underline{\hspace{2cm}}$

b) $\sin^{-1} \frac{\sqrt{3}}{2} = \underline{\hspace{2cm}}$

c) $\cos^{-1} 0 = \underline{\hspace{2cm}}$

d) $\tan^{-1} \frac{\sqrt{3}}{3} = \underline{\hspace{2cm}}$

e) $\sin^{-1} (-1) = \underline{\hspace{2cm}}$

f) $\sin^{-1} \left(\sin \frac{11\pi}{9} \right) = \underline{\hspace{2cm}}$

13) SIMPLIFY THE FOLLOWING EXPRESSIONS:

7
pts

a) $\frac{5}{3} x^{2/3} - \frac{5}{3} x^{-4/3}$

b) $x^2 \cdot \frac{1}{2} (3x+5)^{-1/2} \cdot 3 + 2x \sqrt{3x+5}$

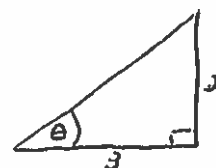
8
pts

c)
$$\frac{(x^2 + 12)^2 (-2x) - (12 - x^2) \cdot 2(x^2 + 12) \cdot 2x}{(x^2 + 12)^4}$$

9
pts

(14) For the angle θ in the right triangle shown, find

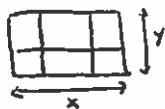
a) $\sin 2\theta$



8
pts

b) $\cos 2\theta$

(15) A farmer has 360 ft of fencing to build 6 rectangular pens, as shown below. Find the largest possible value for the total enclosed area.



9
pts

(16) If $f(x) = \frac{1}{x^3 + 8x}$, find and simplify $\frac{f(t) - f(x)}{t - x}$.

9
pts

17) SOLVE THE EQUATION $4x (\ln x)^3 - 18x \ln(x^4) = 0$.

9
pts

18) FIND THE MAXIMUM POSSIBLE AREA OF A CIRCULAR SECTOR WITH A PERIMETER OF 36 CM.



9
pts

19) EXPRESS $\sin^4 3\theta$ IN A FORM THAT DOES NOT INVOLVE POWERS OR PRODUCTS OF TRIG FUNCTIONS.

8
pts

20 Find all angles θ (in radians) with $0 \leq \theta < 2\pi$ which satisfy $\sin 2\theta + \cos \theta = 0$.

P. 7

7
PTS

21 Solve the equation $e^{2x} - 12e^x - 29 = 0$.

9
PTS

22 If a bacterial culture which is growing exponentially increases from 5 mg to 6 mg in 4 hours, find how long the culture takes to increase by 80%.

10
PTS
(EXTRA
CREDIT)

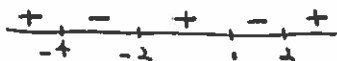
23) For the function $y = \frac{(x-2)(x+4)}{(x-1)(x+2)}$,

a) Find equations for the asymptotes to the graph.

4
PTS

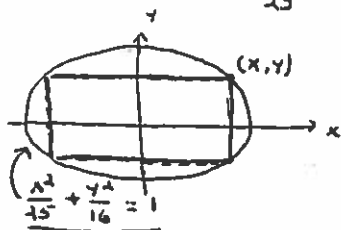
VERTICAL: _____ HORIZONTAL: _____

b) Use the sign chart below to sketch the graph, showing the asymptotes, intercepts, and any points of intersection with the horizontal asymptote.



10
PTS

24) Find the maximum possible area for a rectangle which is inscribed in the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$.



4
PTS
(Extra
Credit)