21B A01/A02 Final Exam

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Do not begin working until we tell you to start. Make sure to **show all of your work**. You do not need to fully simplify your answers, but make sure to evaluate all integrals and trig functions unless told otherwise.

The exam features 8 questions worth 10 points each, plus a bonus problem worth 5 points.

Name ________________________________

ID Number ________________________________

1. Evaluate the following integral:

$$\int_4^5 \frac{2x^2 - 5x + 1}{(x-1)(x-2)(x-3)}\, dx$$

2. For a positive integer n, let S_n be the Riemann sums estimate for the area under the curve $f(x) = \cos(x)$ from $x = 0$ to $x = \pi/2$ using n equal width rectangles and **left endpoints**.

   (a) What is S_3?

   (b) Using sigma notation, give a formula for S_n which works for any positive integer n.

   (c) Using calculus ideas, find $\lim_{n\to\infty} S_n$.

3. Solve the following integrals:

(a)

$$\int e^{6x} \cos(0)\, dx$$

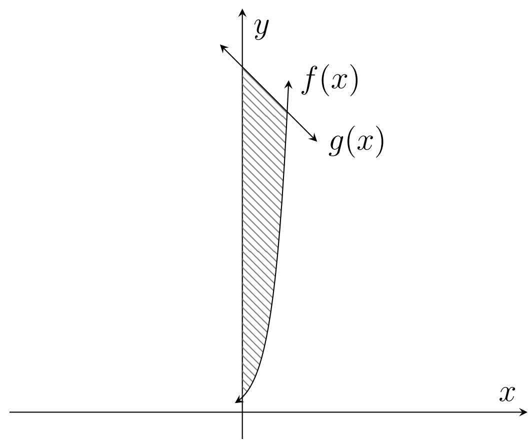
(b)

$$\int e^{6x} \cos(x)\, dx$$

(c)

$$\int e^{6x} \cos(e^{6x})\, dx$$

4. In the diagram below, $f(x) = e^x$ and $g(x) = -(x-3) + e^3$. Use the **shell method** to find the volume of the solid formed when the shaded region is revolved around the y-axis.



5. Evaluate the following integrals if they converge or write **Diverges** if they diverge. (Hint: notice that all 3 integrals are improper integrals).

(a)

$$\int_0^1 \frac{1}{x^{2/5}}\,dx$$

(b)

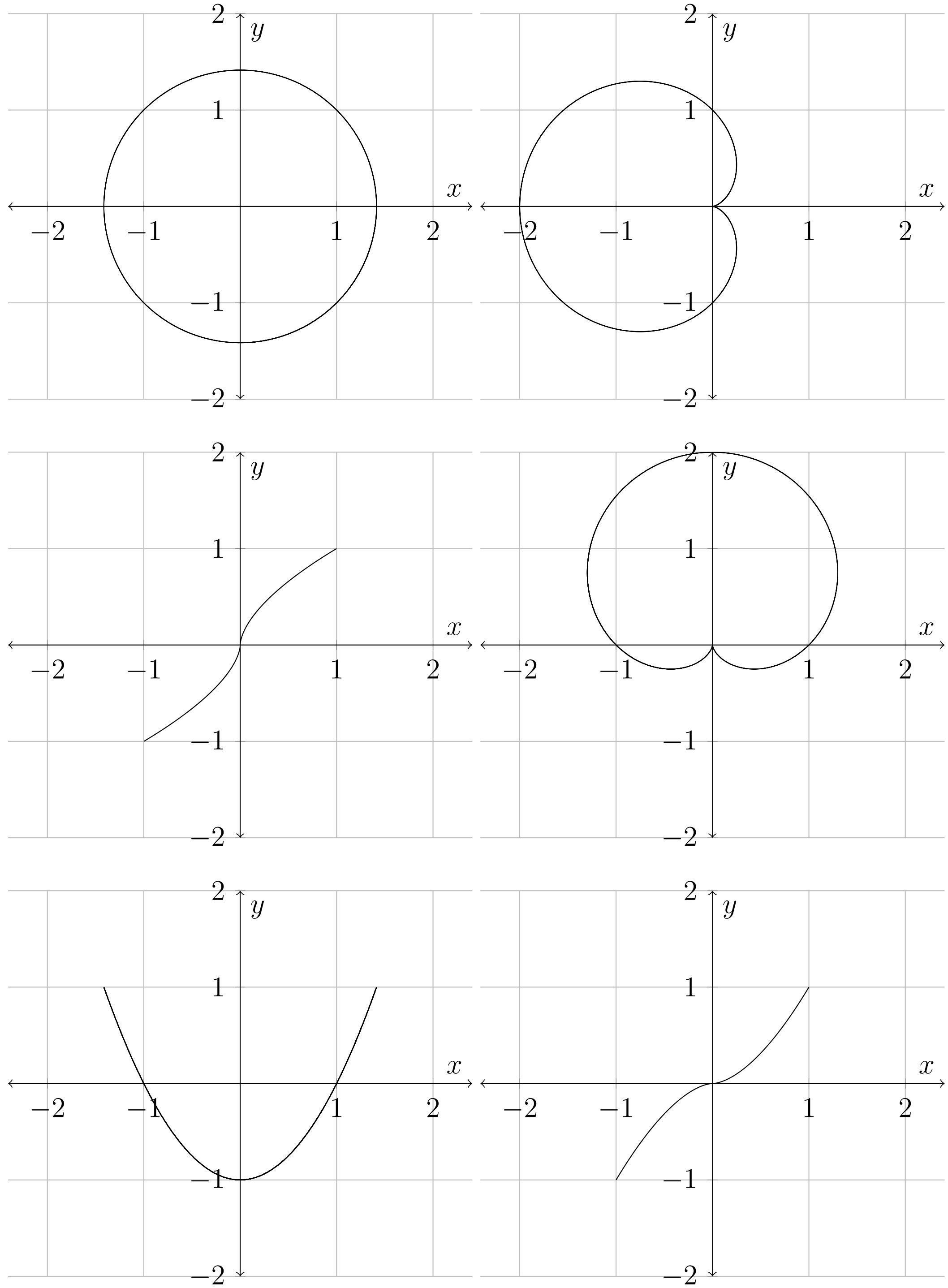
$$\int_{-\infty}^{\infty} x^3\,dx$$

(c)

$$\int_0^5 \frac{1}{(x-2)^3}\,dx$$

6. The graphs of the curves $y = (x-1)(x-2)$ and $y = -(x-1)(x+4)$ together divide the plane into four unbounded regions and one bounded region. What is the area of the bounded region?

7. For each of the parametric or polar equations on the next page, write the letter (A through F) of the corresponding graph. Note that some graphs may be used more than once, or not at all. **For this problem only, you do not need to show your work, and all answers should be written on the next page** .



(a) $r = 1 - \cos(\theta), \qquad 0 \le \theta \le 2\pi$

(b) $x = \cos(t) + \sin(t), \qquad y = \sin(t) - \cos(t), \qquad 0 \le t \le 2\pi$

(c) $r = \sin(\theta) - 1 \qquad 0 \le \theta \le 2\pi$

(d) $x = t^5, \qquad y = t^3, \qquad 0 \le t \le 1$

(e) $x = \cos(t) - \cos^2(t), \qquad y = \sin(t) - \sin(t)\cos(t), \qquad -\pi \le t \le \pi$

8. What is the length of the curve defined by $e^y = \sec(x)$ from the point $(0, 0)$ to the point $(\pi/4, \ln(\sqrt{2}))$? **Hint**: Start by rewriting the equation to give y as a function of x. It's possible to use parametric equations to get a solution, but I would not recommend it for this problem.

9. **Bonus:** Find the following sum:

$$\sum_{k=1}^{1000} \frac{1}{k^2 + k}$$

Hint: Replace the fraction with an equivalent expression using a trick we learned in this class. Write out the first few terms of the sum and see if you notice a pattern.