

University of California, Davis
Department of Mathematics

Fall Quarter 2019
MAT 180
Special Topics

Course Title: Finite Reflection Groups

Professor: Monica Vazirani

Prerequisites:

MAT 150A and a solid background in linear algebra.

Required Text:

Finite Reflection Groups

Authors L. C. Grove, C. T. Benson

Part of the Graduate Texts in Mathematics book series
(GTM, volume 99)

See: <https://link.springer.com/book/10.1007%2F978-1-4757-1869-0>

Course Description:

A reflection is a linear transformation describing the mirror image about a line in the plane, about a plane in 3-dimensional space, or generally, about a hyperplane in n -dimensional space.

A reflection group is a discrete group which is generated by a set of reflections of a finite-dimensional (Euclidean) space. Examples of finite reflection groups include the symmetry groups of regular polytopes (e.g., Platonic solids) and of regular tilings (e.g., wallpaper patterns), the symmetric group, and the Weyl groups of simple Lie algebras.

The theory of reflection groups links linear algebra, abstract algebra (particularly group theory), Lie algebras, and representation theory in a beautiful way.

The following Course Topics are planned to be covered:

- Orthogonal transformations
- Reflections in real Euclidean space

- Coxeter groups
- Crystallographic groups
- Root systems
- The classification of finite Coxeter groups

One aim of the course will be to understand the classification of finite reflection groups using Coxeter-Dynkin diagrams. While the course is primarily targeted at students interested in studying higher mathematics, the subject matter may be of interest (and possible use) in subjects such as chemistry, computer science, materials science, and theoretical physics.

As such, we can observe examples of the applications of reflection groups. A kaleidoscope is an example of an optical device for visualizing certain reflection groups in the plane.

Additionally, examples of infinite reflection groups include the triangle groups corresponding to regular tessellations of the Euclidean plane and the hyperbolic plane, and Weyl groups of infinite-dimensional Kac-Moody algebras.

Course Grade:

Grading criteria for this course is still to be determined.