Math 21C, CRN: 38564-8, Final Friday, Dec 7 Fall 2022

Name:	

Student ID:									

You may not use a calculator.
You may use one page of notes.
You may not use the textbook.
Please do not simplify answers.

1. (5 points)(Sequence)
   Find the following limit or show that it does not exist:

$$\lim_{n\to\infty} \frac{3n^3 + \sin n}{n(n+1)(n+2)}$$

2. (20 points)(Series)
   Determine whether each of the following infinite sums converges and briefly explain your reasoning:

   (a)

$$\sum_{n=1}^{\infty} \sqrt{\frac{n^2 - 1}{n^2 + 1}}$$

(b)

$$\sum_{n=1}^{\infty} \frac{n^3}{2^n}$$

(c)

$$\sum_{n=1}^{\infty} \frac{2^n}{n^3}$$

(d)

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(n)}$$

(e)

$$\sum_{n=1}^{\infty} \frac{(-1)^{2n}}{\ln(2n)}$$

(just the even terms from the previous one.)

3. (20 points)(Interval)
Determine for which values of x the following series converges absolutely, converges conditionally and diverges.

$$\sum_{n=0}^{\infty} \frac{2^n}{\sqrt{n^3}}(x-3)^n.$$

4. (20 points)(Taylor)

(a) Find the degree 2 Taylor polynomial $P_2(x)$ for the function

$$f(x) = (x + 1)e^x.$$

(b) Use Taylor's theorem with remainder to bound the error in approximating $(0.9)e^{-0.1}$ by $P(-0.1)$.

5. (20 points)(Planes)
Find an equation for the plane containing the points $(1,1,1)$, $(0,1,2)$ and $(1,2,2)$.

6. (20 points)(Chain Rule)
Find

$$\frac{\partial z}{\partial s}(1,1)$$

if $z(s,t)$ is some unknown function but

$$z(1,1) = 1,$$

$$\frac{\partial f}{\partial s}(1,1) = 12,$$

$$f(x,y,z) = x^2 + yz,$$

$$x(s,t) = s^2 + t^2,$$

$$y(s,t) = st.$$

7. (20 points)(Gradients)
   Find the Maximum value of the directional derivative of $f(x, y, z) = xyz$ at the point $(1, 2, 3)$.

8. (25 points)(Critical)
Find all the critical points of the function

$$f(x,y) = \frac{1}{3}x^3 + xy - \frac{1}{2}y^2$$

and determine whether each is a local Maximum, a local minimum or a saddle point.

9. (25 points)(Global)
Find the (global) Maximum and (global) minimum values of the function

$$f(x, y) = \frac{1}{3}x^3 - \frac{1}{2}y^2$$

in the triangle with sides $y = 0$, $x = 4$ and $x = y$.

10. (25 points)(Lagrange)
Find the Maximum sum of the coordinates of a point on the ellipsoid

$$x^2 + \frac{1}{2}y^2 + \frac{1}{4}z^2 = 7.$$

11. (25 points)(Extra Credit: You do not need to do this problem.)
Consider the function $f(x, y) = ax^2 - xy + by^2$ depending on constants a and b.

(a) For which values of a and b does $f(x, y)$ have more than one critical point?

(b) In these cases find all the critical points and determine whether any are global Maxima or global minima.