1. (5 points) (Sequence)
   Find the following limit or show that it does not exist:
   \[ \lim_{n \to \infty} \frac{3n^3 + \sin n}{n(n + 1)(n + 2)} \]

2. (20 points) (Series)
   Determine whether each of the following infinite sums converges and briefly explain your reasoning:
   
   (a) \[ \sum_{n=1}^{\infty} \sqrt{\frac{n^2 - 1}{n^2 + 1}} \]
(b) \[
\sum_{n=1}^{\infty} \frac{n^3}{2^n}
\]

(c) \[
\sum_{n=1}^{\infty} \frac{2^n}{n^3}
\]
(d) \[
\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(n)}
\]

(e) \[
\sum_{n=1}^{\infty} \frac{(-1)^{2n}}{\ln(2n)}
\]

(just the even terms from the previous one.)
3. (20 points)(Interval)
   Determine for which values of \( x \) the following series converges absolutely, converges conditionally and diverges.

   \[
   \sum_{n=0}^{\infty} \frac{2^n}{\sqrt{n^3}} (x - 3)^n.
   \]
4. (20 points)(Taylor)

(a) Find the degree 2 Taylor polynomial \( P_2(x) \) for the function

\[
f(x) = (x + 1)e^x.
\]

(b) Use Taylor’s theorem with remainder to bound the error in approximating \((0.9)e^{-0.1}\) by \(P(-0.1)\).
5. (20 points) (Planes)
    Find an equation for the plane containing the points (1, 1, 1), (0, 1, 2) and (1, 2, 2).
6. (20 points) (Chain Rule)

Find \( \frac{\partial z}{\partial s}(1,1) \)

if \( z(s,t) \) is some unknown function but

\[ z(1,1) = 1, \]
\[ \frac{\partial f}{\partial s}(1,1) = 12, \]
\[ f(x,y,z) = x^2 + yz, \]
\[ x(s,t) = s^2 + t^2, \]
\[ y(s,t) = st. \]
7. (20 points) (Gradients)

Find the Maximum value of the directional derivative of \( f(x, y, z) = xyz \)
at the point \((1, 2, 3)\).
8. (25 points) (Critical)

Find all the critical points of the function

\[ f(x, y) = \frac{1}{3}x^3 + xy - \frac{1}{2}y^2 \]

and determine whether each is a local Maximum, a local minimum or a saddle point.
9. (25 points) (Global)

Find the (global) Maximum and (global) minimum values of the function

\[ f(x, y) = \frac{1}{3}x^3 - \frac{1}{2}y^2 \]

in the triangle with sides \( y = 0, x = 4 \) and \( x = y \).
10. (25 points)(Lagrange)

Find the Maximum sum of the coordinates of a point on the ellipsoid

\[ x^2 + \frac{1}{2} y^2 + \frac{1}{4} z^2 = 7. \]
11. (25 points)(Extra Credit: You do not need to do this problem.)
Consider the function \( f(x, y) = ax^2 - xy + by^2 \) depending on constants \( a \) and \( b \).

(a) For which values of \( a \) and \( b \) does \( f(x, y) \) have more than one critical point?

(b) In these cases find all the critical points and determine whether any are global Maxima or global minima.