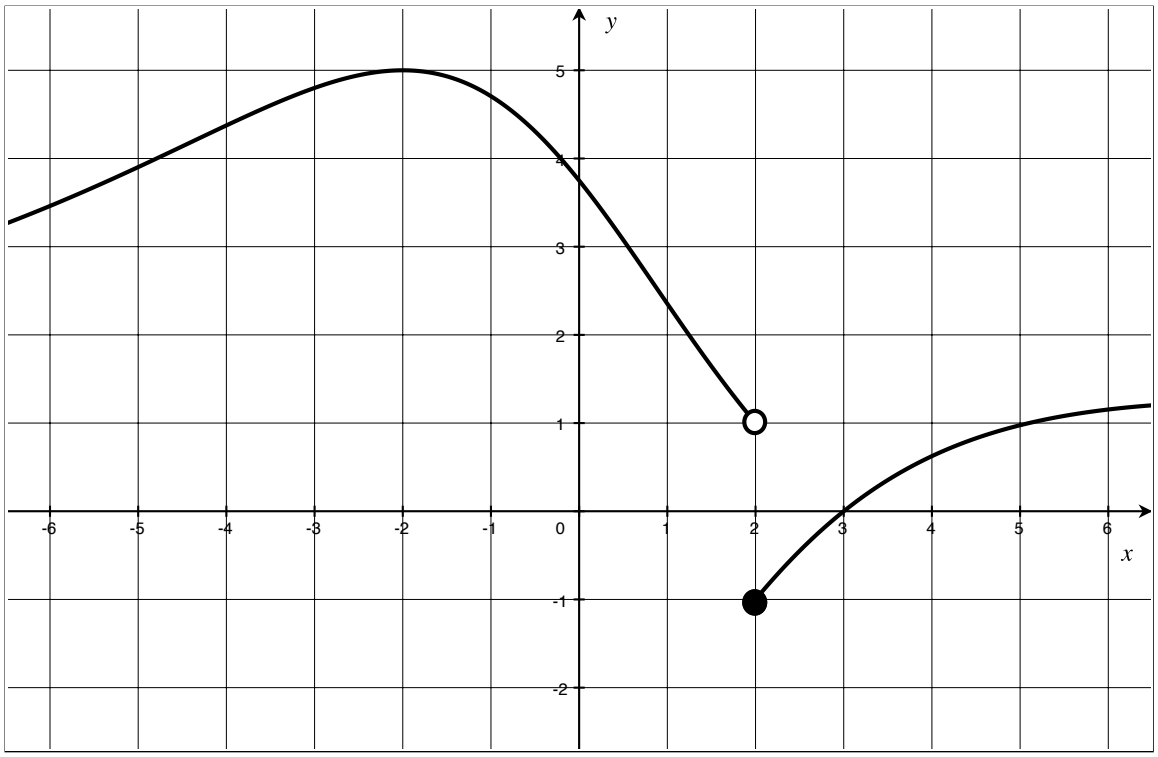
Math 16A
Final Exam

(1) Based on the graph below, label each of the following as either "+" (positive), "-" (negative), "0" (zero), or "DNE" (does not exist). (40 points)

_____ $\lim_{x \to 2^-} f(x)$ _____ $\lim_{x \to 2^+} f(x)$

_____ $f(-2)$ _____ $f'(-2)$ _____ $f''(-2)$

_____ $f(3)$ _____ $f'(3)$ _____ $f''(3)$



(2) Evaluate each of the following limits. (40 points)

(a) $\lim_{x \to -3} \dfrac{x^3 + 2x^2 - 3x}{x + 3}$

(b) $\lim_{x \to 1} \dfrac{\sqrt{x^2 + 3} - 2}{x - 1}$

(c) $\lim_{x \to 1} \sqrt{x^2 + 8x}$

(d) $\lim_{x \to \infty} \dfrac{4x^2 + 2x + 1}{2x + 1000}$

(3) Find the derivative of each of the following functions. (40 points)

(a) $f(x) = 3x^5 - \sqrt{x} + \sqrt{3} - \dfrac{1}{x}$

(b) $f(x) = (\sin(x) + \cos(x))^3$

(c) $f(x) = \dfrac{x^3 - \sqrt{x}}{x+1}$

(d) $f(x) = (5x+7)\sqrt{x^2+2x+2}$

(4) Find the equation for the tangent line to the given function at the specified point. (10 points)

$f(x) = 2\sqrt{x^2 - 2x + 1}, \quad x = 5$

(5) Find $\frac{dy}{dx}$ using implicit differentiation. (10 points)

$x^2y + xy^2 = (1 + y)^2$

(6) Suppose an airplane is flying east from an airport at 2000 ft/sec, and a second plane is flying north from the same airport at 3000 ft/sec. How fast is the distance between the planes changing when the first plane is 3000 ft from the airport and the second plane is 4000 ft from the airport? (20 points)

(7) Sketch a graph of the following function. Be sure to list all zeros, asymptotes, critical numbers, and points of inflection, as well as where the function is increasing, where it is decreasing, where it is concave up, and where it is concave down. (40 points)

$$f(x) = \frac{12}{x^2 + 3}$$

Tip: showing your work in an *organized* fashion makes it easier to get partial credit.

(Bonus) What was the most interesting topic we covered this term, and why?

Trigonometric Identities

$$\begin{aligned}
\sin(A+B) &= \sin(A)\cos(B)+\cos(A)\sin(B)\\
\sin(A-B) &= \sin(A)\cos(B)-\cos(A)\sin(B)\\
\cos(A+B) &= \cos(A)\cos(B)-\sin(A)\sin(B)\\
\cos(A-B) &= \cos(A)\cos(B)+\sin(A)\sin(B)
\end{aligned}$$

$$\begin{aligned}
\sin(2A) &= 2\sin(A)\cos(A)\\
\cos(2A) &= \cos^2(A)-\sin^2(A)
\end{aligned}$$

$$\sin^2(A)+\cos^2(A)=1$$

$$\tan^2(A)+1=\sec^2(x)$$

$$1+\cot^2(A)=\csc^2(x)$$