FINAL EXAM

Winter Quarter 2017
MAT 16B: Short Calculus
Section 004

KLEIBER HALL 003
Tuesday, March 21, 2017
10:30AM-12:30PM

READ and SIGN
the back of this page

INSTRUCTIONS

1. Read these instructions!

2. **Write your Student ID Number on EVERY page of this exam.**

3. Any exam that does not have a Student ID Number on every page will be discarded.

4. You may begin the exam as soon as you take your seat.

5. You will be asked to present your Student ID card during this exam.

6. Have your cellphones turned off or on silent mode.

7. Do NOT use any electronic aids (such as calculators or any device connected to the internet or phone service).

8. Do NOT use any notes, books, or any written or reading material.

9. This final exam packet has six (6) double-sided printed pages, including [a] this first cover and instruction page, [b] a forumlas page, and [c] four (4) exam pages printed front and back.

10. **SHOW ALL WORK.** Points will be deducted if not enough work is shown. Arithmetic errors will be ignored ONLY if all other work shown is correct.

11. Any formulas derived in class may only be used if they are stated completely. In other words, in addition to the forumla, explain the variables used in the formula and how they relate to the problem at hand.

12. Students who have completed the exam must hand in their exam and exit the exam room quickly. The exams of all remaining students will be collected at 12:30PM.

13. Department policy dictates that all completed mathematics final exams are the property of the Department. Students may look over their final exams after all the exams have been graded, but may not keep them.

14. Final exam scores and preliminary course grades will be posted on Canvas by Thursday, March 23, 12PM. The overall course grade will be submitted to the Registrar on Friday, March 24, 12PM. Students will only have those 24 hours to contact the instructor with any questions about their grade. The instructor will be in MSB 2149 on Thursday, March 23, 10-5PM and Friday, March 24, 10-12PM.

15. All graded quizzes and midterms must be collected before Friday, March 24. Grade discrepencies will only be corrected if the student can provide a physical copy of the original exam. All quizzes and midterms that have not been collected by students will be destroyed after Friday, March 24, 1PM.

16. Any student suspected of cheating will be reported to the Office of Student Support & Judicial Affairs.
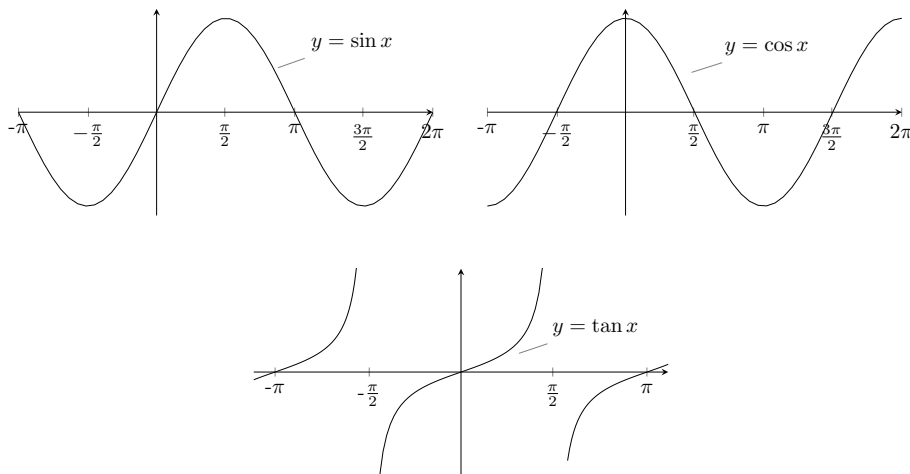

I have read and understand the above instructions and I declare that I personally completed this exam without any outside assitance, including course material, other source material, or assitance from any person(s) or device(s).

_____   _____
Signature of Student   Date

FORMULAS

$y = \sin x$

$y = \cos x$

$y = \tan x$

$$\frac{d}{dx}(\sin u) = \cos u \, \frac{du}{dx} \qquad\qquad \frac{d}{dx}(\csc u) = -\csc u \, \cot u \, \frac{du}{dx}$$

$$\frac{d}{dx}(\cos u) = -\sin u \, \frac{du}{dx} \qquad\qquad \frac{d}{dx}(\sec u) = \sec u \tan u \, \frac{du}{dx}$$

$$\frac{d}{dx}(\tan u) = \sec^2 u \, \frac{du}{dx} \qquad\qquad \frac{d}{dx}(\cot u) = -\csc^2 u \, \frac{du}{dx}$$

$$\int \sin u \, du = -\cos u + C \qquad\qquad \int \csc u \, du = -\ln|\csc u - \cot u| + C$$

$$\int \cos u \, du = \sin u + C \qquad\qquad \int \sec u \, du = \ln|\sec u + \tan u| + C$$

$$\int \tan u \, du = -\ln|\cos u| + C \qquad\qquad \int \cot u \, du = \ln|\sin u| + C$$

$$\int e^u \, du = e^u + C \qquad\qquad \int a^u \, du = \frac{1}{\ln a} a^u + C$$

$$\int u \, e^u \, du = e^u(u - 1) + C \qquad \int (u+1)^2 \, e^u \, du = e^u \left(u^2 + 1\right) + C$$

Hints:
- Don't forget $+C$ for indefinite integrals.
- Notation matters: $\int f(x)$ is incorrect. Be sure to write $\int f(x)\,dx$.

[May be used for scratch work.]

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1. **Exponential and Logarithmic Functions**

    (a) Find the limit. Show ALL work. (Hint: To use L'Hôpital's Rule, be sure to verify "something" first.)

    $$\lim_{x \to 1} \frac{\ln x}{1 - x^2}$$

    / 5 pts

    (b) Find the exponential function $y = C\,e^{k\,t}$ that passes through the points $\left(0, \frac{1}{2}\right)$ and $\left(\frac{1}{5}, \frac{e}{2}\right)$. Show ALL work.

    / 5 pts

2. **Trigonometric Functions**

(a) Use u-substitution to find the indefinite integral. Show ALL work.
(Hint: Remember to use absolute values where appropriate.)

$$\int \frac{e^{-x} \sin e^{-x}}{\cos e^{-x}} \, dx$$

/ 10 pts

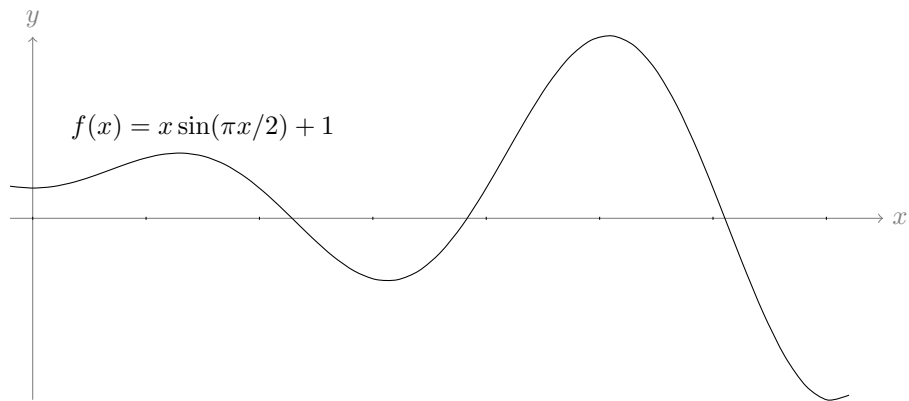# FINAL

3. **Partial Fractions**

Find the indefinite integral. Show ALL work.

(Hint: 10pts to find the partial fraction, 5 pts to compute the integral.
Remember to use absolute values where appropriate.)

$$\int \frac{x+8}{x^3 - 4x^2 + 4x}\, dx$$

/ 15 pts

4. **Numerical Integration**

$f(x) = x\sin(\pi x/2) + 1$

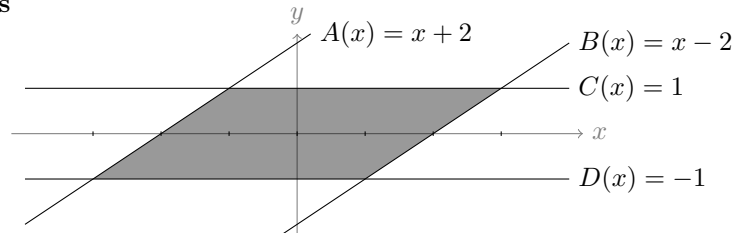| x	0	1	2	3	4	5	6	7
$f(x) = x\sin(\pi x/2) + 1$	1	2	1	-2	1	6	1	-6

Use Simpson's Rule with $n = 4$ to find the approximate value of the definite integral. Show ALL work. (Hint: Pay attention to the limits!)
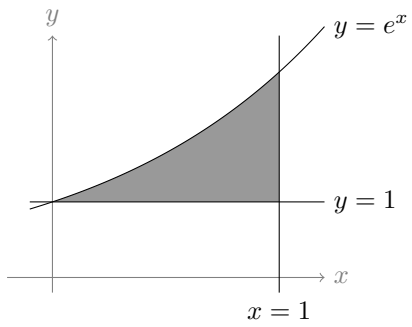
$$\int_1^5 f(x)\,dx$$

/ 10 pts

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5. **Areas and Volumes**

$A(x) = x + 2$ $B(x) = x - 2$

$C(x) = 1$

$D(x) = -1$

(a) Find the definite integral which represents the area of the shaded region. Do not evaluate the integral. Show ALL work.

(Hint: 4pts for finding all 4 vertices of the parallelogram, 6pts for finding the correct definite integral.)

/ 10 pts

(b) Find the definite integral which represents the volume of the solid obtained by rotating the shaded region about the y-axis. Do NOT evaluate the integral. Show ALL work.

/ 10 pts

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6. **Discrete Random Variables**

The following table is the probability distribution for a discrete random variable X.

x	0	1	2	3	4	5
$P(X = x)$	$\frac{1}{20}$	$\frac{3}{20}$	$\frac{2}{20}$	$\frac{5}{20}$	$\frac{6}{20}$	$\frac{3}{20}$

(a) Determine the probability that X takes on a value greater than or equal to 3. Show ALL work.

/ 5 pts

(b) Find the center, or expected value $\mu = \mathbb{E}[X]$, of the data set. You do not need to simplify. Show ALL work.

/ 10 pts

(c) Assuming that the expected value is μ, find the variance of X. Show ALL work.

/ 10 pts

7. **Continuous Random Variables**

(a) Find the expected value of a continuous random variable X with probability density function $f(x) = e^{-x}$ over the interval $[0, \infty)$. Show ALL work.

(Hint: You may find a useful formula on the "Formulas" sheet. Try using $u = -x$.
     To use L'Hôpital's Rule, be sure to verify "something" first.)

/ 10 pts

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(b) Find the standard deviation of a continuous random variable X with probability density function $f(x) = e^{-x}$ over the interval $[0, \infty)$. Show ALL work.

/ *5 pts

Enjoy your spring break!

[May be used for scratch work.]