MAT 16B Prof. Fu Liu Final June 9, 2008

Your PRINTED name is: _____

student ID number is: _____

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Instructions:

1. Do NOT open your exam until you are told to.
2. You have 120 minutes for this exam.
3. Before starting the exam, PRINT your name and student ID in the space above.
4. NO notes or books are allowed.
5. Calculators may NOT be used for this exam. Do not replace precise quantities such as $\sqrt{3}, e^2, \ln 2$, with decimal approximations.
6. Show all your work clearly on the pages provided.
7. Please raise your hand if you have any questions during the exam.

1 (20 pts.) Determine whether each statement is true (T) or false (F). Then CIRCLE the appropriate answer. You do not need to show work. Assume that x and y are positive numbers.

(a) $\ln(x-y) = \ln x - \ln y$ T F

(b) $e^x e^y = e^{x+y}$ T F

(c) $\int \frac{1}{t}\, dt = \ln t + C$ T F

(d) The integral $\int_0^1 \frac{1}{t^2}\, dt$ diverges. T F

(e) The integral $\int_{-\infty}^0 t^2 e^{-t^3}\, dt$ diverges. T F

2 (12 pts.) Let P be the amount of money initially deposited, $A(t)$ the balance in the account after t years, and r the nominal interest rate (in decimal form). Fill in the following table for bank interests. You do not need to show the work.

	Compounded n times per year	Compounded continuously
$A(t)$		
The effective rate		
When the balance will be doubled		

3 (20 pts.) Let $f(x) = \frac{1}{\sqrt{4\pi}} e^{-(\frac{x}{2}-3)^2}$ over $(-\infty, \infty)$.

(a) What type of probability density distribution is $f(x)$? Find the mean, variance, standard deviation, and median WITHOUT integration.

(b) Determine the extreme points and points of inflection of the graph of $f(x)$. If there are extreme points, determine whether they are maxima or minima.

4 (10 pts.) The cumulative sales S (in thousands of units) of a new product after it has been on market for t years are modeled by

$$S = 10(1 - 2^{kt}).$$

During the first year, 2000 units were sold.

(a) Solve for k in the model.

(b) What is the saturation point for this product? (Recall the saturation point is the limit of S as t approaches ∞.)

(c) How many units will be sold after 3 years?

5 (8 pts.) Find a function f that satisfies the conditions.

$$f''(x) = x^2 + x - 1, f'(0) = 5, f(0) = 10.$$

6 (8 pts.) Set up the integral (but DO NOT EVALUATE) for finding the area of the region bounded by the graphs of $y = x^4 - 2x^3 + 5x^2 - 10x$ and $y = x^3 + 15x^2 - 10x$.

7 (12 pts.) Let R be the region bounded by the graphs of $y = -2x + 1, x = 0$ and $y = 5$. (For both parts, set up but do NOT evaluate the integral(s).)

(a) Find the volume of the solid formed by revolving R about the x-axis.

(b) Find the volume of the solid formed by revolving R about the y-axis.

8 (42 pts.) Evaluate the following indefinite integrals. Simplify your answers as much as possible. Show your work.

(a) $\int \frac{1-e^{-x}}{1+e^{-x}}\, dx$

(b) $\int \csc^4 x\, dx$

(c) $\int \frac{2x^2+3x+1}{x^3+4x^2+4x}\, dx$

(d) $\int (3x^2+1)\tan(x^3+x)\ dx$

(e) $\int x \sin x\ dx$

(f) $\int \frac{\ln x}{x}\ dx$

(g) $\int \frac{x^2+1}{\sqrt{x/2-1}}\ dx$

9 (21 pts.) Evaluate the following definite integrals or improper integrals. (If it is an improper integral, determine whether it converges or not. If it does, then evaluate it.) Simplify your answers as much as possible. Show your work.

(a) $\int_0^2 \frac{1}{(x-1)^{4/3}}\, dx$

(b) $\int_1^e x^3 \ln x\, dx$

(c) $\int_1^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}\, dx$

10 (10 pts.) For some number b, $f(x) = 6x(1-x)$ is a probability density function over $[0, b]$.

(a) What is the value of b?

(b) Set up the integrals or formulas to find the mean, variance and standard deviation of the random variable. (Do NOT evaluate.)

11 (10 pts.) The chance of raining in the morning is 20%, and the chance of raining in the afternoon is 30%. Assume that whether it rains in the morning and whether it rains in the afternoon are independent.

(a) What is the probability of raining both in the morning and in the afternoon?

(b) What is the probability of not raining at all today?

12 (17 pts.) Consider the definite integral $\int_0^{\pi} \sin^2 x \, dx$.

(a) Calculate the definite integral directly.

(b) How large should n be in order to guarantee that the absolute error in the Simpson's Rule estimate of the definite integral is at most $\frac{\pi}{360}$? (Recall: $|E| \leq \frac{(b-a)^5}{180n^4} \left[\max_{a \leq x \leq b} |f^{(4)}(x)|\right]$ and $\pi \approx 3.14$.)

13 (10 pts.) Set up the formulas but do NOT evaluate to estimate the definite integral

$$\int_2^4 \sqrt{x^4 - 2}\, dx$$

by using midpoint rule and Trapezoidal rule with $n = 6$.