Q1__________ **Last Initial** ___________________________________

Q2__________ **Student ID** ___________________________________

Q3__________ **FULL Name** ___________________________________

Q4__________

Q5__________

Q6__________

Σ __________

FINAL EXAMINATION

21A §A01-07, 8:00-10:00 am

Wednesday December 12, 2018

Declaration of honesty: I, the undersigned, do hereby swear to uphold the very highest standards of academic honesty, including, but not limited to, submitting work that is original, my own and **unaided by** notes, peeking at the person next to me whose answer is probably wrong anyway, books, calculators, mobile phones, blackberries, apples, artificial intelligence or any other electronic device.

Well-organized and explained responses will receive more credit.

Signature ____________________ **Date** _______________

Q1 scratch/extra space (do not erase your scratch computations, they might earn partial credit):

Question 1
Define what the symbols

$$\lim_{x \to a^+} f(x) = L$$

mean (include a picture in your answer). Use your definition to *prove* that

$$\lim_{x \to a^+} x = a\,.$$

Q2 scratch/extra space (do not erase your scratch computations, they might earn partial credit):

Question 2 Calculate city! *Compute* the following quantities *(show your work and avoid l'Hôpital's rule)*:

(i) $\lim_{\theta \to 0} \dfrac{\cos(\theta) - 1}{\theta}$

(ii) $d(\arcsin(\cos(x)))$

(iii) $\lim_{x \to 0} \dfrac{\sqrt{1+x} - 1}{x}$

(iv) $d\left(x^{x^x}\right)$

Q3 scratch/extra space (do not erase your scratch computations, they might earn partial credit):

Question 3

The product rule says $d(fg) = fdg + gdf$, and the relative change of a quantity Q is defined to be dQ/Q. Employ this information to *show* that the relative change in the ratio P/Q of two quantities P and Q is the difference $dP/P - dQ/Q$ of their relative changes. Now use this fact to *estimate* the ratio

$$\frac{10{,}000{,}000{,}013}{10{,}000{,}000{,}012}.$$

Explain why most calculators give the answer 1 for the above quantity.

Q4 scratch/extra space (do not erase your scratch computations, they might earn partial credit):

Question 4

Give a pictorial/graphical explanation of the Newton–Raphson method then use this method[1] to *approximate* $\sqrt{2}$.

[1] Accurate results will receive more credit, but two recursions can already give an excellent result.

Q5 scratch/extra space (do not erase your scratch computations, they might earn partial credit):

Question 5

Analyze the critical points of the function $f : \mathbb{R} \to \mathbb{R}$ where

$$f(x) = e^x - x\,.$$

What is the *range* of f? *Sketch* the graph of this function. Make sure your sketch indicates interesting features such as the concavity of the graph, extrema, roots and asymptotes (possibly oblique).

Q6 scratch/extra space (do not erase your scratch computations, they might earn partial credit):

Question 6 A ladder leans against the side of a building. The base of the ladder sits in an oil slick which causes the ladder to slide down the wall. Find a formula relating the rate of change of the angle formed by the ladder and the ground in terms of the rate of change of the distance of the base of the ladder from the wall. (*Hint:* First draw a picture. Introduce symbols labelling any quantities that you think are relevant for the problem.)