Q1_____ **Last Initial** _____

Q2_____ **Student ID** _____

Q3_____ **FULL Name** _____

Q4_____

Q5_____

Q6_____

$\Sigma$ _____

# FINAL EXAMINATION

## 21A §E01-07, 3:30-5:30 pm

## Monday December 11, 2017

**Declaration of honesty:** I, the undersigned, do hereby swear to uphold the very highest standards of academic honesty, including, but not limited to, submitting work that is original, my own and **unaided by** notes, peeking at the person next to me whose answer is probably wrong anyway, books, calculators, mobile phones, blackberries, blueberries, boysenberries, raspberries, artificial intelligence or any other electronic device. Volcanic-emotional-support-pet rocks without tattoos permitted.

Well-organized and explained responses will receive more credit.

**Signature** _____ **Date** _____

*Q1 scratch/extra space (do not erase your scratch computations, they might earn partial credit):*

**Question 1**

*Define* what the symbols

$$\lim_{x \to a^+} f(x) = \infty$$

mean (include a picture in your answer). Use your definition to *prove* that

$$\lim_{x \to 0^+} \frac{1}{x} = \infty \, .$$

*Q2 scratch/extra space (do not erase your scratch computations, they might earn partial credit):*

**Question 2** Calculate city! *Compute* the following quantities *(do show your work)*:

(i) $\lim_{x\to\infty} \dfrac{x^2+2x+2}{x^2+2x+1}$

(ii) $d(x^x)$

(iii) $\lim_{x\to 0} \dfrac{\sqrt{1+x}-1}{x}$

(iv) $\dfrac{d\big(x(x+1)(x+2)(x+3)(x+4)(x+5)(x+6)(x+7)(x+8)\big)}{dx}$

*Q3 scratch/extra space (do not erase your scratch computations, they might earn partial credit):*

**Question 3**

A 4’ tall turkey stands 2’ away from the base of a lamppost at night and casts a 1’ shadow. The turkey then saunters[1] away from the lamppost at 1’/second. At what rate does the turkey’s shadow lengthen?

[1] This means to walk in a relaxed fashion!

*Q4 scratch/extra space (do not erase your scratch computations, they might earn partial credit):*

**Question 4**

Use the Newton–Raphson method to compute an approximation[2] for $\sqrt[3]{3}$.

[2]Note: $144^3 = 2985984$ & $145^3 = 3048625$; a well-explained result accurate to 2 decimal places will earn full credit for this problem.

*Q5 scratch/extra space (do not erase your scratch computations, they might earn partial credit):*

**Question 5**

Let $n$ be a positive integer, and $x$ and $y$ be positive numbers that obey

$$x + y = 1 \, .$$

*Find*[3] the values of $x$ and $y$ that minimize $x^n + y^n$.

[3] You might be able to guess the answer using a symmetry argument. To earn full credit, use calculus to show that your guess is correct.

*Q6 scratch/extra space (do not erase your scratch computations, they might earn partial credit):*

**Question 6**

Let $f : \mathbb{R} \to \mathbb{R}$ where

$$f(x) = \frac{1}{1+x^2}\,.$$

Sketch the curve $y = f(x)$. The better your sketch, the more credit you will earn.