Math 21B
Final Exam

Printed Name _____
(FIRST) (LAST)

Signature _____

ID Number _____

Please Show All Your Work, and Mark Your Answers Clearly.
No Calculators -- No Scratch Paper -- No Cell Phones

There are **8 pages** of problems. (The last 2 problems are for extra credit.)

You are expected to do your own work, and to adhere to the UCD Code of Academic Conduct.

Simplify all numerical answers.

Simplify your answers to problems involving trig substitution.

In **#6 and #11**, you just have to **set up** the integrals; you do **not** have to evaluate them.

Please indicate clearly if you continue work on the back of a page.

Please stop working **immediately** when time is called.

Have a Good Summer!

① EVALUATE THE FOLLOWING INTEGRALS:

A) $\int_0^3 x\sqrt{9-x^2}\,dx$

9 PTS

B) $\int_4^{12} \frac{60}{\sqrt{x}\,(x+4)}\,dx$

9 PTS

② THE SPEED OF A PARTICLE MOVING ALONG A LINE AFTER T SECONDS IS GIVEN BY $f(\tau) = \frac{20\tau}{(\tau^2+4)^2}$ CM/SEC. FIND ITS AVERAGE SPEED FOR THE FIRST 4 SECONDS.

9 PTS

③ FIND THE FOLLOWING INTEGRALS:

A) $\int x^2 \cos 5x \, dx$

9 PTS

B) $\int \frac{\sqrt{16 - x^2}}{x} \, dx$

13 PTS

④ FIND THE SLOPE OF THE TANGENT LINE TO THE CURVE $x = t^2 - 5t + 7$, $y = t^3 - 3t^2 - 14$ AT THE POINT $(3, 2)$.

8 PTS

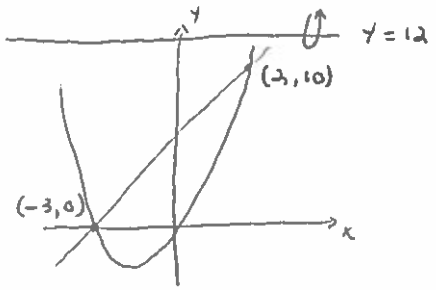
⑤ FIND $\int \frac{4x^4 - 3x^3 + 2x - 16}{x^3 - 2x^2}\, dx$.

12 PTS

⑥ LET R BE THE REGION BOUNDED BY THE GRAPHS OF $y = x^2 + 3x$ AND $y = 2x + 6$.

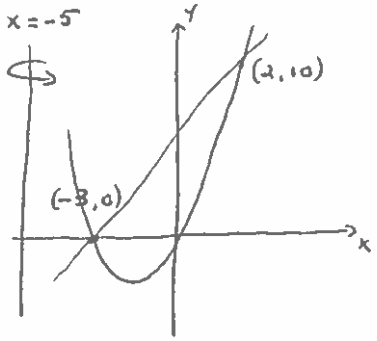
SET UP AN INTEGRAL FOR THE VOLUME OF THE SOLID GENERATED BY REVOLVING R

A) AROUND THE LINE $y = 12$.



8 PTS

B) AROUND THE LINE $x = -5$.



8 PTS

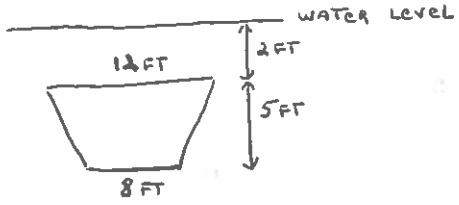
(9) FIND $\int \frac{5x+7}{x^2-6x+13}\,dx$.

9 PTS

(10) FIND THE AREA OF THE SURFACE GENERATED BY REVOLVING THE CURVE $x = 2t$, $y = t^2 + 3$, $0 \le t \le \sqrt{15}$, ABOUT THE Y-AXIS.

10 PTS

(11) SET UP AN INTEGRAL FOR THE FORCE EXERTED BY WATER ON ONE SIDE OF THE ISOSCELES TRAPEZOID SHOWN: (THE WEIGHT-DENSITY OF WATER IS 62.4 LB/FT^3.)



8 PTS

(12) Evaluate $\int_0^{1/2} \frac{6}{x(\ln x)^2}\,dx$, or show that it diverges.

9 pts

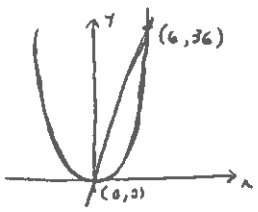
(13) Use Pappus's Theorem to find the volume of the solid obtained by revolving the region inside the circle $(x-3)^2+(y+8)^2=36$ about the line $4x+3y=38$.

8 pts

(14) Find $\int \frac{\tan^{-1} x}{(x+2)^2}\,dx$.

13 pts

(15) FIND THE CENTROID OF THE REGION BOUNDED BY THE CURVES $Y = X^2$ AND $Y = 6X$, GIVEN THAT ITS AREA $A = \int_0^6 (6x - x^2)\,dx = 36$.



12 PTS

(16) FIND THE LENGTH OF THE CURVE $Y = \ln X$, $1 \le X \le \sqrt{35}$.

14 PTS

(17) FIND $\lim_{n\to\infty}\left[\frac{1}{n+1}+\frac{1}{n+3}+\frac{1}{n+5}+\cdots+\frac{1}{3n-1}\right]$, AND JUSTIFY YOUR ANSWER.

9 PTS (EXTRA CREDIT)

(18) FIND THE VOLUME OF THE SOLID GENERATED BY REVOLVING THE REGION UNDER THE CURVE $x = 5 - e^{2t}$, $y = \sin t$, $0 \le t \le \pi$, ABOUT THE LINE $x = 5$.

14 PTS (EXTRA CREDIT)