MATH 21B, Sections D01 and D02
Final exam

December 7, 2016

Name:_____

ID:_____

Section:_____

DO NOT OPEN THIS EXAM YET

(1) Fill in your name, ID and section number (D01 or D02).

(2) This exam is closed-book and closed-notes; no calculators, no phones.

(3) Please write legibly to receive credit. Circle or box your final answers. If your solution to a problem does not fit on the page on which the problem is stated, please indicate on that page where in the exam to find (the rest of) your solution.

(4) You may continue your solutions on additional sheets of paper provided by the proctor. If you do so, please write your name and UNI at the top of each of them and staple them to the back of the exam (stapler available); otherwise, these sheets may get lost.

(5) Anything handed in will be graded; incorrect statements will be penalized even if they are in addition to complete and correct solutions. If you do not want something graded, please erase it or cross it out.

(6) Show your work; correct answers only will receive only partial credit (unless noted otherwise).

(7) Be careful to avoid making grievous errors that are subject to heavy penalties.

(8) If you need more blank paper, ask a proctor.

Out of fairness to others, please stop working and close the exam as soon as the time is called. A significant number of points will be taken off your exam score if you continue working after the time is called. You will be given a two-minute warning before the end.

1	2	3	4	5	6	7	8	Total

1. (20 points) Compute the following indefinite integrals:

a) $\int \sin(2x + 3)dx$

b) $\int \frac{xdx}{\sqrt{x^2+1}}$

c) $\int \frac{dx}{(x+1)(x+2)(x+3)}$

d) $\int \frac{\sqrt{1-x^2}}{x^2}dx$

2. (20 points) Compute the following definite integrals:

a) $\int_0^1 \sqrt{x} \cdot (x+1)dx$

b) $\int_1^2 \ln x \, dx$

c) $\int_0^\pi \sin^3 t\, dt$

d) $\int_0^1 \frac{2x+1}{x^2+1} dx$

3. (10 points) Solve the differential equation $y' = \frac{1}{xy}$.

4. (10 points) Determine if the improper integral $\int_0^{+\infty} \frac{dx}{x^2+4x+5}$ converges or diverges. If it converges, compute its value.

5. (20 points) Consider the region R bounded by the curves

$$y = e^x, y = 0, x = 0 \text{ and } x = 1.$$

a) Find the area of R

b) Find the volume of the solid of revolution obtained by rotation of R about the x-axis

c) Find the volume of the solid of revolution obtained by rotation of R about the y-axis

d) Find the coordinates of the center of mass of R.

6. (10 points) Compute the length of the curve defined by the equations:

$$x(t) = \cos t + \sin t, \ \ y(t) = \cos t - \sin t, \ \ 0 \le t \le 2\pi.$$

7. (10 points) A fish tank has the shape of parallelepiped (all faces are rectangles) with height a, width b and length c. It is full of water. Compute the total force of water pressure on each of its faces (4 sides, top and bottom), assuming that the water density equals ρ and the gravity acceleration equals g.

This is a bonus problem. Please start this problem only if you completed the rest of the exam.

8*. (10 points) Find a function $f(x)$ such that

$$\int f^2(x)dx = \left(\int f(x)dx \right)^2.$$

Hint: define $F(x) = \int f(x)dx$ and write a differential equation for $F(x)$.