MAT 128 B: On-Line Final Exam : 03/18/2020

Please read first: (i) To get full credit show all your work. (ii) No outside help or internet searches. (iii) You can use your class notes and book. (iv) You have until 10:15 am to upload the test

Name:

1 (20 points) A variant of the secant method defines two sequences u_k and v_k such that $f(u_k)$ has one sign and $f(v_k)$ has the opposite sign. From these sequences and the secant method one can derive the expression

$$w_k = \frac{u_k f(v_k) - v_k f(u_k)}{f(v_k) - f(u_k)}, k = 1, 2, 3, ... \tag{1}$$

We define $u_{k+1} = w_k$ and $v_{k+1} = v_k$ if $f(w_k)f(u_k) > 0$ and $u_{k+1} = u_k$ and $v_{k+1} = w_k$ otherwise. Suppose that f'' is continuous on the interval $[u_0, v_0]$ and that for some K, f'' has a constant sign in $[u_K, v_K]$. Explain why either $u_k = u_K$ for all $k \geq K$ or $v_k = v_K$ for all $k \geq K$. Deduce that the methods converges linearly.

2. (30 points) Suppose m linear systems $Ax(p) = b(p)$, $p = 1, 2, ..., m$, are to be solved, each with the $n \times n$ coefficient matrix A.

1. Construct an algorithm using Gaussian elimination with backward substitution ,
2. Show that the required multiplications/divisions are $\frac{1}{3}n^3 + mn^2 - \frac{1}{3}n$
3. Show that the required additions/subtractions are $\frac{1}{3}n^3 + mn^2 - \frac{1}{2}n^2 - mn + \frac{1}{6}n$.

3. (40 points) The Frobenius norm of a square matrix is defined as $||A||_F = (\sum_{i=1}^n \sum_{j=1}^n a_{i,j}^2)^{\frac{1}{2}}$

1. (10 points) Show that the Frobenius norm is a matrix norm
2. (10 points) Show that if A is a symmetric matrix, then $||A||_F^2 = trace(A^2)$, where the trace is the sum of the elements in the diagonal.
3. (10 points) Show that $||A||_2 \leq ||A||_F \leq n^{\frac{1}{2}}||A||_2$
4. (10 points) Show that $||Ax||_2 \leq ||A||_F||x||_2$

4. (10 points) Show that if B is singular ($det(B) = 0$), then $\frac{1}{K(A)} \leq \frac{||A-B||}{||A||}$

[Hint: There is a vector with $||x|| = 1$, such that $Bx = 0$. Derive the estimate using $||Ax|| \geq \frac{||x||}{||A^{-1}||}$.