MAT 16A

Final

March 23, 2023

1 sheet of paper is allowed. No calculators. All incidents of cheating or the appearance thereof will be brought to the attention of Student Judicial Affairs.

Name:
ID:

There are two sections: Section A contains 15 problems worth 25 points.
Section B contains 11 questions in total. Attempt any 8 problems. The rest of the 3 questions, if attempted, will be counted as extra credits.

a) Section A _____

b) Section B

1 _____
2 _____
3 _____
4 _____
5 _____
6 _____
7 _____
8 _____
9 _____
10 _____
11 _____

Total (out of 105): _____ Total (out of 35): _____

Section A

Multiple Choice Questions$[10 \times 1 = 10]$

[Circle the letter of choice]
eg:

b

1) Relative Maxima of a function $f(x)$ could be found by

   a) Finding solutions to $f'(x) = 0$ (say at $x = c$) and checking $f''(c)$ exists and $f(c) < 0$

   b) Finding solutions to $f'(x) = 0$ (say at $x = c$) and checking $f''(c) < 0$

   c) Finding solutions to $f'(x) = 0$ (say at $x = c$) and checking $f''(c) > 0$

   d) Finding solutions to $f'(x) = 0$ (say at $x = c$) and checking $f(c)$ exists.

2) For a polynomial function $y = p(x)$,

$$\lim_{x \to 3} p(x) =$$

   a) $p(3)$

   b) May or may not exist.

   c) 0

   d) $\frac{0}{0}$ form-indeterminate form.

3) Which functions are differentiable only on their domain in $\mathbb{R}$

a) Rational functions

b) Polynomial functions

c) $f(x) = \sqrt{x}$

d) Polynomial and rational functions

4) We say

$$\lim_{x \to a} f(x)$$

is unbounded if

a) a is not in the domain of $f(x)$

b) Left hand limit of $f(x)$ or the Right hand limit of $f(x)$ approaches $\pm$ infinity as x approaches a.

c) $y = a$ is a horizontal asymptote.

d) it's an indeterminate form $\left(\frac{0}{0} \text{ or } \frac{\infty}{\infty}\right)$

5)

$$\lim_{x \to -\infty} \frac{(2x-1)^2}{3x^2+7} = ? \qquad [x \text{ approaches negative infinity.}]$$

a) 0

b) 4/3

c) 2//3

d) Does not exist.

6) True or False:

$$x^2 \sin x + 3 \cos x + 2$$

is a polynomial function.

a) True

b) False

7)

$$\lim_{x \to 3} f(x) = -\infty$$

then $x = 3$ is a:

a) Vertical Asymptote

b) Horizontal Asymptote

8) Let

$$f(x) = \frac{(x^3 - 1) + (x^2(2x^2 - 2))}{x^3 + 3x^4 - 1}$$

be a rational function then we can say the horizontal asymptote of $f(x)$ is

a) $2/3$

b) 1

9) $\tan(4\pi/3) =$

a) undefined

b) 0

10) If $y = \cot(x)$ then $\frac{dy}{dx} =$

a) $-\csc x \cot x$

b) $-\csc^2 x$

Short answers $[5 \times 3 = 15]$

a) Show that the limit

$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$$

exists.

b) Write the quotient rule of derivatives. Give an example of a function $f(x)$ which uses the quotient rule. Take the derivative of the function $f(x)$ using the quotient rule.

c) Let $h(x) = f(g(x))$

$$g'(2) = 3; g(2) = 1 \qquad \text{and} \qquad f'(1) = 2,$$

then find the value of

$$h'(2) = ?$$

d) Give the definition of $f(x)$ increasing in an interval I.

e) Give an example of a function $y = f(x)$ which is increasing on $\mathbb{R}$.

Section B

Exercise 1(10 points)

Evaluate the following limits:

a)(4 points)

$$\lim_{x \to 3} \frac{\frac{1}{x} - \frac{1}{3}}{x - 3}$$

b)(3 points)

$$\lim_{x \to 0} \frac{x}{\sqrt{x+4}-2}$$

c)(3 points)

$$\lim_{x \to 0} \frac{\sin x - x \cos x}{x}$$

Exercise 2(10 points)

Differentiate the following functions:

a)

$$f(x) = \frac{\tan(x^2 + 1)}{\sqrt{1 - x}}$$

b)

$$f(x) = \sec(\sqrt{\tan(x^2) + 1})$$

Exercise 3(10 points)

a)

Find the tangent line for the following function at the given point:

$$x^2y^2 + \cos(\pi xy) = x - y; \qquad \text{at } (1, 1)$$

b)

Use the definition of derivative to find the derivative of

$$f(x) = 2x + 7.$$

Exercise 4(10 points)

Consider the function:

$$f(x) = \frac{x^3}{x^3 - 8}.$$

Find the following:

- x-intercept

- y-intercept

- Domain of f

- Find the interval where $f(x)$ is continuous.

- $$f'(x) = \frac{(-24x^2)}{(x^3-8)^2}.$$

  Find the interval where $f(x)$ is differentiable.

- Find the critical points of the function $f(x)$.

Exercise 5(10 points)

Consider the function:

$$f(x) = \frac{x^2}{x^2 - 9}.$$

Find the following (using the previous problem or otherwise):

- Find the extrema's of f (as a tuple (x,y)) and classify whether they are maxima or minima.

- Find the interval where f is increasing or decreasing.

$$f''(x) = \frac{(x^3 - 8)(96x)[(4 + x^3)]}{(x^3 - 8)^4}$$

$$\sqrt[3]{-4} \approx -1.587; \qquad f''(-1.75) \approx -304.78$$

- points of inflection (as a tuple (x,y))

- the intervals of concave upward and concave downward.

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Exercise 6(10 points)

Consider the same function as above:

$$f(x^2) = \frac{x}{x^2 - 9}.$$

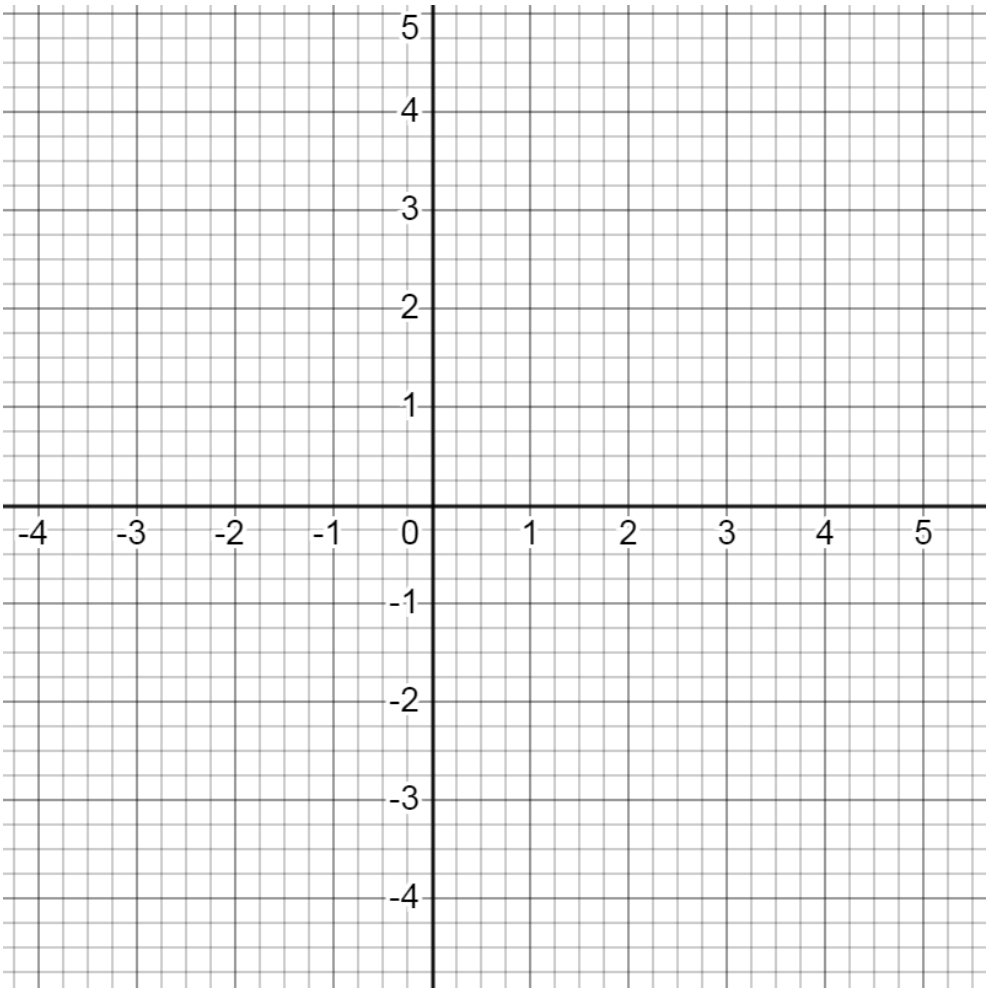
Find the following:

- Vertical and Horizontal Asymptotes

- Fill in the chart for properties in the test intervals:

Test Intervals	Properties (Inc/Dec , Concavity)

Draw the approximate graph of the function using the above information.

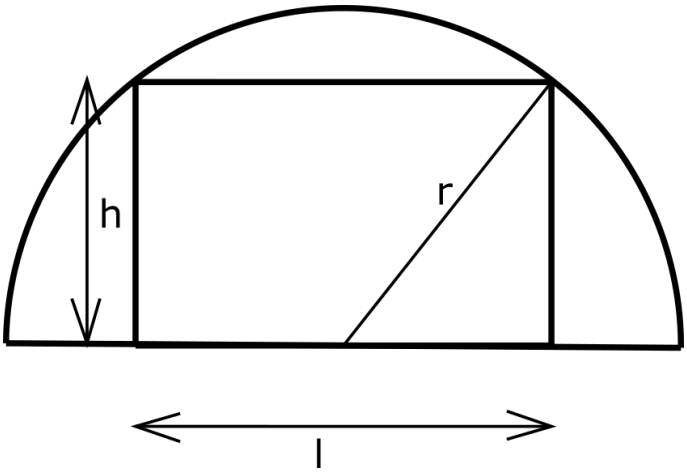


Exercise 7(10 points)

Find the dimensions of the rectangle of largest area that can be inscribed in a semicircle

$$y = \sqrt{36 - x^2}$$

if one side of the rectangle lies on the diameter and the other two vertex lies on the semicircle.



Exercise 8(10 points)

Find the points on the ellipse

$$4x^2 + y^2$$

that are farthest away from the point $(1, 0)$.

Exercise 9(10 points)

My friend Addie and Subho started from North Safeway (A)(due north of MU) and Davis Downtown(B) (due east of MU) respectively. They will meet in Memorial Union (MU). Addie is 3 miles from MU and has a speed of 12 miles per hour. Subho is 4 miles from MU and has a speed of 16 miles per hour.
At what rate is the distance between the two of them changing? Think carefully about whether this rate should be positive or negative.

Exercise 10(10 points)

a)

Approximate the value of $\sqrt{26}$ using differentials.

b)

Find the inverse of the following function.

$$f(x) = x - \frac{3x^2}{3x+2}$$

Exercise 11(10 points)

a)

What can be said about the sign of $f'(1)$, $f'(-1)$ and $f''(-1)$, $f''(1)$?

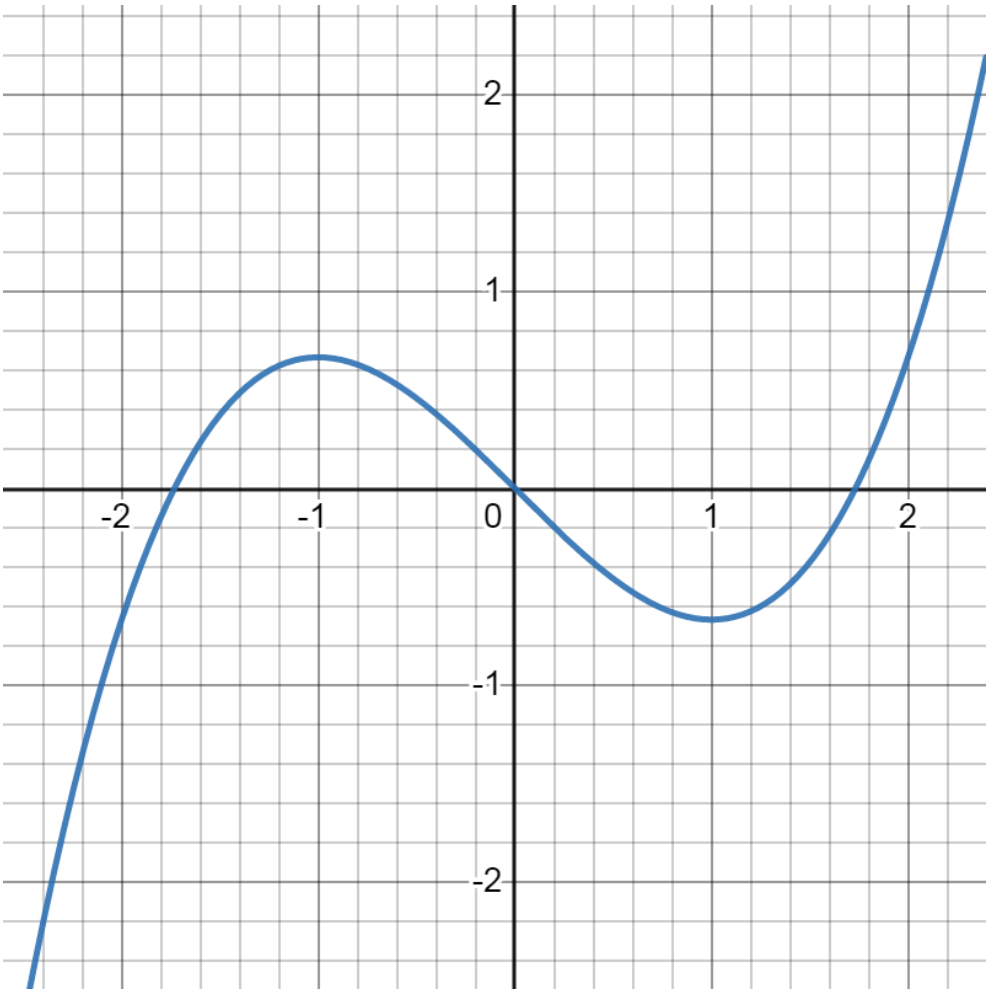
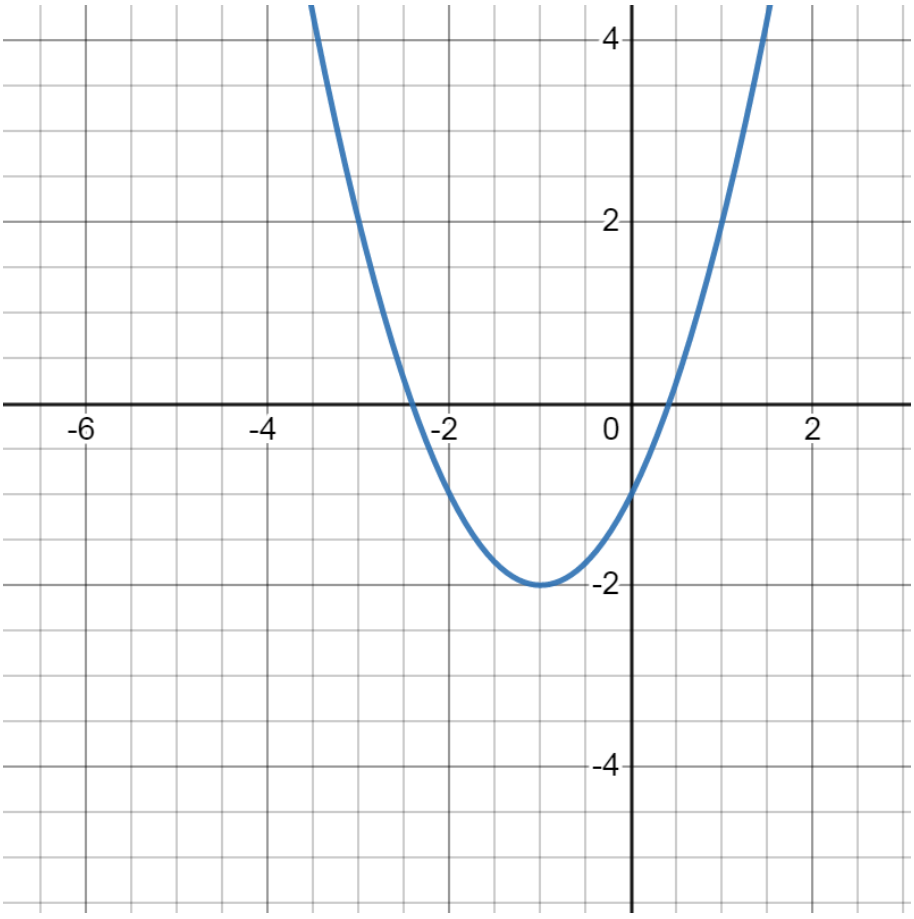
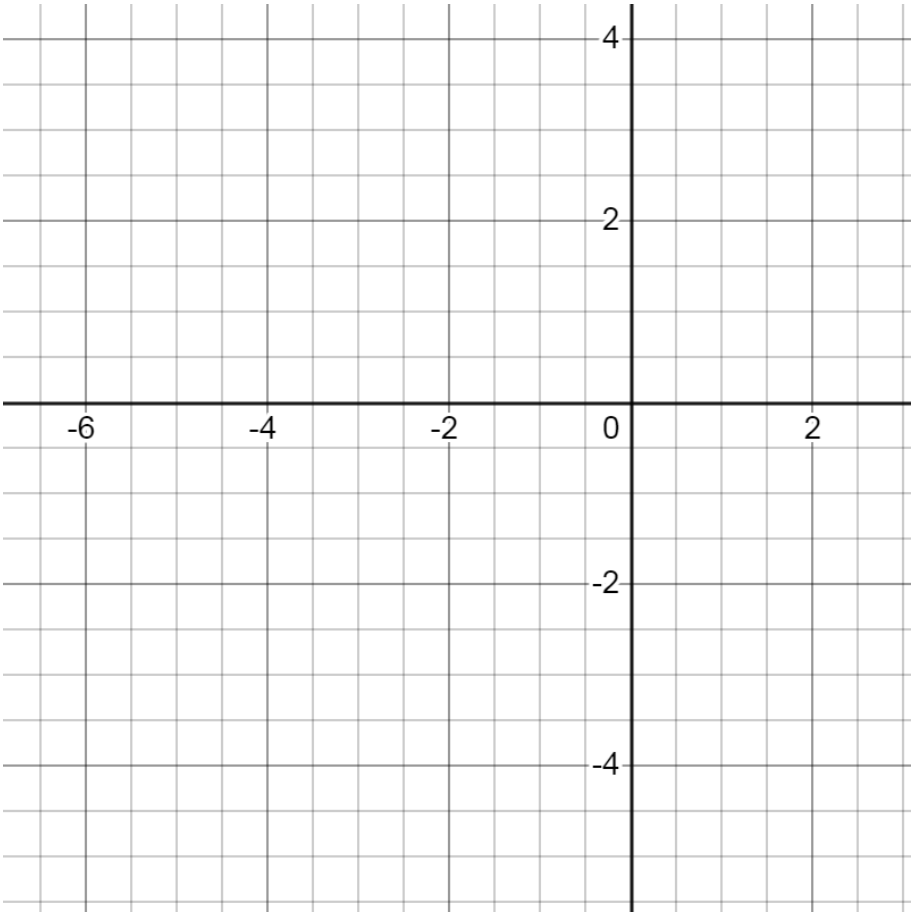


Figure 1:

b)

Try to draw the graph of $f'(x)$ for the following graph of $f(x)$.




4
2
-6
-4
-2
0
2
-2
-4