JUNE 2026 TOPOLOGY PRELIM EXAM

Name:

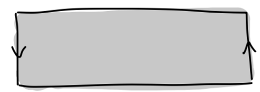
Student ID:

This exam has 6 problems. Each problem is worth 10 points. You should give full proofs or explanations of your solutions. Remember to state or cite theorems that you use in your solutions.

Please write your solutions in this exam booklet. If you need an extra page for part of your solution that you want graded, append the extra page to the end of your exam, write a note saying continued on extra page in the page within the test booklet for that problem, and clearly write the problem number and your student ID on the extra page.

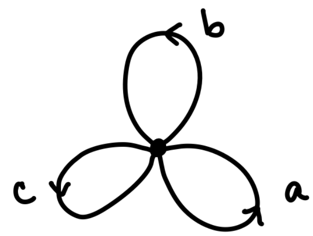
(1) Give an explicit deformation retraction from $\mathbb{R}^n - \{0\}$ to the unit sphere S^{n-1}.

(2) Let M be a Möbius band:



Let X be the space obtained by gluing three copies of M together by identifying their boundaries by the identity map. Compute $\pi_1(X)$.

(3) Let X be the wedge of three circles. Let a, b, c be paths based at the wedge point p, each traveling exactly once along one of the loops, as shown below:



Consider the subgroup $H = \langle a^2, b^2, aba^{-1}, bab^{-1}, c\rangle$ of $\pi_1(X, p)$.

Draw the covering space $\tilde{X}_H$ of X corresponding to H, clearly indicating the lifts of a, b, c. Is this covering normal? Explain why or why not.

(4) Consider two subsets $X = \{x^4 + y^4 + z^4 = 1\}$ and $Y = \{x + y + z = c\}$ in $\mathbb{R}^3$.

a) Prove that X is a smooth manifold and find its dimension.

b) Prove that Y is a smooth manifold (for any choice of c) and find its dimension.

c) Find all values of c such that X and Y intersect transversally.

(5) Consider the 1-form $\alpha = xdy - dz$ in $\mathbb{R}^3$ and the map

$$f : [0, +\infty) \times \mathbb{R}^2 \to \mathbb{R}^3,\ \ f(r, \varphi, z) = (r\cos(\varphi), r\sin(\varphi), z).$$

a) Compute the 2-form $d\alpha$ and the 3-form $\alpha \wedge d\alpha$.

b) Compute the 1-form $f^*\alpha$.

(6) Consider the vector field $v = (1 - x^2, -y)$ (equivalently, $v = (1 - x^2)\frac{\partial}{\partial x} - y\frac{\partial}{\partial y}$) on $\mathbb{R}^2$.

a) Find all zeros of v.

b) Sketch the vector field v in the square $[-3, 3] \times [-3, 3]$.

c) Find the index of v at each of its zeros.