Name:

Student ID #:

Final Exam

Math 21A, December 12, 2013

Please answer all questions in the space provided. Write neatly and clearly. Show your work. Box your answers clearly. An extra sheet of paper is stapled to the back for scratchwork and for extra room for answers. If you need the extra room, write a clear note in your solution where the question is asked and clearly identify the solution on the scratchwork.

Question	Total Possible	Score
1	15	
2	15	
3	14	
4	12	
5	14	
6	16	
7	20	
8	14	
Total	120	

1. 15 Points. Calculate the following limits. Then include the precise definition that is satisfied for the limit to be true.

a.$\lim_{x\to 2}(5x^2 - 3x)$

Definition satisfied:

b.$\lim_{x\to\infty} \frac{1}{x}$

Definition satisfied:

b.$\lim_{x\to 3^-} \frac{x}{x-3}$

Definition satisfied:

2. 15 points. Consider the function $f(x) = \frac{x^2+2x-2}{x-1}$.

a. Perform polynomial long division on $f(x)$ to rewrite $f(x)$ into a simpler form.

b. Does this function have any horizontal asymptotes? If yes, what are they? In any case, for there to an asymptote what limit is satisfied?

c. Does this function have any vertical asymptotes? If yes, what are they? In any case, for there to an asymptote what limit is satisfied?

d. Does this function have any slant asymptotes? If yes, what are they? In any case, for there to an asymptote what limit is satisfied?

3. 14 points.
a. What is the definition of the derivative of $f(x)$ at a point $x = x_0$?

b. Use the definition in part a to calculate the derivative of $f(x) = \sqrt{x}$ at $x_0 = 9$ using algebra and limit laws.

c. Find the tangent line to $f(x)$ at $x = 9$. Use this to estimate $\sqrt{9.12}$.

d. Graph $f(x)$ and the tangent line calculated part c on the same set of axes.

4. 12 points. When a person stands on a scale, their weight pushes down against a spring. The weight of a person, W, in pounds, is given by how far the scale displaces downward, x, in milimeters, by the function $W(x) = 20x^2$. Suppose your weight is exactly 180 pounds. What are the exact bounds on x that give a value of $W(x)$ between 179 and 181 pounds?

Find the Linearization of $W(x)$ around the value $x = 3$.

Find the differential of $W(x)$ for general x.

If the absolute error on the weight is 1 pound when $x = 3$, what does the differential approximate the absolute error on the displacement to be?

5. 14 points. Calculate the following derivatives. There is no need to simplify long polynomials, but clearly state the rule you are using.

a. $\frac{d}{dx}\left[(3 - x^2)(x^3 - x + 1)\right]$

b. $\frac{d}{dx}\left[\frac{3x^2-4}{2x^2-8}\right]$

c. Calculate $\frac{d}{dx}[\tan(x)]$ Don't state a memorized answer. Give the simplest form as a final solution.

d. $\frac{d}{dx}\left[e^{(-x^2)}\right]$

6. 16 Points. The population of fish in the Lake Spafford and Putah creek is a function of time $P(t)$, where P is the number of fish and t is in years. a. What is an appropriate range of populations? What does the derivative $\frac{dP}{dt}$ represent?

b. The function $P(t)$ satisfies the following differential equation:

$$\frac{dP}{dt} = P(500 - P)$$

which means that $\frac{dP}{dt}$ depends on the current population of fish. Use implicit differentiation to calculate the second derivative of P with respect to t.

c. Calculate $\frac{dP}{dt}$ and $\frac{d^2P}{dt^2}$ when the population is 400 fish. What do these quantities mean for the fish population? Sketch the approximate local graph of $P(t)$ on axes labeled P and t, given that the population is approximately 400 fish at time $t = 10$.

7. 20 Points. Let $f(x) = xe^{(-x^2)}$.

a. What are the critical points of f(x)?

b. On what intervals is $f(x)$ increasing? Where is it decreasing? Classify the critical points of $f(x)$.

c. Justify an dUse L'Hopital's Rule to calculate the limit of $f(x)$ as x goes to ∞.

d. Where is $f(x)$ concave up? Where is it concave down? Where are the points of inflection of $f(x)$?

e. Graph $f(x)$. Label critical points, points of inflection, and concavity.

8. 14 Points. a. What is the definition for a function $f(x)$ to be continuous at x_0, an interior point of its domain?

b. Show that $g(x) = x^3 + x + 1$ is continuous at any real value $x = x_0$.

c. Because $g(x)$ is continous on the interval $[-1, 1]$, what values must $g(x)$ take on that interval?

d. What does the Mean Value Theorem tell you about $g(x)$ on $[-1, 1]$