INSTRUCTIONS

(1) Write your name and ID in the indicated space above.

(2) Answer the FIRST question and ANY FOUR from the remaining questions.

(3) The numbers at the top of each question indicate the distribution of points for all parts of the question. All questions carry 10 points.

(4) You must show your work/calculation wherever it is necessary to obtain the answer.

(5) Write answers only in the space provided. If necessary use separate sheets for rough work. If extra sheets are needed please staple them with the main answer sheet before submitting.

(6) State your answers clearly. For example you can draw a box around your answer after you finish computations.

(7) Use of all electronic gadgets, e.g. mobiles, pagers, smartphones, calculators are prohibited during the exam.

(8) Discussion among students during the exam is prohibited.

(9) Use of textbooks, or class notes are strictly prohibited. This is a closed-book exam. Any student trying to use unfair means at any time of the exam may be asked to leave the exam hall and she/he may get a zero score for the exam in such a case.

(10) Put down your signature at the bottom of this page.

Student’s Signature:
Question A. \(10 \times 1 = 10\)
State whether the following are TRUE or FALSE. No explanations are required.

(1) If we know how to draw the curve (i) \(y = f(x + 1)\) then to draw the curve (ii) \(y = f(x)\) we need to translate the curve (i) through a distance 1 to the left hand side.

(2) The curve \(y = x^{\frac{1}{3}}\) is obtained by reflecting the curve \(y = x^3\) in the x-axis.

(3) The numbers 1, 2, 3 are all in the domain of the function \(\log_3(\frac{2x-1}{2x-3})\).

(4) The number 4 is the range of the function \(3e^{2x} + 5\).

(5) We have \(\ln 2 < 1\).

(6) If \(f(x) = 1 - x\) for all \(x\), then \((f \circ f)(x) = 1 - 2x\).

(7) There is a function \(g(x)\) such that \(g(5x) = g(x)^5\) for all real number \(x\).

(8) Suppose \(a > 1\); then for all numbers \(x\) one has \(a^{2\log_a x} = 2x\).

(9) For any positive number ‘\(a\)’ we have the conclusion: \(a^x = a^y \Rightarrow x = y\).

(10) The number \(1/e\) is rational.
Question B. \((2 + 2 + 3 + 3)\)
Suppose \(f(x) = x^3(x - 1)^2\) for all number \(x\).

(1) What can you say about the domain and range of \(f(x)\)?

(2) Find the \(x\)-intercepts and the \(y\)-intercepts of the curve \(y = f(x)\).

(3) Determine the sign of \(f(x)\) on each of the following intervals: \((−∞, 0), (1, ∞), (0, 1)\).

(4) Draw a rough sketch of the graph of \(f(x)\) using (2), (3).
   Indicate the nature of the curve near the \(x\)-intercepts as properly as possible.
Question C. (2 + 3 + 2 + 3)
Consider the function \( g(x) = \frac{x}{(x+1)(x-2)} \).

(1) Find the \( x \)-intercepts and \( y \)-intercepts of the curve \( y = g(x) \).
(2) Which lines are asymptotes of the the curve \( y = g(x) \).
(3) Determine the sign of \( g(x) \) on the following intervals:
\( (-\infty, -1), (-1, 0), (0, 2), (2, \infty) \).
(4) Draw a rough sketch of the graph of the function \( g(x) \)
using (1), (2), (3).
Question D. (4 + 6)

(1) A rectangle is inscribed in a circle of radius 6. Express the area of the rectangle in terms of the width $x$ (say).

(2) Find the point of the line $x - 2y - 5 = 0$ closest to the origin.
Question E. (4 + 3 + 3)
Solve the following equations and inequalities below. (You must show all the intermediate steps through which you arrive at the solutions.)

(1) $\log_2(x - 1) + \log_2(x + 2) = 2$
(2) $2^{3x} = 3^{2x}$
(3) $\ln x < \ln(2 - x)$
Question F. (3 + 2 + 1 + 4)
Consider the function $G(x) = -\ln(x - 2)$.

(1) What can you say about the domain and the range of the function $G(x)$? Show your work.
(2) Where does the graph of $G(x)$ intersect the $x$-axis and the $y$-axis?
(3) Which lines are asymptotes of the curve $y = G(x)$.
(4) Draw a rough sketch of the graph of the function $G(x)$. 
Question G. \((3 + 2 + 1 + 4)\)
Consider the function \(F(x) = 1 - 2^x\).

(1) What can you say about the domain and the range of the function \(F(x)\)? Show your work.

(2) Where does the graph of \(F(x)\) intersect the \(x\)-axis and the \(y\)-axis?

(3) Which lines are asymptotes of the curve \(y = F(x)\).

(4) Draw a rough sketch of the graph of the function \(F(x)\)?