Preliminary Exam in Algebra

Spring, 2016

Instructions:

(1) All problems are worth 10 points. Explain your answers clearly. Unclear answers will not receive credit. State results and theorems you are using.
(2) Use separate sheets for the solution of each problem.

Problem 1. Let $\mathbf{k}$ be a field and let $R = Mat_{n,n}(\mathbf{k})$ be the ring of $n \times n$ matrices with entries from $\mathbf{k}$. Let $f : R \to S$ be any ring homomorphism. Show that f is either injective or zero.

Problem 2. Let R be a ring with the identity, consisting of p^2 elements. Show that R is commutative.

Problem 3. Let G be a group generated by elements a, b each of which has order 2. Prove that G contains a subgroup of index 2.

Problem 4. Prove that every finite group G of order > 2 has a nontrivial automorphism.

Problem 5. Find all possible Jordan canonical forms for a matrix $A = T((123))$ if T is a two dimensional complex linear representation of the symmetric group S_3.

Problem 6. Find the smallest nonnegative integer $C \geq 0$ for which $R_c = Z[x]/(c, x^2 - 2)$ is

(a) a domain

(b) a field.