1. How many lines through the origin are there in the $\mathbb{F}_5$-vector space $\mathbb{F}_5^3$? How many lines are there that may or may not pass through the origin? (The latter are defined as cosets of lines through the origin.)

2. Let $V$ be the quotient vector space $\mathbb{R}[[a,b,c,d]]/(a+2b+3c+4d)$. Find a basis for $V$, find a non-zero dual vector $\phi : V \to \mathbb{R}$, and find its values on the vectors $a, b, c,$ and $d$.

3. The real vector space $V = \mathbb{R}^3$ is also a vector space over the rational numbers $\mathbb{Q}$, if we forget part of scalar multiplication. Give an example of three vectors in $V$ that are linearly independent over $\mathbb{Q}$, but linearly dependent over $\mathbb{R}$.

4. The quotient ring $\mathbb{Q}[x]/(x^3 + 2x^2 + 4x + 2)$ is a field because the polynomial in the denominator is irreducible. Find the reciprocal $\frac{1}{x+1}$ in this ring.

5. Draw a diagram of the ideal $I = (1 + i\sqrt{5}, 1 - i\sqrt{5})$ in the ring $R = \mathbb{Z}[\sqrt{-5}]$, and describe the quotient ring $R/I$. (It is isomorphic to a much more standard ring which you should find.)

6. Let $R = \mathbb{R}[[x]]$ be the ring of formal power series over the real numbers. If $a$ or $a(x)$ is an element of $R$, let $h(a)$ be the degree of the first non-zero term of $a$. Is $h$ a Euclidean height for this ring?

7. Let $R = M_3(\mathbb{Z})$ be the ring of $3 \times 3$ matrices over the integers. Does $R$ have a non-zero nilpotent element? Does it have a zero divisor which is not nilpotent?

8. Suppose that an abelian group $A$ has presentation matrix

$$M = \begin{pmatrix} 97 & 137 \\ 73 & 103 \end{pmatrix}.$$  

This is pretty complicated, so I will give you the hint that the determinant of this matrix is $-10$. Prove that just from that hint, there is only one possibility for the Smith normal form of $M$. What, then, is the isomorphism type of $A$?