180 : Linear Algebra over Finite Fields

Prerequisites: MAT 22A or MAT 67. MAT 108 strongly recommended.

Course Description:
Everything you learn to do with systems of equations, matrices, and vector spaces over $\mathbb{R}$ can be done over a finite field $\mathbb{F}_q$.

In other words, instead of the scalars being real numbers, the scalars now come from a finite set, for which addition and multiplication still make sense.

Two points still determine a line, but now a line contains a finite number of points. We can now count things like how many points are on a line or on a plane, and so we get interesting connections to combinatorics. There are some card games based on these ideas that we will investigate. There are also applications to cryptography and error correcting codes.

We will explain what finite fields are, redo the highlights of linear algebra in this new context, and hopefully see some of the interesting applications.

Textbook: There is not a set textbook. Some material may follow parts of Combinatorics of Symmetric Designs by Ionin and Shrikhande or parts of Discrete Mathematics by Biggs or other online resources.

A rough outline of the quarter:

Week 1:
Card game
rules of game
given 2 cards complete to valid triple uniquely?
counting — how many cards? , one card in how many valid triples
Week 2:
counting — how many cards? , one card in how many valid triples
make grids, Latin squares

Week 3:
Modular arithmetic
Focus on n=3
Lines, points, planes over F_3
How does this relate to the card game?

Week 4 and 5:
equations for lines
how do you tell if two lines same?
count "all" lines.
lines through origin, affine lines , sub spaces
count all lines containing given point, parallel lines
planes - same questions, more dimensions
Designing other games?

Week 6 (or 7):
points, lines , planes, matrices, linear transformation, basis
counting
Vector spaces in general, axioms.

Week 7 (or 6):
F_p , linear algebra over F_p
counting
Possibly: applications (codes, designs, Latin squares, projective geometry, games)

Week 8:
more applications
other finite fields such as F_4, F_8, F_9 and how to construct them

Week 9, 10:
matrices over F_q, invertible matrices, counting
more applications

Grading Scheme: A combination of class attendance/participation, homework, and/or final exam. Precise weights among these are yet to be determined.