A Branch & Cut Based Hybrid Method for the Multi-Depot Ring Star Problem

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A The Multi-Depot Ring Star Problem (MDRSP)

- A ***ring star*** is a cycle paired with assignments of customers to the ring vertices.
- For a solution of the MDRSP connect all the customers to depots by disjoint ring stars – optionally, use ***Steiner vertices***.
- Use at most m ring stars per depot and at most q customers per ring star.
- Minimize the overall edge and assignment costs.

Applications in ***Reliable Telecommunication Network Design*** and ***Transportation Network Planning***.

B Branch & Cut Method

$$\min \quad \sum_{k\in D}\sum_{e\in E} c_e x_e^k + \sum_{k\in D}\sum_{ij\in A} c_{ij} z_{ij}^k$$

$$s.\ t. \quad \sum_{e\in\delta(k)} x_e^k \le 2m_k \quad \forall\ k \in D,$$

$$\sum_{e\in\delta(i)} x_e^k = 2z_{ii}^k \quad \forall\ i \in U, \forall\ k \in D,$$

$$\sum_{e\in\delta(j)} x_e^k = 2w_j^k \quad \forall\ j \in W, \forall\ k \in D,$$

$$\sum_{k\in D}\sum_{j\in R(i)} z_{ij}^k = 1 \quad \forall\ i \in U,$$

$$\sum_{k\in D} w_j^k \le 1 \quad \forall\ j \in W,$$

$$\sum_{i\in U}\sum_{j\in S\cap R(i)} z_{ij}^k \le \frac{q_k}{2}\sum_{e\in\delta(S)} x_e^k \quad \forall\ S \subseteq V\setminus D : S \ne \emptyset, \forall\ k \in D,$$

$$x_e^k \in \{0,1\} \quad \forall\ e \in E, \forall\ k \in D,$$

$$z_{ij}^k \in \{0,1\} \quad \forall\ ij \in A, \forall\ k \in D,$$

$$w_j^k \in \{0,1\} \quad \forall\ j \in W, \forall\ k \in D.$$

Dynamic separation of the ***fractional capacity*** inequalities and valid ***connectivity*** and ***ring multi-star*** inequalities using the *Network Simplex Algorithm*. B & C carried out in the CPLEX 12.2 framework.

- **Just capable to solve small instances (~30 customers) efficiently.**

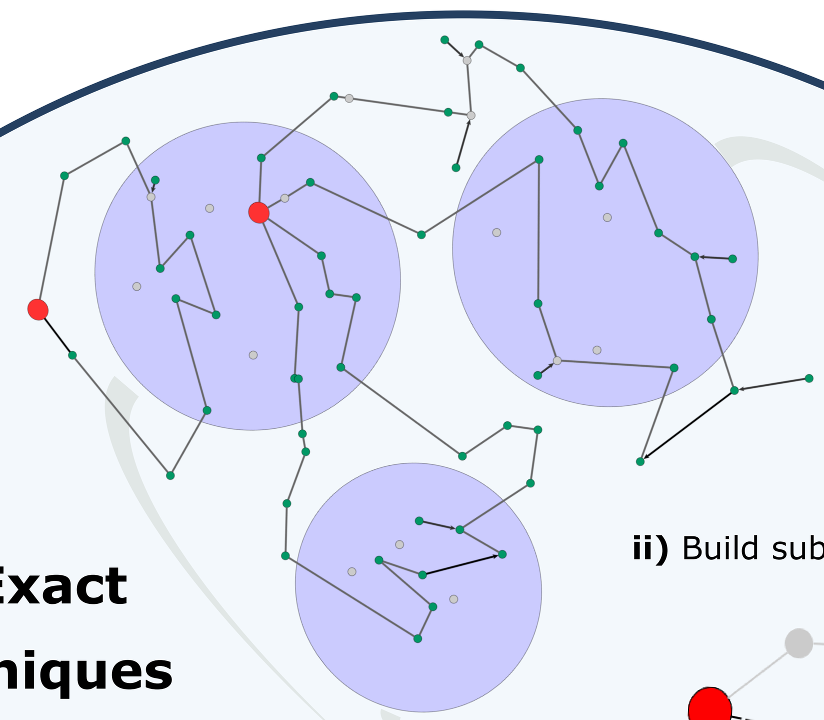
C Hybrid Heuristic

Iterative improvement algorithm combining *locally exact searches* with *exact contraction-based perturbation techniques.*

1. Start Solution

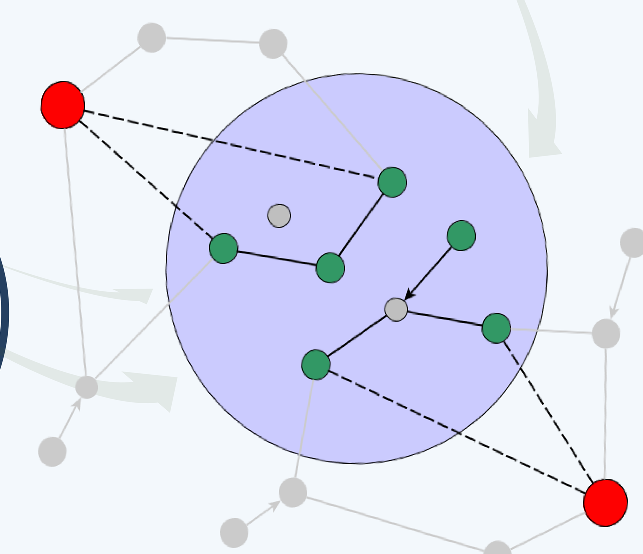
We hierarchically apply variants of the contraction techniques in 3.

2. Locally Exact Search Techniques

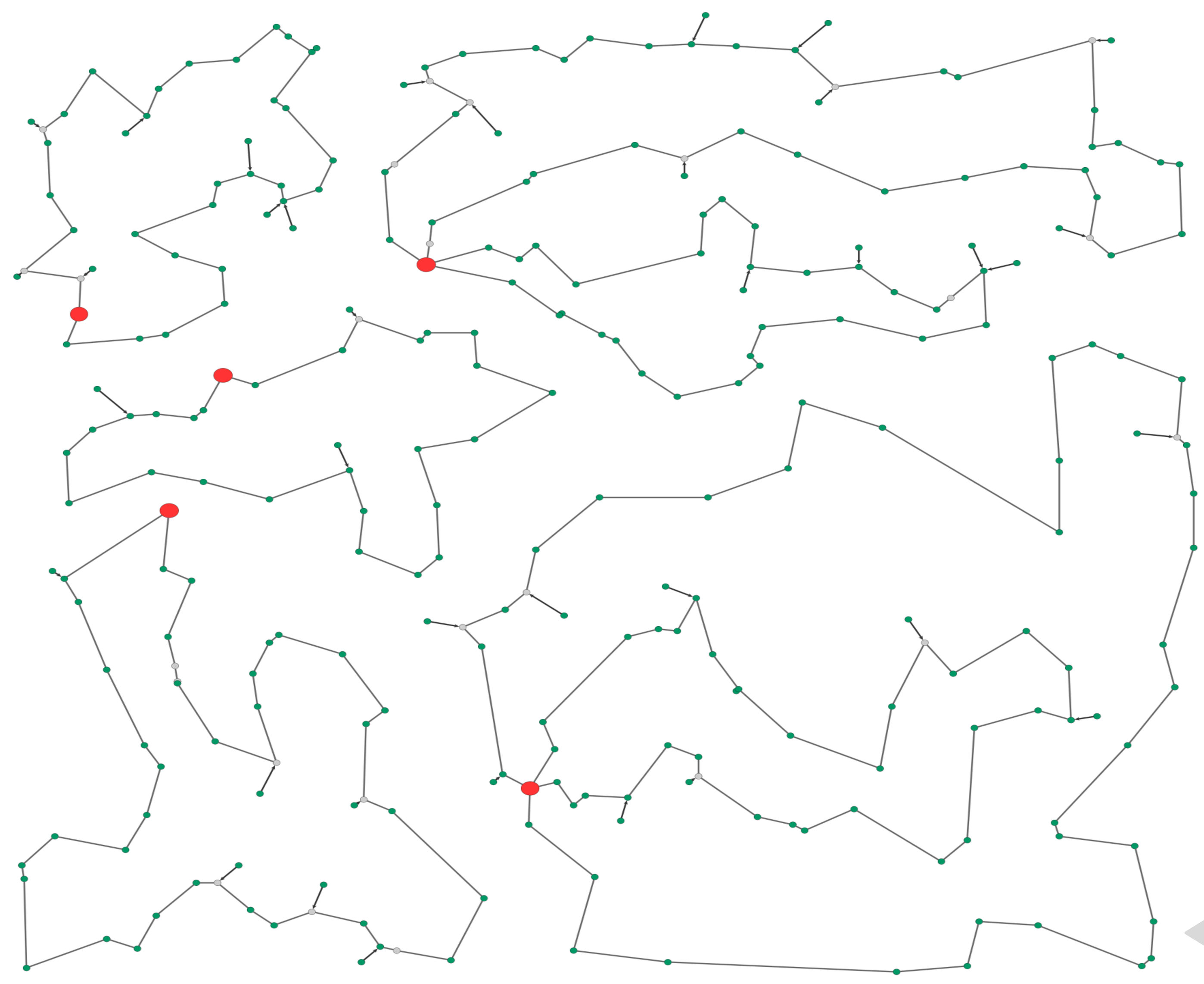


i) *Focus* on solution substructures: ***ball***, ***path star***, ***ring star***, ***long edge***, ***path star exchange***.

ii) Build subproblems of type **MDRSP**.



iii) Apply our **Branch & Cut** method in *black-box* fashion. Refine iteratively by grasping the solution using each substructure idea. Order the neighborhoods by increasing subproblem complexity.

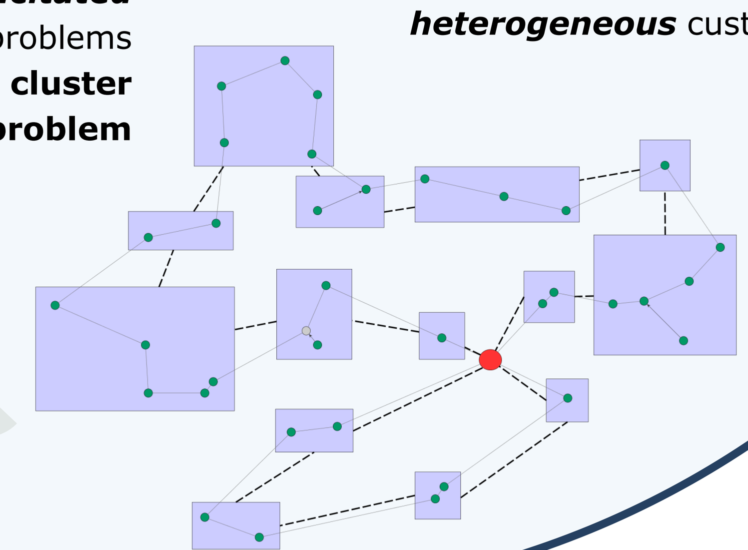


Identification of *ring-in-ring* structures for instance B100 (250 customers, 625 Steiner vertices, 5 depots, m=2, m=40) with a total cost of 68473.

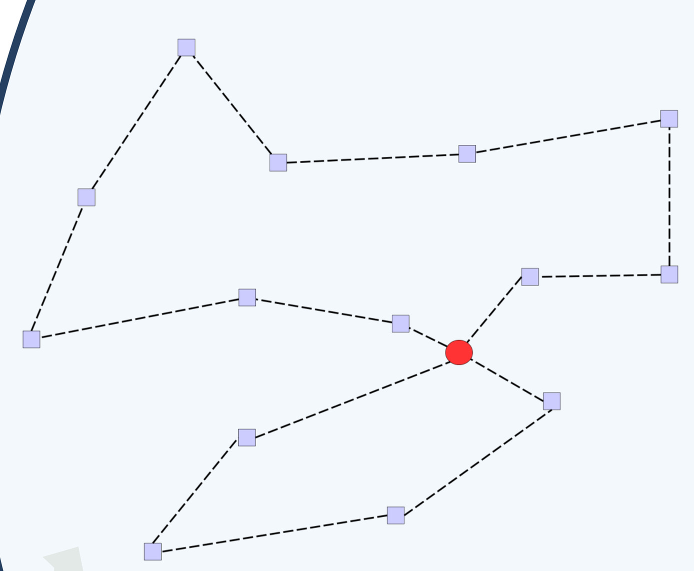
3. Exact Global Contraction Techniques

i) Consider a **single ring star, a depot's ring stars** or **all ring stars** in the current solution. Build ***homogeneous*** or ***heterogeneous*** customer clusters.

ii) *Defocus* - contract clusters and build ***Capacitated Vehicle Routing*** subproblems using **solution cluster distances** or **min problem edge costs**.



iii) Apply exact optimization – **Branch & Cut** – and refine using a subset of the locally exact search techniques before evaluating.



4. Computational Results

276 capacity-tight euclidean literature instances with up to 1000 vertices and various *m/q* combinations. Comparison with heuristical (tabu-neighborhood search) results by Baldacci et al. The C++ implementation was tested on a standard 1.33 GHz machine.

- Competitive runtime **<1500s**.
- Set maximal subproblem size to 25 customers.
- Up tp **5000 MIP** solver runs per instance.
- **Improved 92%** of previous results **up to 12 %**; **3% on average**.

D Conclusions

- **Very efficient after designing suitably structured neighborhoods.**
- **Mathematical Programming outperforms locally heuristic searches on this complex problem structure.**
- **Scalable solution quality.**
- **Generic solution technique applicable to any problem size.**

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