1. Circle T or F, depending on the truth value of the following statements. For statements with quantifies the universal set is \( \mathbb{R} \).

a) \( A \neq \{A\} \). T F

b) \( (\exists x)(\forall y)(x - y = 0) \). T F

c) \( (\forall y)(\exists x)(x - y = 0) \). T F

d) \( (\exists x)(\forall y)((xy = 0) \land (x + y = y)) \). T F

e) There exists a set \( A \) such that \( A \subset A \). T F

f) \( (P \Rightarrow Q) \equiv (Q \Rightarrow P) \). T F

g) For all sets \( A \) we have \( \emptyset \in A \). T F

h) If \( P \) is a tautology, then \( Q \Rightarrow P \) is a tautology for all statements \( Q \). T F

i) No set is equal to its power set. T F

j) \( (\exists x) \sim (\exists y)(x + y > y) \). T F
2. Let $A_1 = \{1, 4\}$, $A_2 = \{1, 2\}$, and $A_3 = \{-1, 0, 1\}$. Let $I = \{1, 2, 3\}$ and consider the universal set $\mathbb{Z}$. Write down the following sets:

a) $\bigcup_{i \in I} A_i$

b) $A_1 - A_2$

c) $\bigcap_{i \in I} A_i$

d) $P(A_2)$

e) $\bigcup_{i \in I} A_i'$
3. i) Prove that the sentence

\[ (P \land Q) \lor (P \land \sim Q) \land (R \lor \sim R) \]

is logically equivalent to a sentence consisting of a single atomic proposition

ii) Is this sentence a tautology? Why or why not?
4. Let $A$ and $B$ be sets. Prove the following:

i) $A \cap B^c \subseteq (A \cap B)^c$

ii) $A \subseteq B \Rightarrow B \cap A = A.$
5. Let $A$ be a set and $\{B_i\}_{i \in I}$ be an indexed family of sets. Prove

$$A - \bigcup_{i \in I} B_i = \bigcap_{i \in I} (A - B_i)$$