Homework 3

due October 24, 2001 in class

1. Artin 2.2.16 (pg. 70)
2. Artin 2.2.21 (pg. 71)
3. Artin 2.3.1 (pg. 71)
4. Artin 2.4.10 (pg. 72)
5. Artin 2.4.16 (pg. 72)
6. Artin 2.4.17 (pg. 72)
7. Let $\varphi : G \to G'$ be a group homomorphism. For $H \leq G$ let $\varphi(H) = \{\varphi(a) \mid a \in H\}$ be the image of H under φ, and for $H' \leq G'$ let $\varphi^{-1}(H') = \{a \in G \mid \varphi(a) \in H'\}$ be the inverse image of H'. Show that $\varphi^{-1}(H')$ is a subgroup of G and $\varphi(H)$ is a subgroup of G'.
8. Let $m, n \in \mathbb{Z}$ be positive integers. Show that $n\mathbb{Z} \leq m\mathbb{Z}$ if and only if m divides n.
9. (a) Show that every subgroup of an abelian group is abelian.

   (b) Show by example that there exists a non-abelian group G such that every proper subgroup of G is abelian.