1. Prove that $\text{SL}_n(\mathbb{R}) \lhd \text{GL}_n(\mathbb{R})$. Here $\text{SL}_n(\mathbb{R})$ is the special linear group defined as $\text{SL}_n(\mathbb{R}) = \{ A \in \text{GL}_n(\mathbb{R}) \mid \det A = 1 \}$.

2. Find the center of $\text{GL}_2(\mathbb{R})$.

3. Let $G$ be a group. Show that if $G/Z(G)$ is cyclic then $G$ is abelian.

4. Let $H \leq G$ with index $[G : H] = n$. Show that $G$ has a normal subgroup of index at most $n!$.

   (Hint: Consider the action of $G$ on the left cosets of $H$).

5. Let $G$ be a group, $N \trianglelefteq G$ and let $\bar{G} = G/N$. Prove that $\bar{x}, \bar{y} \in \bar{G}$ commute if and only if $x^{-1}y^{-1}xy \in N$. The element $x^{-1}y^{-1}xy$ is called the commutator of $x$ and $y$ denoted by $[x, y]$.

6. Show that all subgroups of index 2 are normal.

7. Find all normal subgroups of $S_4$.

8. Dummit, Foote Section 2.1 Exercise 6 (page 49)

9. Dummit, Foote Section 3.1 Exercise 14 (page 87)