Homework 5

due November 5, 2003

1. Rosen 4.1 #3, pg. 135

For which positive integers m is each of the following statements true?

$$(a) \quad 27 \equiv 5 \pmod{m}$$
$$(b) \quad 1000 \equiv 1 \pmod{m}$$
$$(c) \quad 1331 \equiv 0 \pmod{m}$$

2. Rosen 4.1 #22, pg. 136

Show by mathematical induction that if n is a positive integer, then $4^n \equiv 1 + 3n \pmod{9}$.

3. Rosen 4.1 #24, pg. 136

Give a complete system of residues modulo 13 consisting entirely of odd integers.

4. Rosen 4.1 #28, pg. 137

Find the least positive residues modulo 47 of each of the following integer.

$$(a) \quad 2^{32} \qquad (b) \quad 2^{47} \qquad (c) \quad 2^{200}.$$

5. Rosen 4.1 #35, pg. 138

Show that for every positive integer m there are infinitely many Fibonacci numbers f_n such that m divides f_n. (*Hint:* Show that the sequence of least positive residues modulo m of the Fibonacci numbers is a repeating sequence.)

6. Rosen 4.2 #2 (a)-(c), pg. 141

Find all solutions of each of the following linear congruences.

$$(a) \quad 3x \equiv 2 \pmod{7}$$
$$(b) \quad 6x \equiv 3 \pmod{9}$$
$$(c) \quad 17x \equiv 14 \pmod{21}.$$